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Determination and Analysis of Wireless Multi-Hop Relay System Performance in the Presence of Fading

-Tutorial-

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- Ten years ago, it was expected that in 2020, mobile and wireless traffic volume will increase thousand-fold over 2010 figures
- Now, we see exponential increase in mobile data traffic and it is considered as a critical trigger towards the new era, or 5G, of mobile wireless networks
- 5G will require very high carrier frequency spectra with massive bandwidths, extreme base station densities, and unprecedented numbers of antennas for supporting the enormous increase in the volume of traffic

- This increase in the number of wirelessly-connected devices in the tens of billions points will have a profound impact on society
- Massive machine communication, a basis for the Internet of Things (IoT), will make our everyday life more efficient, comfortable and safer, through a wide range of applications including traffic safety and medical services
- The variety of applications and data traffic types will be significantly larger than today, and will result in more diverse requirements on services, devices and networks

- Wireless sensor networks (WSNs) will play a fundamental role in the realization of Internet of Things (IoT) and Industry 4.0
- Arising from the presence of spatially distributed sensor nodes in a sensor network cooperative diversity can be achieved by using the sensor nodes between a given source-destination pair as intermediate relay stations

- So, multi-hop wireless relaying technique has recently received significant attention especially in cellular, modern ad-hoc, and wireless sensor networks for its performance benefits
- This is an efficient technology for increasing the coverage with respect to the channel path-loss, and including hotspot throughput improvements
- These advantages of multi-hop relaying are particularly pronounced for rural areas with small population and low level of traffic density

- The transmission characteristics of multi-hop relaying systems have been widely investigated
- Significant attention is dedicated to cascaded fading channels which appear in wireless multi-hop transmission

Some references:

Rau´l Cha´vez-Santiago, Michał Szydełko, Adrian Kliks, Fotis Foukalas, Yoram Haddad, Keith E. Nolan, Mark Y. Kelly, Moshe T. Masonta, Ilangko Balasingham, "5G: The Convergence of Wireless Communications", Wireless Pers. Commun. 2015, 83:1617–1642, DOI 10.1007/s11277-015-2467-2

Afif Osseiran, Volker Braun, Taoka Hidekazu, Patrick Marsch, Hans D. Schotten, Hugo Tullberg, Mikko Uusitalo, Malte Schellmann

"The Foundation of the Mobile and Wireless Communications System for 2020 and Beyond: Challenges, Enablers and Technology Solutions", IEEE 77th Vehicular Technology Conference (VTC Spring), 2-5 June 2013, Dresden, Germany, DOI: 10.1109/VTCSpring.2013.6692781

- The received signals are created as the products of a large number of rays reflected via *N* statistically independent scatters
- Therefore, the statistical analysis of products of two or more random variables (RVs) is intensified because of their applicability in performance analysis of wireless relay communication systems with more hops (sections)

- The wireless relay system output signal is a product of signal envelopes from each system sections
- Because of that, the cascade fading models are developed by the product of independent but not necessarily identically distributed random variables

- Many researchers are currently working in this area and new cascade fading models have been suggested recently in the literature
- Due to this fact, the performance of products of a higher number of random variables has become an important topic over the past decade

- The products of RVs are applied not only in wireless channel modeling, multi-hop relay systems, multiple input multiple output (MIMO) keyhole systems, cascaded channels with fading, but also in other natural sciences, such as biology and physics (especially quantum physics), and also in social sciences, econometrics, ...

- In order to fill the gap areas in the literature tied to cascaded models, an analysis has been done here
- This effort will surely help the researchers working in this area, to be able to identify the most appropriate fading channel model for an efficient wireless relay communication system design

- In this tutorial, mostly a wireless multi-hop relay communication system operating in a multipath fading environment will be analyzed
- The processes of derivation the system performance of the first-order (probability density function (PDF), cumulative distribution function (CDF), outage probability (OP), moments, amount of fading (AoF)) and the second-order (level crossing rate (LCR) and average fade duration (AFD)) will be presented for different fading distributions

- The impact of the specific parameters will be analyzed
- Based on this analysis, it is possible to estimate the behavior of real systems in the presence of fading

About fading

- In wireless communications, fading is deviation of the attenuation affecting a signal over propagation media
- The fading may vary with time, geographical position or frequency, and it is modeled as a random process
- In wireless systems, fading may either be due to multipath propagation, called multipath fading, or due to shadowing from obstacles affecting the wave propagation

Multipath fading

- Short-term fading (multipath fading) is propagation phenomena caused by atmospheric, ionospheric reflection and refraction, and reflection from water bodies and terrestrial objects
- Multipath fading degrades the system performance and limits the system capacity
- Received signal experiences fading resulting in signal envelope variation

Shadowing

- Shadowing is the result of the topographical elements and other structures in the transmission path such as trees, tall buildings...
- A log-normal or gamma distribution model the average power to account for shadowing

- There are more distributions that can be used to describe signal envelope variation in fading channels, which are dependent on propagation environment and communication scenario
- They depend on existence of line of sight component, nonlinearity of propagation environment, the number of clusters in propagation channel, inequality of quadrature components power and signal envelope power variation

- Various statistical models explain the nature of fading and several distributions describe the envelope of the received signal: Rayleigh, Rician, Nakagami-*m*, Hoyt or Nakagami-q, Weibull, α-μ, α-κ-μ, κ-μ, η-μ, … which was originally derived for reliability study purposes

- A multipath fading channel is a communication channel containing multipath fading
- The communication wireless relay mobile radio systems will be consider in this lecture
- A radio transmission system in which intermediate radio stations or radio repeaters receive and retransmit radio signals is known as relay system

- Wireless relay system can have several (two or more) sections
- The desired signal in sections is subjected to some kind of multipath fading

Outage probability

- We need to determined the first and the second order system performance
- The outage probability is important the first order performance measure of wireless communication system which is defined as probability that receiver output signal envelope falls below of the specified threshold and can be calculated from cumulative distribution function

- Mathematically, the outage probability is the CDF of the signal and is given by:

$$P_{out}\left(\gamma_{th}\right) = P\left(z < \gamma_{th}\right)$$

with γ_{th} being the threshold value

- There are two ways to define the outage probability in wireless relay systems
- For the first case, the outage probability is defined as probability that the signal envelope in any sections falls below the specified threshold
- The outage probability for this case can be calculated as Cumulative Distribution Function (CDF) of minimum of signal envelopes from sections

- For the second case, the outage probability is defined as probability that output signal envelope falls down the determined threshold
- The outage probability for this case is equal to the CDF of product of signal envelopes at sections

Level crossing rate

- Also, useful closed form expression for average level crossing rate (LCR) are calculated for some cases
- The level crossing rate is the second order system performance

Average fade duration

- The resulting integrals are solved for example by using the Laplace approximating formula or some other method
- Later, the expression for LCR can be used for calculating the average fade duration (AFD) of proposed relay system

Wireless Relay System with Two Sections in κ-μ Short-Term Fading Channel

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- The κ-μ distribution describes signal envelopes in channels with dominant components
- The κ-μ distribution is characterized by two parameters
- The parameter κ is Rician factor, which is defined as the ratio of dominant component power and scattering components power

- The parameter μ is related to the number of clusters in propagation channel
- The κ-μ small scale fading is more severe for less values of parameter μ
- Also, the κ-μ multipath fading is more severe for lesser value of dominant component power and higher values of scattered components power

- The κ-μ distribution is general distribution
- The κ-μ distribution reduces
- -to Nakagami-m distribution for κ=0,
- -to Rician distribution for μ=1, and
- -to Rayleigh distribution for κ=0 and μ=1

- When Rician factor goes to infinity, κ-μ multipath fading channel becomes no fading channel
- Also, when parameter μ goes to infinity, there is no fading in the channel

PROBABILITY DENSITY FUNCTION AND CUMULATIVE DISTRIBUTION FUNCTION OF MINIMUM OF TWO κ-μ RANDOM VARIABLES

- The probability density function of κ-μ random variable x_1 is:

$$p_{x_1}\left(x_1\right)=\frac{2\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\,\Omega_1^{\frac{\mu+1}{2}}}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k\left(k+1\right)}{\Omega_1}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\,\Gamma\left(i_1+\mu\right)}\cdot x_1^{2i_1+2\mu-1}e^{-\frac{\mu\left(k+1\right)}{\Omega_1}x_1^2},x_1\geq 0$$

where κ is Rician factor, μ is severity parameter and Ω_1 is signal envelope average power

-The random variable x_2 follows also κ-μ distribution

- Cumulative distribution function of x_1 is:

$$F_{x_1}(x_1) = \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega_1^{\frac{\mu+1}{2}}} \cdot \sum_{i_1=0}^{\infty} \left(\mu \sqrt{\frac{k(k+1)}{\Omega_1}} \right)^{2i_1+\mu-1} \frac{1}{i_1! \Gamma(i_1+\mu)} \cdot$$

$$\cdot \frac{1}{2} \left(\frac{\Omega_1}{\mu(k+1)} \right)^{i_1+\mu} \cdot \gamma\left(i_1+\mu, \frac{\mu(k+1)}{\Omega_1} x_1^2 \right), x_1 \geq 0$$

where $\gamma(n, x)$ is incomplete Gamma function of argument x and order n

- Cumulative distribution function of x_2 has the same shape

- When the signal level at any section falls below the defined threshold, the outage probability can be calculated from cumulative distribution function of the minimum of the signal envelopes at sections of wireless communication system
- Let analyze the minimum x of two random variables x_1 and x_2, which is define as:

$$x = \min(x_1, x_2)$$

- Probability density function of minimum *x* of two random variables is:

$$p_x(x) = p_{x_1}(x)F_{x_2}(x) + p_{x_2}(x)F_{x_1}(x)$$

- Cumulative distribution function of minimum *x* of two random variables is:

$$F_x(x) = 1 - \left(1 - F_{x_1}(x)\right)\left(1 - F_{x2}(x)\right)$$

CDF of minimum x of two κ-μ random variables is:

$$F_x(x) = 1 - \left(1 - \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega_1^{\frac{\mu+1}{2}}} \cdot \sum_{i_1=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1} \frac{1}{i_1!\Gamma(i_1+\mu)} \cdot \right.$$

$$\left. \cdot \frac{1}{2}\left(\frac{\Omega_1}{\mu(k+1)}\right)^{i_1+\mu} \gamma\left(i_1+\mu, \frac{\mu(k+1)}{\Omega_1}x^2\right)\right) \cdot$$

$$\left(1 - \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega_2^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2+\mu)} \cdot \right.$$

$$\left. \cdot \frac{1}{2}\left(\frac{\Omega_2}{\mu(k+1)}\right)^{i_2+\mu} \gamma\left(i_2+\mu, \frac{\mu(k+1)}{\Omega_2}x^2\right)\right)$$

- The outage probability is the probability that the receiver output signal envelope is below a given threshold γ_{th}
- It can be straight calculated as:

$$P_{out}\left(\gamma_{th}\right) = F_x\left(x\right)$$

where $F_x(x)$ is presented in the previous slide

- Here, CDFof minimum of two κ-μ random variables is the outage probability of wireless relay communication system with two sections over κ-μ multipath fading channel

Probability Density Function and Cumulative Distribution Function of Product of Two κ-μ Random Variables

- By the second definition, the outage probability of wireless relay system can be calculated as probability that signal envelope at the output of wireless relay communication system falls below the predetermined threshold
- For this case, the outage probability can be calculated as cumulative distribution function of product of two κ-μ distributed signals.

- Product of two κ-μ random variables x_1 and x_2 is:

$$x = x_1 \cdot x_2$$

$$x_1 = \frac{x}{x_2}$$

- Conditional PDF of x is:

$$p_x(x / x_2) = \left|\frac{dx_1}{dx}\right| p_{x_1}\left(\frac{x}{x_2}\right) = \frac{1}{x_2} p_{x_1}\left(\frac{x}{x_2}\right)$$

- After integration, the expression for $p_x(x)$ becomes:

$$p_x(x) = \int_0^\infty dx_2 \frac{1}{x_2} p_{x_1}\left(\frac{x}{x_2}\right) p_{x_2}(x_2) =$$

$$= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega_1^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1} \cdot \frac{1}{i_1!\Gamma(i \ +\mu)} \cdot$$

$$\cdot \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega_2^{\frac{\mu+1}{2}}} \cdot \sum_{i_2=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1} \cdot \frac{1}{i_2!\Gamma(i_2+\mu)}$$

$$\cdot x^{2i_1+2\mu-1} \left(\frac{\Omega_2 x^2}{\Omega_1}\right)^{i_2-i_1} K_{2i_2-2i_1}\left(2\sqrt{\frac{\mu^2 (k+1)^2 x^2}{\Omega_1 \Omega_2}}\right)$$

- Cumulative distribution function of *x* is:

$$F_x(x)=\int_0^x dt\, p_x(t)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}}\cdot\sum_{i_2=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1}\cdot\frac{1}{i_1!\Gamma(i+\mu)}\cdot$$

$$\cdot\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}}\cdot\sum_{i_2=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1}\cdot\frac{1}{i_2!\Gamma(i_2+\mu)}\cdot$$

$$\cdot\int_0^{\infty}dx_2\, x_2^{2i_2-2i_1-1}e^{-\frac{\mu(k+1)}{\Omega_2}x_2^2}\cdot\frac{1}{2}\left(\frac{\Omega_1}{\mu(k+1)x_2^2}\right)^{i_1+\mu}\gamma\left(i_1+\mu,\frac{\mu(k+1)}{\Omega_1}\frac{x^2}{x_2^2}\right)\Bigg)$$

- After substituting, the previous expression for CDF becomes:

$$F_x\left(x\right)=\frac{2\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}}\cdot\sum_{i_2=0}^{\infty}\left(\mu\sqrt{\frac{k\left(k+1\right)}{\Omega_1}}\right)^{2i_1+\mu-1}\cdot\frac{1}{i_1!\Gamma\left(i \quad +\mu\right)}\cdot$$

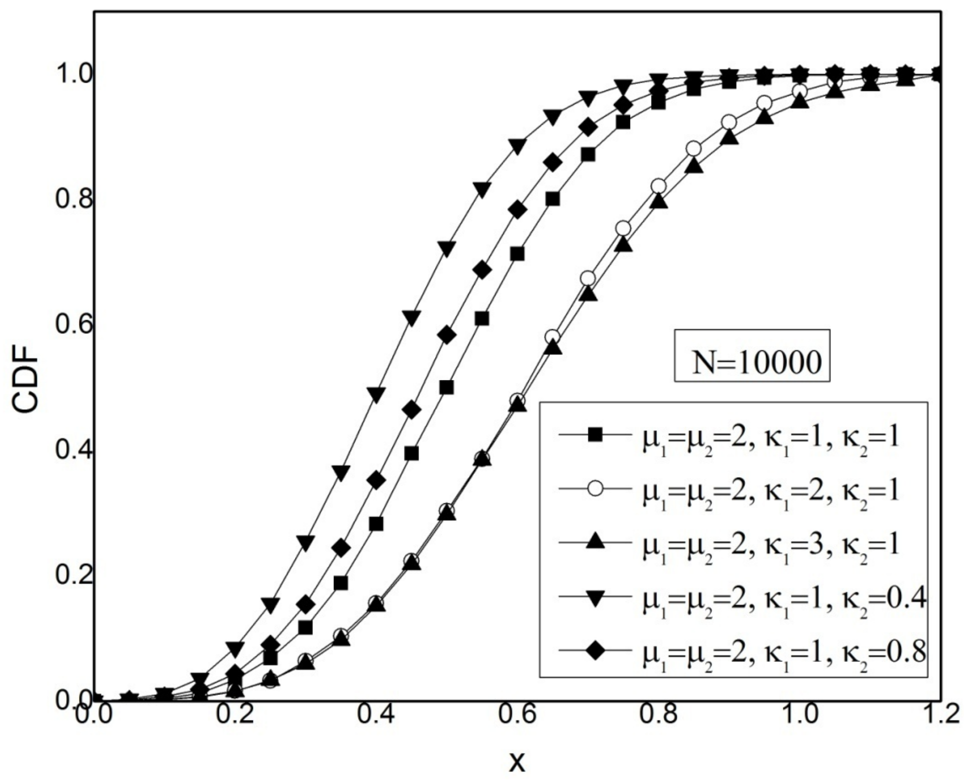
$$\cdot\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_2^{\frac{\mu+1}{2}}}\cdot\sum_{i_2=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_2}}\right)^{2i_2+\mu-1}\cdot\frac{1}{i_2!\Gamma(i_2+\mu)}\cdot$$

$$\cdot\frac{1}{2}\frac{1}{i_1+\mu}x^{2i_1+2\mu}\cdot\sum_{j_1=0}^{\infty}\frac{1}{(i_1+\mu+1)(j_1)}\cdot\left(\frac{\mu(k+1)x^2}{\Omega_1}\right)^{j_1}\cdot$$

$$\cdot\left(\frac{\Omega_2x^2}{\Omega_1}\right)^{\frac{i_2}{2}-\frac{i_1}{2}-\frac{j_1}{2}}K_{i_2-i_1-j_1}\left(2\sqrt{\frac{\mu^2\left(k+1\right)^2x^2}{\Omega_1\Omega_2}}\right)$$

$K_n(x)$ is the modified Bessel function of the second kind

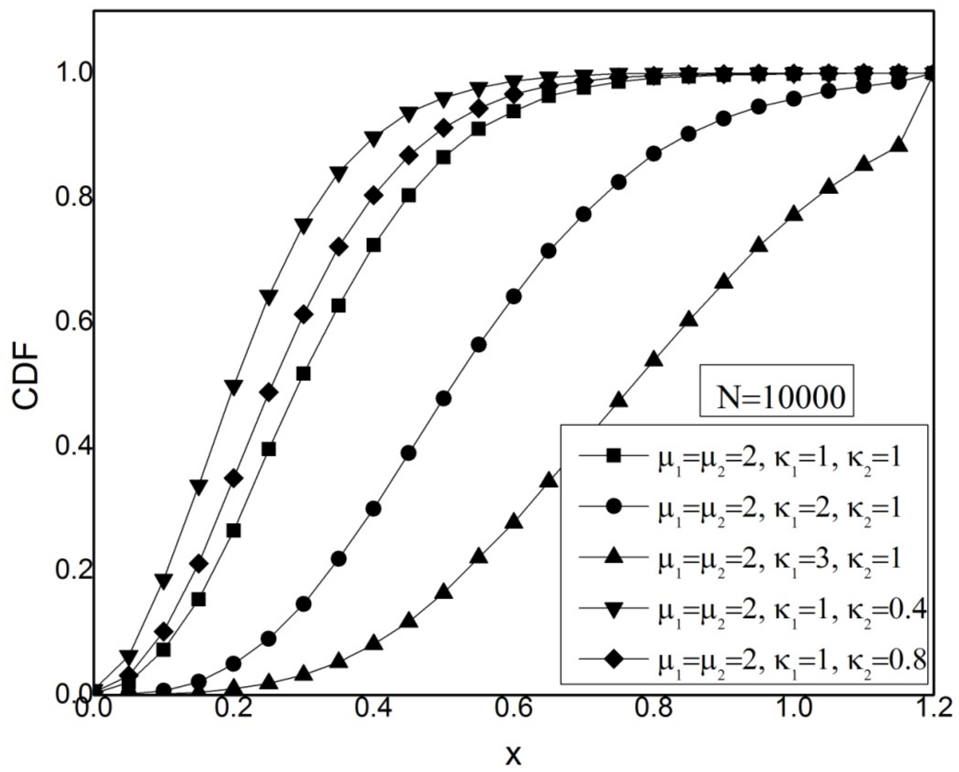
The cumulative distribution function of minimum of two κ-μ random variables



Numerical Results

- The cumulative distribution function of minimum of two κ-μ random variables versus signal envelope is presented in previous Figure
- The CDF is plotted for $\mu_1=\mu_2=2$ and variable parameters κ_1 and κ_2
- It is visible that CDF increases with increasing of the signal envelope
- The cumulative distribution function decreases for larger values of Rician factor κ_1
- Also, one can see from this figure that CDF is bigger for higher values of Rician factor κ_2

The cumulative distribution function of the product of two κ-μ random variables



- The cumulative distribution function of product of two κ-μ random variables depending on the signal envelope is presented in new Figure
- The parameters μ_1 and μ_2 are equal to each other and have a value of 2
- It is possible to see from this Figure that CDF becomes bigger with increasing of the signal envelope
- It shows that CDF is less for higher values of Rician factors κ_1 and κ_2

- The system performance is better for smaller values of the outage probability, i.e., cumulative distribution function
- This can be achieved by increasing the Ricean factors κ_1 and κ_2
- Since Rician factor is the ratio of dominant component power and scattering components power, it is evident that bigger dominant component powers and smaller scattering components give better system performance

Conclusion 1

- In this article, the communication relay radio system with two sections exposed to κ-μ multipath fading is analyzed
- The outage probability is calculated in the closed forms for the minimum and product of two random variables

- The obtained formulas for the outage probability for relay (κ-μ)*(κ-μ) channels could be used for calculation the outage probability of other relay channels
- For $\kappa_1=0$ and $\kappa_2=0$, (κ-μ)*(κ-μ) relay channel reduces to Nakagami- Nakagami relay channel;
- for $\mu_1=1$ and $\mu_2=1$, (κ-μ)*(κ-μ) channel becomes Rician*Rician channel
- Because of that, this article has general importance

- Results of this analysis can be used by designers of relay systems in the case of presence of fading with κ-μ distribution
- The designers of these systems can choose optimal parameters for given value of the outage probability
- Because of the generality of the results, the presence of other types of fading can also be covered with this investigation

Wireless Relay System with Two Sections in the Presence of κ-μ and η-μ Multipath Fading

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Probability Density Functions of κ-μ Random Variable and η-μ Random Variable

- The κ-μ random variable follows distribution:

$$p_{x_1}\left(x_1\right)=\frac{2\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}}\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k\left(k+1\right)}{\Omega}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma\left(i_1+\mu\right)}\cdot x_1^{2i_1+2\mu-1}e^{-\frac{\mu\left(k+1\right)}{\Omega_1}x_1^2},x_1\geq 0$$

- The η-μ random variable follows distribution:

$$p_{x_2}(x_2) = \frac{4\sqrt{\pi}\,\mu^{\mu+1/2}h^{\mu}x_2^{2\mu}}{\Gamma(\mu)H^{\mu-1/2}\Omega_2^{\mu+1/2}}e^{-\frac{2\mu h}{\Omega_2}x^2} \cdot I_{\mu-1/2}\left(\frac{2\mu h}{\Omega_2}x_2^2\right) =$$

$$= \frac{4\sqrt{\pi}\,\mu^{\mu+1/2}h^{\mu}}{\Gamma(\mu)H^{\mu-1/2}\Omega_2^{\mu+1/2}} \cdot \sum_{i_2=0}^{\infty}\left(\frac{\mu h}{\Omega_2}\right)^{2i_2+\mu-1/2}\frac{1}{i_2!\,\Gamma(i_2+\mu)} \cdot x_2^{4i_2+4\mu-1}e^{-\frac{2\mu h}{\Omega_2}x_2^2}, \quad x_2 \ge 0$$

- The parameters are:

$$H = \frac{\eta^{-1} - \eta}{4}, \quad h = \frac{2 + \eta^{-1} - \eta}{4}, \quad \eta \geq 0$$

- The variances of in-phase and quadrature independent Gaussian processes are arbitrary with the ratio *η*

Cumulative Distribution Functions of κ-μ Random Variable and η-μ Random Variable

- Cumulative distribution function of κ-μ random variable x_1 is:

$$F_{x_1}(x_1)=\int_0^{x_1} dt\, p_{x_1}(t)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}}\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma(i_1+\mu)}\cdot\int_0^{x_1} dt\, t^{2i_1+2\mu-1}e^{-\frac{\mu(k+1)}{\Omega_1}t^2}=$$

$$=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}}\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma(i_1+\mu)}\cdot\frac{1}{2}\left(\frac{\Omega_1}{\mu(k+1)}\right)^{i_1+\mu}\cdot\gamma\left(i_1+\mu,\frac{\mu(k+1)}{\Omega_1}x_1^2\right),\ x_1\geq 0$$

- where *γ(n,x)* is incomplete Gamma function:

$$\gamma\left(n,x\right)=\frac{1}{n}e^{-x}x^{n}\cdot\sum_{j=0}^{\infty}\frac{1}{\left(n+1\right)\left(j\right)}x^{j}$$

- Cumulative distribution function of η-μ random variable x_2 is:

$$F_{x_2}(x_2) = \int_0^{x_2} dt\, p_{x_2}(t)dt =$$

$$= \frac{4\sqrt{\pi}\,\mu^{\mu+1/2}h^{\mu}}{\Gamma(\mu)H^{\mu-1/2}\Omega_2^{\mu+1/2}} \cdot \sum_{i_2=0}^{\infty}\left(\frac{\mu h}{\Omega_2}\right)^{2i_2+\mu-1/2} \cdot \frac{1}{i_2!\,\Gamma(i_2+\mu)} \cdot \int_0^{x_2} dt\, t^{4i_2+4\mu-1} e^{-\frac{2\mu h}{\Omega_2}t^2} =$$

$$= \frac{4\sqrt{\pi}\,\mu^{\mu+1/2}h^{\mu}}{\Gamma(\mu)H^{\mu-1/2}\Omega_2^{\mu+1/2}} \times \sum_{i_2=0}^{\infty}\left(\frac{\mu h}{\Omega_2}\right)^{2i_2+\mu-1/2} \times \frac{1}{i_2!\,\Gamma(i_2+\mu)} \times \frac{1}{2}\left(\frac{\Omega_2}{2\mu h}\right)^{2i_2+2\mu} \times \gamma\left(2i_2+2\mu, \frac{2\mu h}{\Omega_2}x_2^2\right), x_2 \geq 0$$

Minimum of κ-μ Random Variable and η-μ Random Variable

- Minimum of random variables x_1 and x_2 is:

$$x = \min\left(x_1, x_2\right)$$

- Cumulative distribution function of x is:

$$F_x(x) = 1 - \left(1 - F_{x_1}(x)\right)\left(1 - F_{x2}(x)\right)$$

- CDF is:

$$F_x(x)=1-\left(1-\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{k\mu}\Omega_1^{\frac{\mu+1}{2}}}\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma(i_1+\mu)}\cdot\right.$$

$$\left.\cdot\frac{1}{2}\left(\frac{\Omega_1}{\mu(k+1)}\right)^{i_1+\mu}\cdot\gamma\left(i_1+\mu,\frac{\mu(k+1)}{\Omega_1}x_1^2\right)\right)\cdot$$

$$\cdot\left(1-\frac{4\sqrt{\pi}\,\mu^{\mu+1/2}h^{\mu}}{\Gamma(\mu)H^{\mu-1/2}\Omega_2^{\mu+1/2}}\cdot\sum_{i_2=0}^{\infty}\left(\frac{\mu h}{\Omega_2}\right)^{2i_2+\mu-1/2}\cdot\frac{1}{i_2!\Gamma(i_2+\mu)}\cdot\frac{1}{2}\left(\frac{\Omega_2}{2\mu h}\right)^{2i_2+2\mu}\cdot\gamma\left(2i_2+2\mu,\frac{2\mu h}{\Omega_2}x_2^2\right)\right)$$

- Since the outage probability is actually the probability that signal envelope becomes smaller than the determined threshold, the outage probability of relay system can be calculated from cumulative distribution function of minimum of κ-μ random variable and η-μ random variable:

$$P_0 = F_x\left(x_0\right)$$

- where x_0 is the determined threshold

Product of κ-μ Random Variable and η-μ Random Variable

- Random variables x_1 follows κ-μ distribution and random variable x_2 follows η-μ distribution
- Product of x_1 and x_2 is:

$$x = x_1 \cdot x_2$$

$$x_1 = \frac{x}{x_2}$$

- The probability density functions of *x* is:

$$p_x(x) = \int_0^\infty dx_2 \frac{1}{x_2} p_{x_1}\left(\frac{x}{x_2}\right) p_{x_2}(x_2) =$$

$$= \frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu} \Omega_1^{\frac{\mu+1}{2}}} \cdot \sum_{i_1=0}^{\infty} \left(\mu\sqrt{\frac{k(k+1)}{\Omega}}\right)^{2i_1+\mu-1} \frac{1}{i_1!\Gamma(i_1+\mu)} \cdot$$

$$\times \frac{4\sqrt{\pi}\,\mu^{\mu+1/2} h^{\mu}}{\Gamma(\mu) H^{\mu-1/2} \Omega_2^{\mu+1/2}} \times \sum_{i_2=0}^{\infty} \left(\frac{\mu h}{\Omega_2}\right)^{2i_2+\mu-1/2} \frac{1}{i_2!\Gamma(i_2+\mu)} \times$$

$$x^{2i_1+2\mu-1} \cdot \int_0^\infty dx_2\, x_2^{-1-2i_1-2\mu+1+4i_2+4\mu-1} e^{-\frac{\mu(k+1)}{\Omega_1}\frac{x^2}{x_2^2} - \frac{2\mu h}{\Omega_2} x_2^2} =$$

- Finally:

$$p_x(x) = \frac{2\mu (k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}} e^{k\mu}\, \Omega_1^{\frac{\mu+1}{2}}} \cdot \sum_{i_1=0}^{\infty} \left(\mu \sqrt{\frac{k(k+1)}{\Omega}} \right)^{2i_1+\mu-1} \frac{1}{i_1!\,\Gamma(i_1+\mu)} \cdot$$

$$\times \frac{4\sqrt{\pi}\, \mu^{\mu+1/2} h^{\mu}}{\Gamma(\mu) H^{\mu-1/2} \Omega_2^{\mu+1/2}} \times \sum_{i_2=0}^{\infty} \left(\frac{\mu h}{\Omega_2} \right)^{2i_2+\mu-1/2} \frac{1}{i_2!\,\Gamma(i_2+\mu)} \times$$

$$\cdot x^{2i_1+2\mu-1} \cdot \left(\frac{\Omega_2 x^2}{\Omega_1} \right)^{2i_2-i_1+\mu-1/2} \cdot K_{4i_2-2i_1+2\mu} \left(2\sqrt{\frac{2\mu^2 (k+1) h x^2}{\Omega_1 \Omega_2}} \right)$$

- where $K_n(x)$ marks the modified Bessel function of the second kind

- The cumulative distribution function of *x* is:

$$F_x(x) = \int_0^x dt\, p_x(t) =$$

$$= \frac{2\mu\left(k+1\right)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}\, e^{k\mu}\, \Omega_1^{\frac{\mu+1}{2}}} \cdot \sum_{i_1=0}^{\infty} \left(\mu\sqrt{\frac{k\left(k+1\right)}{\Omega}}\right)^{2i_1+\mu-1} \frac{1}{i_1!\,\Gamma\left(i_1+\mu\right)} \cdot$$

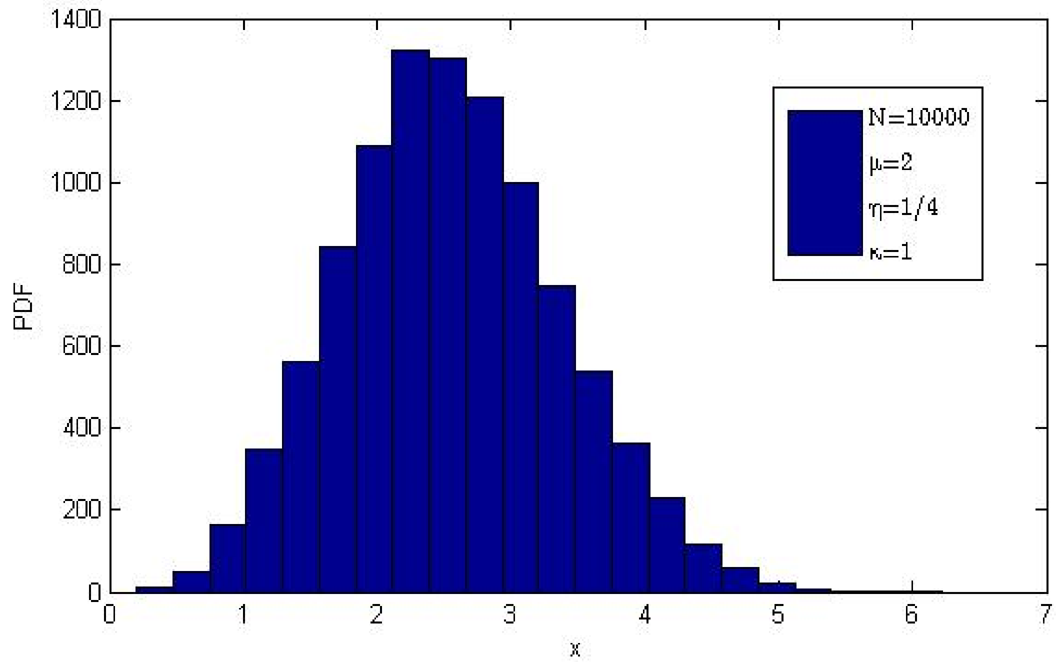
$$\times \frac{4\sqrt{\pi}\,\mu^{\mu+1/2}h^{\mu}}{\Gamma(\mu)H^{\mu-1/2}\Omega_2^{\mu+1/2}} \times \sum_{i_2=0}^{\infty} \left(\frac{\mu h}{\Omega_2}\right)^{2i_2+\mu-1/2} \frac{1}{i_2!\,\Gamma(i_2+\mu)} \times$$

$$\cdot \int_0^{\infty} dx_2\, x_2^{4i_2-2i_1+2\mu-1} e^{-\frac{2\mu h}{\Omega_2}x_2^2} \cdot \frac{1}{2}\left(\frac{\Omega_1 x_2^2}{\mu\left(k+1\right)}\right)^{i_1+\mu_1} \cdot \gamma\left(i_1+\mu_1, \frac{\mu\left(k+1\right)}{\Omega_1}\frac{x^2}{x_2^2}\right)$$

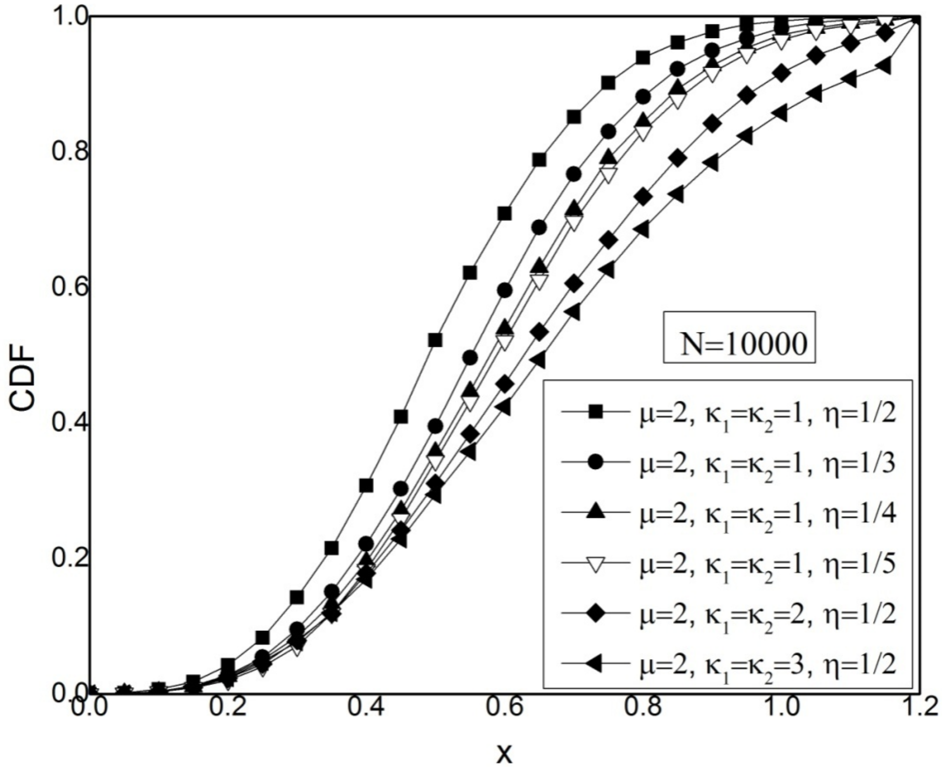
- *γ(n,x)* is incomplete Gamma function defined as:

$$\gamma\left(n,x\right)=\int_{0}^{x} dt\; t^{n-1}e^{-t}$$

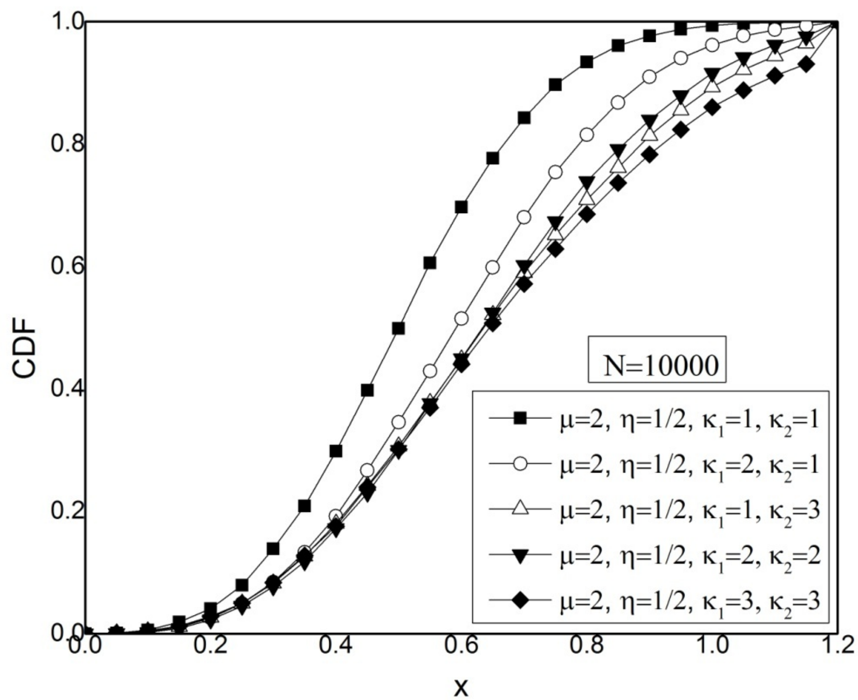
Probability density function of minimum of κ-μ random variable and η-μ random variable for κ=1, μ=2 and η=1/4



The cumulative distribution function of minimum of κ-μ random variable and η-μ random variabl for μ=2 and variable parameters $\kappa_1=\kappa_2$ and η

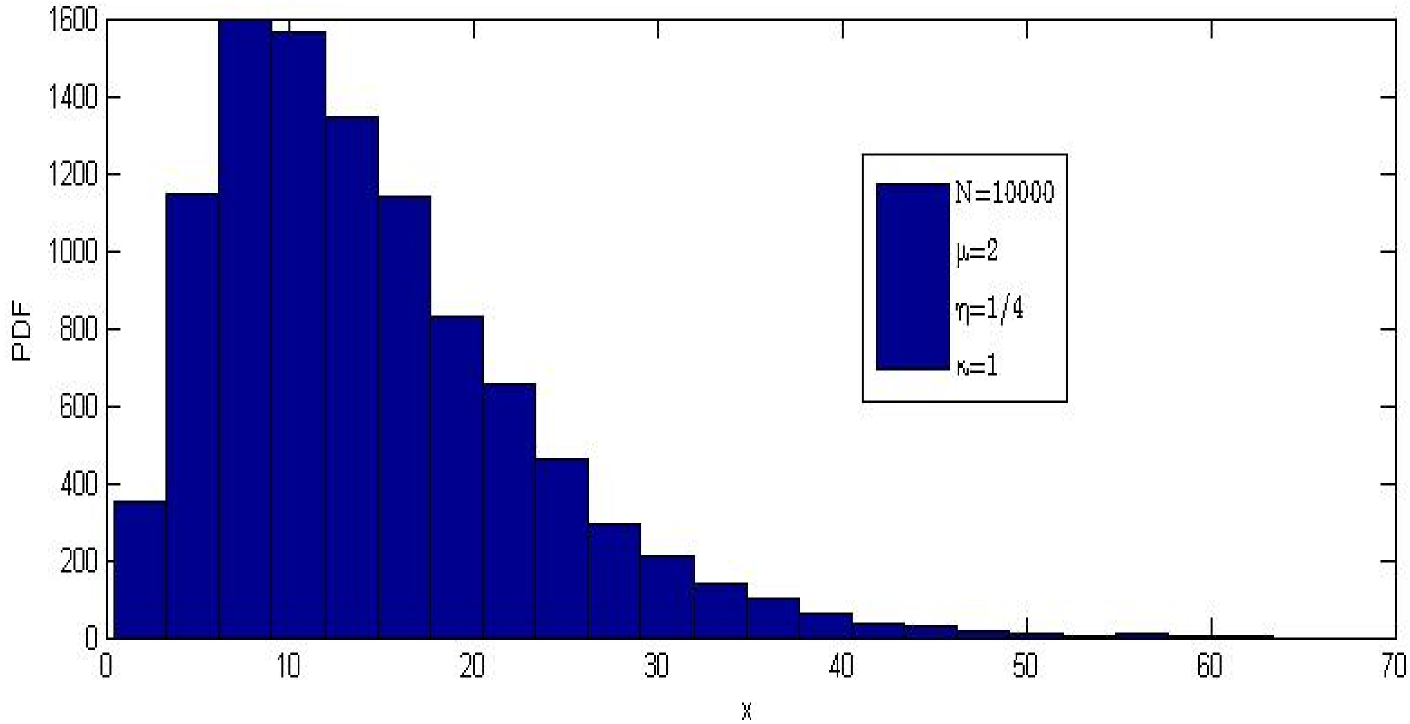


The cumulative distribution function of minimum of κ-μ random variable and η-μ random variabl for μ=2, η=1/2 and changable parameters κ_1 and κ_2

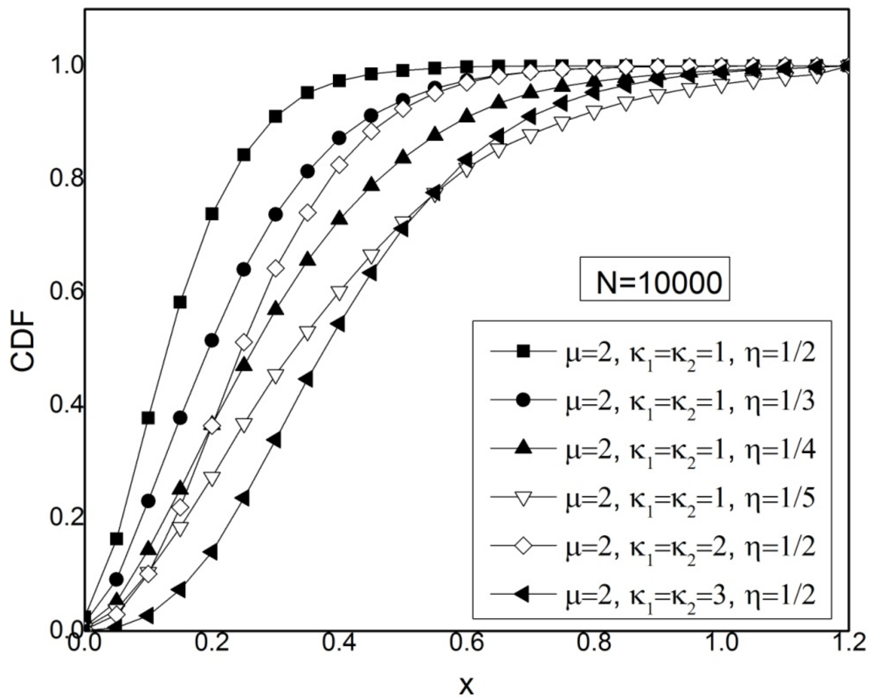


- The cumulative distribution function is smaller for lesser values of Rician factor κ
- Also, one can see that CDF is bigger for bigger values of fading parameter η
- It can be observed from Fig. 3 that CDF decreases for larger values of Rician factors κ_1 and κ_2

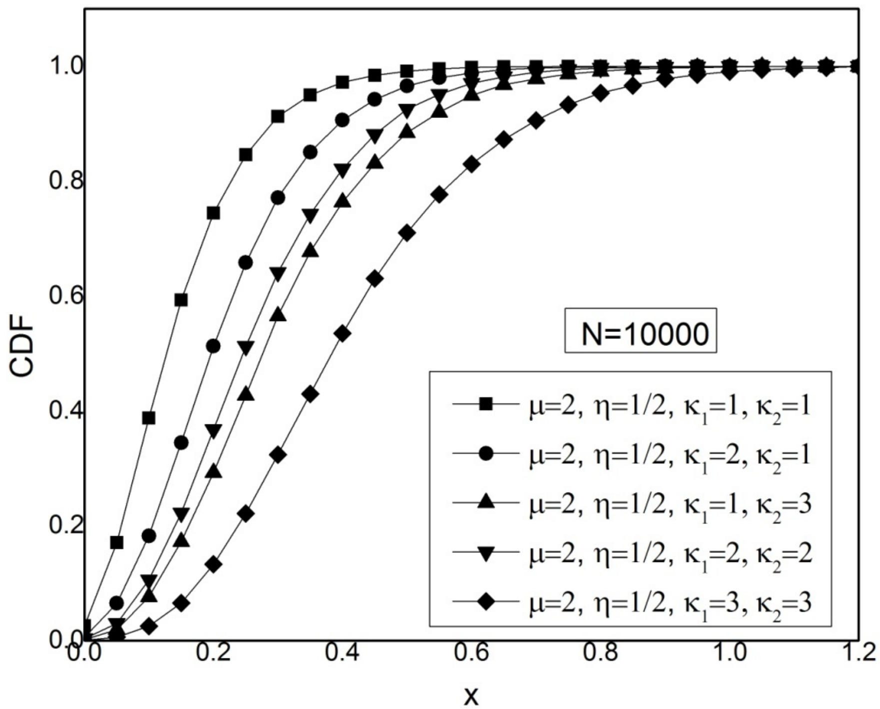
The probability density function of product of κ-μ random variable and η-μ random variable for κ=1, μ=2 and η=1/4



The cumulative distribution function of product of κ-μ random variable and η-μ random variable for μ=2 and variable parameters η and $\kappa_1=\kappa_2$



The cumulative distribution function of product of κ-μ random variable and η-μ random variable for μ=2, η=1/2 and changable parameters κ_1 and κ_2



- It can be noticed CDF grows with increasing of signal envelope till saturation
- The CDF is smaller for higher values of Rician factors κ_1 and κ_2
- Also, one can see 5 that CDF is bigger for greater values of fading parameter η
- It could be remarked also that CDF decreases for larger values of Rician factors κ_1 and κ_2

Conclusion 2

- In this part, communication mobile relay radio system with two sections is analyzed when the first section is exposed to κ-μ multipath fading and the second section is upset by η-μ multipath fading
- In the channel of the first section the dominant component is present
- In the second section, the powers of in-phase and quadrature components are different

- The κ-μ random variable and η-μ random variable are general random variables, so that obtained expressions for the outage probability for relay channel (κ-μ), (η-μ) can be used for evaluation the outage probability for other relay channels

- For κ=0 and η=1, (κ-μ), (η-μ) relay channel reduces to Nakagami, Nakagami relay channel;
- for μ_1=1 and η=1, (κ-μ), (η-μ) distribution becomes Rician, Nakagami-*m* relay channel;
- for κ=0 and μ_2=1, Nakagami-*m*, Nakagami-*q* distribution may be obtained from (κ-μ), (η-μ) distribution

B. Talha, M. Patzold:

On the statistical properties of double Rice channels,

Proceedings of 10th International Symposium on Wireless Personal Multimedia Communications, WPMC 2007, Jaipur, India, 517-522

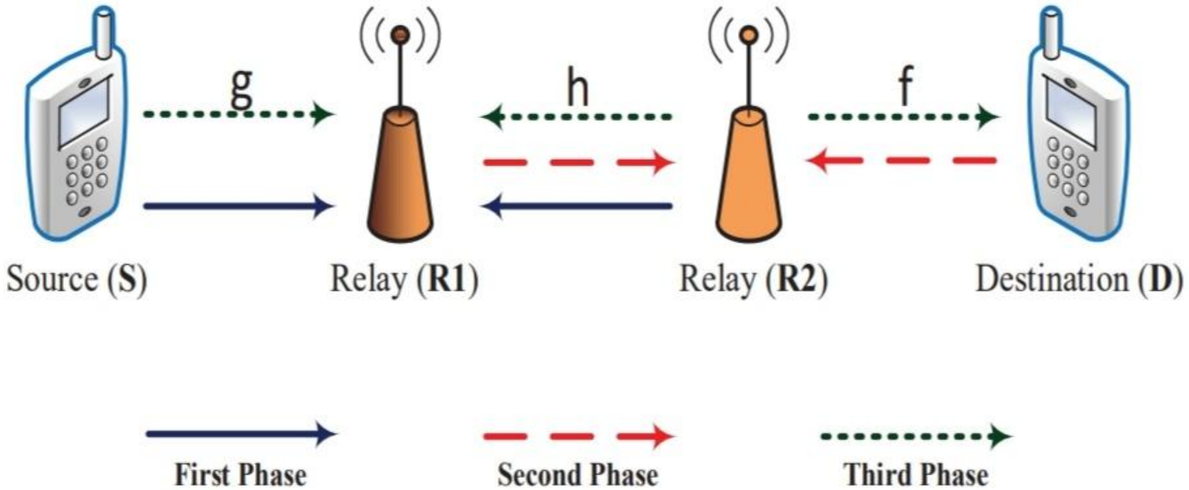
Some Performance of Three-hop Wireless Relay Channels in the Presence of Rician Fading

Dragana Krstic, Petar Nikolic, Sinisa Minic, Zoran Popovic

Outline

- THE FIRST ORDER PERFORMANCE OF PRODUCT OF THREE RICIAN RANDOM VARIABLES
  - *PDF of Product of Three Rician RVs*
  - *CDF of Product of Three Rician RVs*
  - *Outage probability of Product of Three Rician RVs*
- THE SECOND ORDER PERFORMANCE OF THE PRODUCT OF THREE RICIAN RANDOM VARIABLES
  - *LCR of Product of Three Rician RVs*
  - *AFD of Product of Three Rician RVs*

- A three-hop communication system, that we analyze, is illustrated in the next figure
- It consists of the source node, denoted by (S), sending the information signal to the destination (D) with the help of two consecutive relays, namely R1 and R2
- The AF relay nodes are assumed to be untrusted and hence, they can overhear the transmitted information signal while relaying



System model of a three-hop wireless relay

- All nodes are equipped with a single antenna operating in half-duplex mode
- The consecutive relays are necessary helpers to deliver the information signal to the destination
- This assumption is valid when the network nodes experience a heavy shadowing, or when the distance between terminals is large, or when the nodes suffer from limited power resources

- For three-hop relay system we will obtain the second order characteristics
- The knowledge of second-order statistics of multipath fading channels (level crossing rate (LCR) and average fade duration (AFD)) can help us better understand and mitigate the effects of fading

- For example, the AFD determines the average length of error bursts in fading channels
- So, in fading channels with relatively large AFD, long data blocks will be significantly affected by the channel fades than short blocks

THE FIRST ORDER PERFORMANCE OF PRODUCT OF THREE RICIAN RANDOM VARIABLES

A) PDF of Product of Three Rician RVs

- Rician fading is a stochastic model for radio propagation where the signal arrives at the receiver by several different paths when one of the paths, typically a line of sight signal or some strong reflection signals, is much stronger than the others
- In Rician fading, the amplitude gain is characterized by a Rician distribution

- Rician RVs x_i have Rician distribution:

$$p_{x_i}(x_i) = \frac{2(\kappa_i+1)}{\Omega_i e^{\kappa_i}} \sum_{j_i=0}^{\infty} \left(\frac{(\kappa_i+1)\kappa_i}{\Omega_i} \right)^{j_i} \frac{1}{(j_i!)^2} \cdot$$

$$\cdot\, x_i^{2j_i+1} e^{-\frac{\kappa_i+1}{\Omega_i} x_i^2}, \; x_i \geq 0$$

- where Ω_i are mean powers of RVs x_i, and
- κ_i are Rician factors

- Rician factor is defined as a ratio of signal power of dominant component and power of scattered components
- It can have values from [0, ∞]

- The output signal from multi-hop relay system is product of random variables (RVs) at hops outputs
- A random variable *x* is product of three Rician RVs:

$$x = \prod_{i=1}^{3} x_i$$

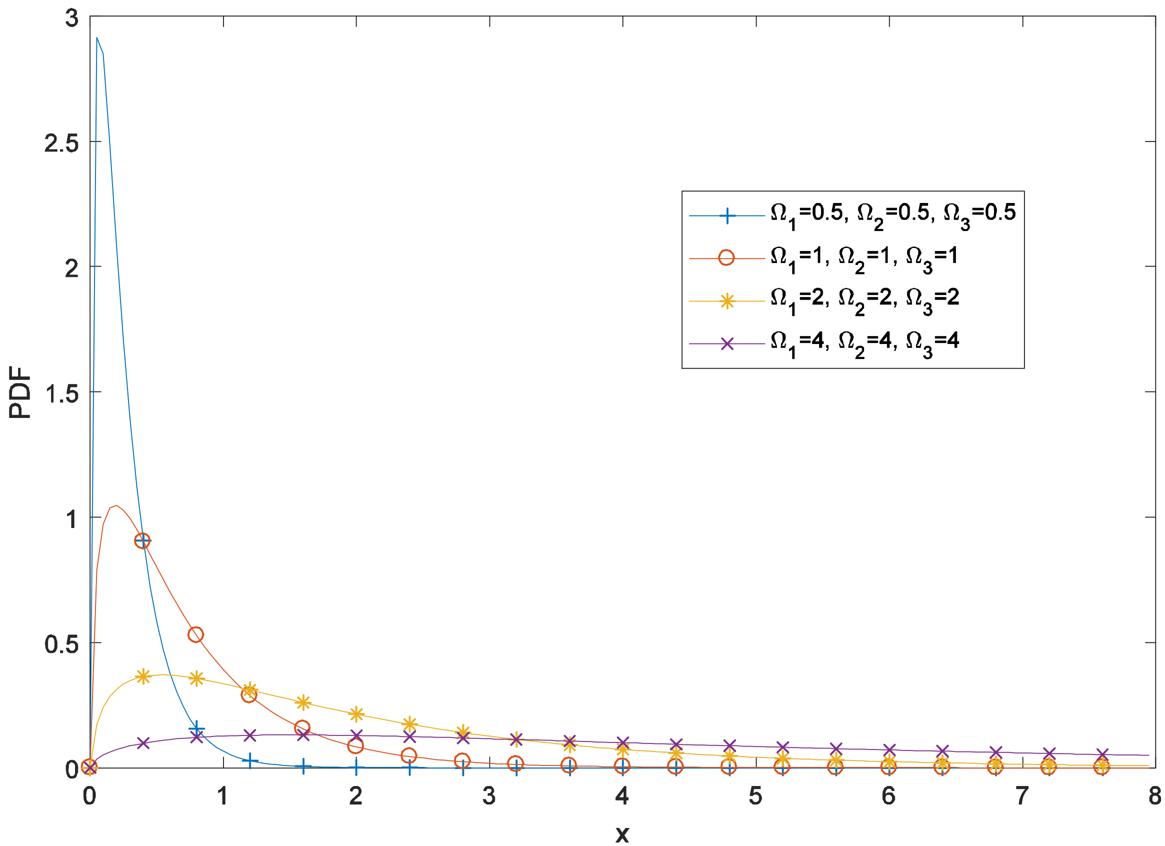
- Probability density function of product of three Rician RVs *x* is:

$$p_x(x) = \frac{2(\kappa_1+1)}{\Omega_1 e^{\kappa_1}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_1+1)\kappa_1}{\Omega_1} \right)^{j_1} \frac{1}{(j_1!)^2} \cdot \frac{2(\kappa_2+1)}{\Omega_2 e^{\kappa_2}} \sum_{j_2=0}^{\infty} \left(\frac{(\kappa_2+1)\kappa_2}{\Omega_2} \right)^{j_2} \frac{1}{(j_2!)^2} \cdot$$

$$\cdot \frac{2(\kappa_3+1)}{\Omega_3 e^{\kappa_3}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_3+1)\kappa_3}{\Omega_3} \right)^{j_1} \frac{1}{(j_3!)^2} \cdot$$

$$\int_0^{\infty} dx_2 \int_0^{\infty} dx_3 x_2^{-1-2j_1+2j_2} x_3^{-1-2j_1+2j_3} \cdot x^{2j_1+1} e^{-\frac{\kappa_1+1}{\Omega_1}\left(\frac{x}{x_2 x_3}\right)^2 - \frac{\kappa_2+1}{\Omega_2}x_2^2 - \frac{\kappa_3+1}{\Omega_3}x_3^2}$$

PDF of product of three Rician RVs for different values of the signal powers for $\kappa_1=\kappa_2=\kappa_3=1$



B) CDF of Product of Three Rician RVs

– Cumulative distribution function (CDF) of product of three Rician RVs is:

$$F_x(x) = \int_0^\infty dt\, p_x(t) =$$

$$= \frac{2(\kappa_1+1)}{\Omega_1 e^{\kappa_1}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_1+1)\kappa_1}{\Omega_1}\right)^{j_1} \frac{1}{(j_1!)^2} \cdot \frac{2(\kappa_2+1)}{\Omega_2 e^{\kappa_2}} \sum_{j_2=0}^{\infty} \left(\frac{(\kappa_2+1)\kappa_2}{\Omega_2}\right)^{j_2} \frac{1}{(j_2!)^2} \cdot$$

$$\cdot \frac{2(\kappa_3+1)}{\Omega_3 e^{\kappa_3}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_3+1)\kappa_3}{\Omega_3}\right)^{j_1} \frac{1}{(j_3!)^2} \cdot \int_0^\infty dx_2 \int_0^\infty dx_3 x_2^{-1-2j_1+2j_2+2j_1+2} x_3^{-1-2j_1+2j_3+2j_1+2}$$

$$\cdot e^{-\frac{\kappa_2+1}{\Omega_2}x_2^2 - \frac{\kappa_3+1}{\Omega_3}x_3^2} \cdot \frac{1}{2}\left(\frac{\Omega_1}{\kappa_1+1}\right)^{j_1+1} \gamma\left(j_1+1, \frac{\kappa_1+1}{\Omega_1}\frac{x^2}{x_2^2 x_3^2}\right) \qquad (*)$$

- Rayleigh fading is a model for stochastic fading when there is no line of sight signal
- Because of that it is considered as a special case of the more generalized concept of Rician fading
- Rayleigh fading is obtained for Rician factor $\kappa=0$
- A case with $\kappa \rightarrow \infty$ present the scenario without fading

- Since this reason, derived expressions for CDF of product of three Rician RVs can be used for evaluation a CDF of product of three Rayleigh RVs, also for CDF of product of two Rayleigh RVs and Rician RV, and CDF of product of two Rician RVs and Rayleigh RV
- Obtained results can be used in performance analysis of wireless relay communication radio system with three sections in the presence of multipath fading

- This means that derived CDFs are used for cases:

1) when Rician fading is present in all three sections ($\kappa_i \neq 0$, i=1,2,3), then

2) when Rayleigh fading is present in all three sections ($\kappa_1=\kappa_2=\kappa_3=0$), the next

3) when Rayleigh fading is present in two sections and Rician in one ($\kappa_1=\kappa_2=0$, $\kappa_3 \neq 0$) and

4) when Rayleigh fading is present in one and Rician fading in two sections ($\kappa_1=0$, $\kappa_2 \neq 0$, $\kappa_3 \neq 0$)

C) Outage probability of Product of Three Rician RVs

- The outage probability is an important performance measure of communication links operating over fading channels
- Outage probability is defined as the probability that information rate is less than the required threshold information rate Γ_{th}

- Pout is the probability that an outage will occur within a specified time period:

$$P_{out} = \int_0^{\Gamma_{th}} p_x(t)\,dt$$

- $p_x(x)$ is the PDF of the signal and
- Γ_{th} is the system protection ratio depending on the type of modulation employed and the receiver characteristics
- Pout can be expressed as:

$$P_{out} = F_x(\Gamma_{th})$$

- Plots of the outage probability, for different values of parameters, are shown in Figs. 2 and 3
- The choice of parameters is intended to illustrate the broad range of shapes that the curves of the resulting distribution can exhibit
- It is evident that performance is improved with an increase in Rician factors κ_l
- Also, higher values of fading powers Ω_i tend to reduce the outage probability and improve system performance, as it is expected

Fig. 2 Outage probability of product of three Rician RVs versus signal envelope *x* for different values of Rician factor κ_1 and signal power Ω=1

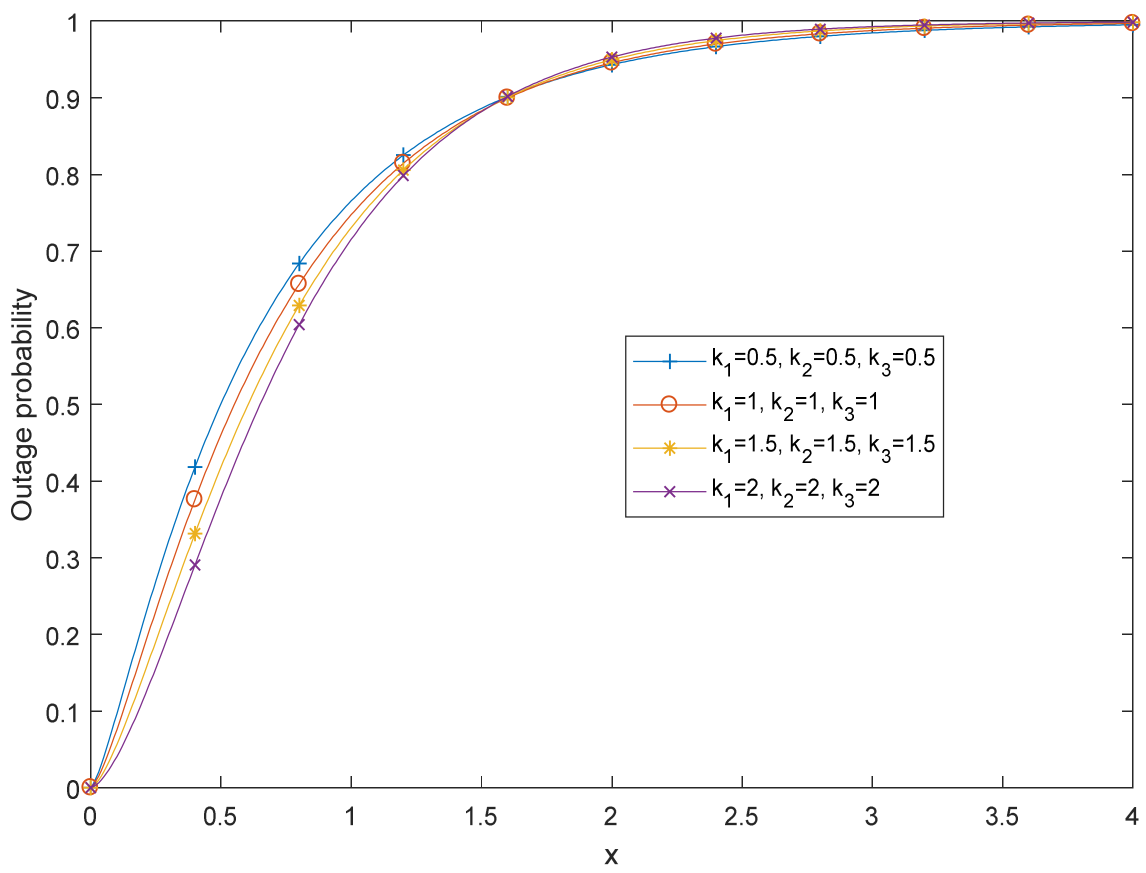
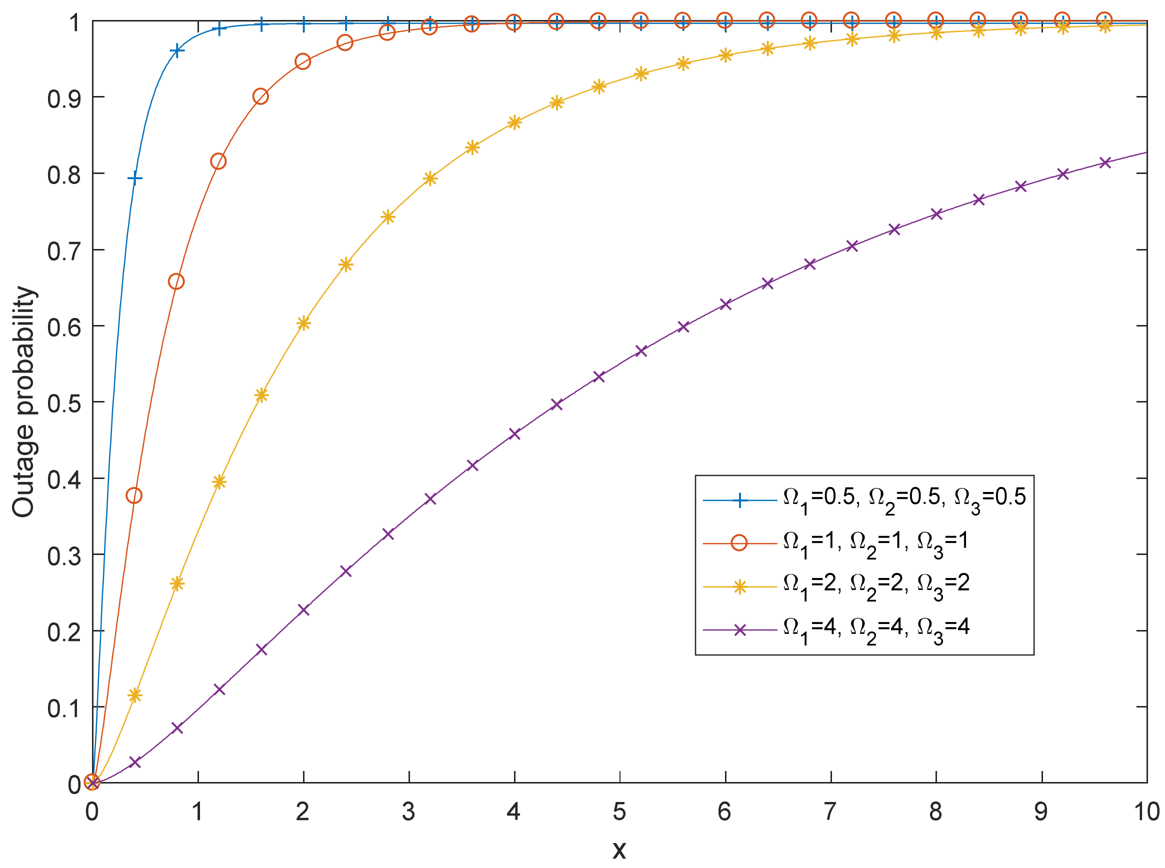


Fig. 3 Outage probability of product of three Rician RVs depending on signal envelope for different values of signal power Ω_i and Rician factor κ=1



Moments of Signals over Wireless Relay Fading Environment with Line-of-Sight

Dragana Krstic, *Petar Nikolic*, *Zoran Popovic*, *Sinisa Minic*, *Mihajlo Stefanovic*

INTRODUCTION

- Moments present quantitative measure of the function’s shape

- They are used in both, mechanics and statistics

- When we deal with probability distribution, then:
  - zero-th moment is total probability,
  - the first one is mean of the signal (or expected value)
  - the second is the variance (or the average power of signal)
  - the third is skewness, and
  - the fourth moment is kurtosis

Moments of Product of Three Rician RVs

The expected value (also called the mean value) of the product of three Rician RVs *x* is defined:

$$m_x = E\left[x\right] = \int_0^{\infty} x p\left(x\right) dx$$

where *E* denotes the statistical expectation operator, since $p_x(x)dx$ is the probability of RV *x* lying in the infinitesimal strip *dx*, m_x is interpreted as the weighted average of *x*, where each weight is the probability of that a specific value of *x* occurring

The expected value of a RV is an average of the values that the RV takes in a large number of experiments and is called the first moment of a RV:

$$m_1 = \overline{x} = \frac{2(\kappa_1+1)}{\Omega_1 e^{\kappa_1}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_1+1)\kappa_1}{\Omega_1}\right)^{j_1} \frac{1}{(j_1!)^2} \cdot$$

$$\cdot \frac{2(\kappa_2+1)}{\Omega_2 e^{\kappa_2}} \sum_{j_2=0}^{\infty} \left(\frac{(\kappa_2+1)\kappa_2}{\Omega_2}\right)^{j_2} \frac{1}{(j_2!)^2} \cdot \cdot \frac{2(\kappa_3+1)}{\Omega_3 e^{\kappa_3}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_3+1)\kappa_3}{\Omega_3}\right)^{j_1} \frac{1}{(j_3!)^2} \cdot$$

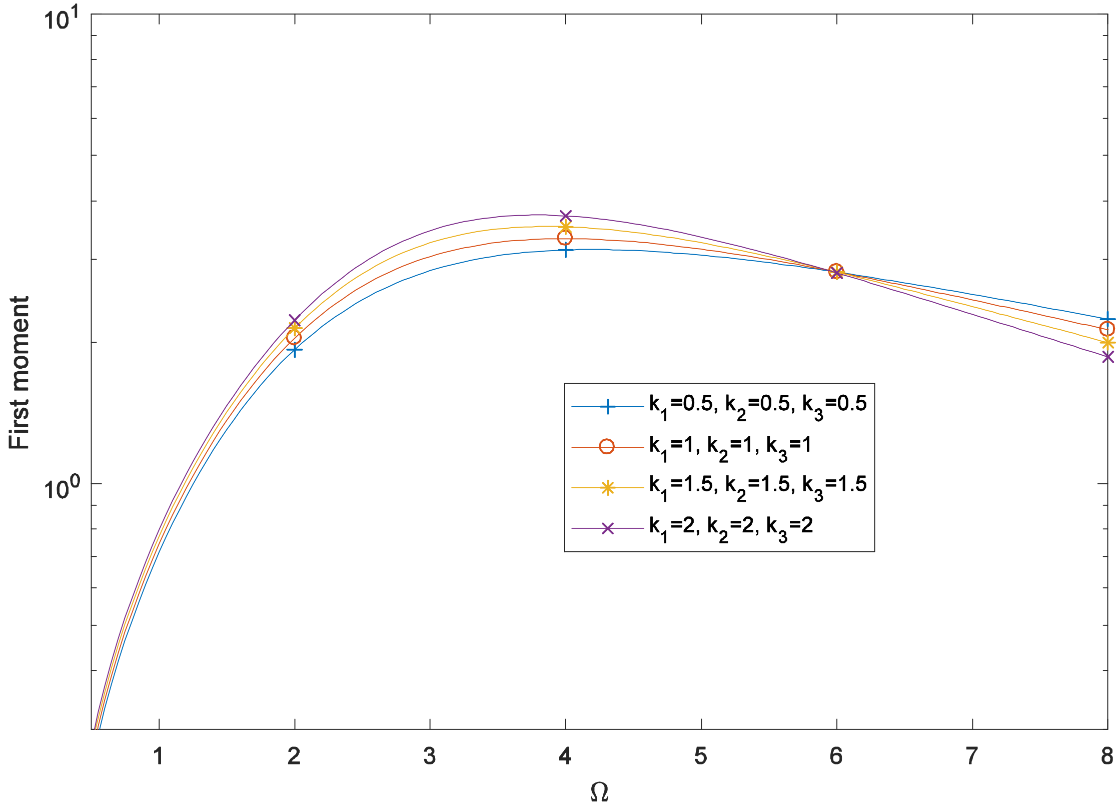
$$\int_0^\infty dx_2 \int_0^\infty dx_3 x_2^{-1-2j_1+2j_2} e^{-\frac{\kappa_2+1}{\Omega_2}x_2^2} x_3^{-1-2j_1+2j_3} e^{-\frac{\kappa_3+1}{\Omega_3}x_3^2} \cdot \frac{1}{2}\left(\frac{\Omega_1}{\kappa_1+1}\right)^{j_1+3/2} \Gamma(j_1+3/2)\left(x_2^2 x_3^2\right)^{j_1+3/2} =$$

$$= \frac{2(\kappa_1+1)}{\Omega_1 e^{\kappa_1}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_1+1)\kappa_1}{\Omega_1}\right)^{j_1} \frac{1}{(j_1!)^2} \cdot \frac{2(\kappa_2+1)}{\Omega_2 e^{\kappa_2}} \sum_{j_2=0}^{\infty} \left(\frac{(\kappa_2+1)\kappa_2}{\Omega_2}\right)^{j_2} \frac{1}{(j_2!)^2} \cdot$$

$$\cdot \frac{2(\kappa_3+1)}{\Omega_3 e^{\kappa_3}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_3+1)\kappa_3}{\Omega_3}\right)^{j_1} \frac{1}{(j_3!)^2} \cdot \frac{1}{2}\left(\frac{\Omega_1}{\kappa_1+1}\right)^{j_1+3/2} \Gamma(j_1+3/2) \cdot \frac{1}{2}\left(\frac{\Omega_2}{\kappa_2+1}\right)^{j_2+3/2} \Gamma(j_2+3/2) \cdot \frac{1}{2}\left(\frac{\Omega_3}{\kappa_3+1}\right)^{j_3+3/2} \Gamma(j_3+3/2)$$

❖The first moment for the product of three Rician RVs is shown graphically in the next two figures

❖One can see that the first moment for product of three Rician RVs depending on Rician factor κ for a few values of signal power $\Omega=\Omega_1=\Omega_2=\Omega_3$ is bigger for higher values of Ω_i

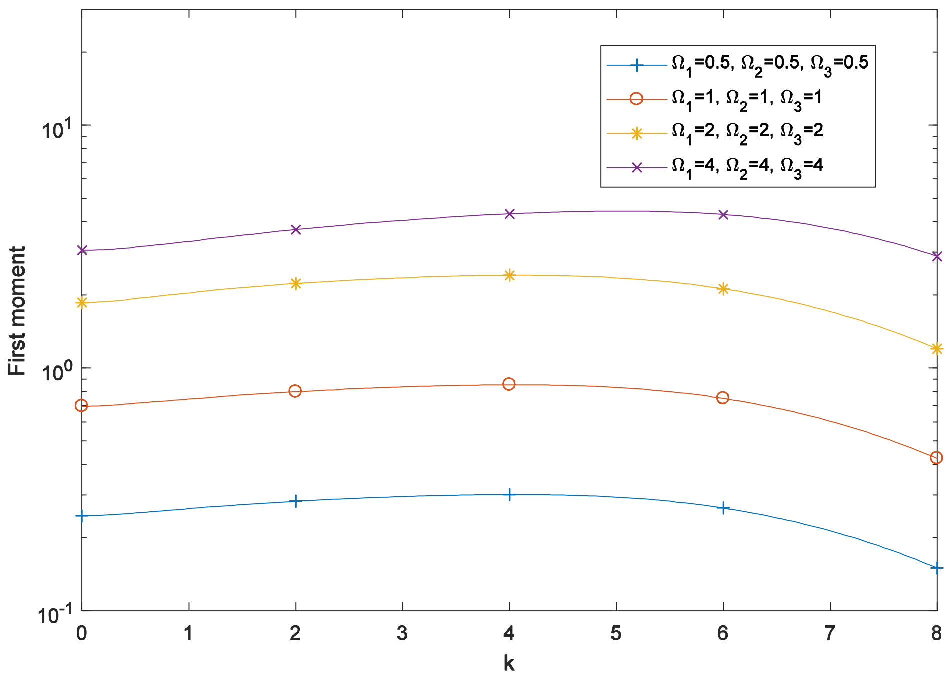


10^1
10^0
First moment
1
2
3
4
5
6
7
8
Ω
k_1=0.5, k_2=0.5, k_3=0.5
k_1=1, k_2=1, k_3=1
k_1=1.5, k_2=1.5, k_3=1.5
k_1=2, k_2=2, k_3=2

❖It is possible to notice from figure below an increasing of the first moment with increasing of Ω till maximal values

❖Then, m_1 start to decline

❖For small Ω, m_1 is higher for higher values of Rician factor κ



Ω_1=0.5, Ω_2=0.5, Ω_3=0.5
Ω_1=1, Ω_2=1, Ω_3=1
Ω_1=2, Ω_2=2, Ω_3=2
Ω_1=4, Ω_2=4, Ω_3=4
First moment
10^{1}
10^{0}
10^{-1}
0
1
2
3
4
5
6
7
8
k

- The second moment is known as the mean-squared value of the RV, or variance or the signal's average power

- The positive square root of the second moment (variance) is the standard deviation

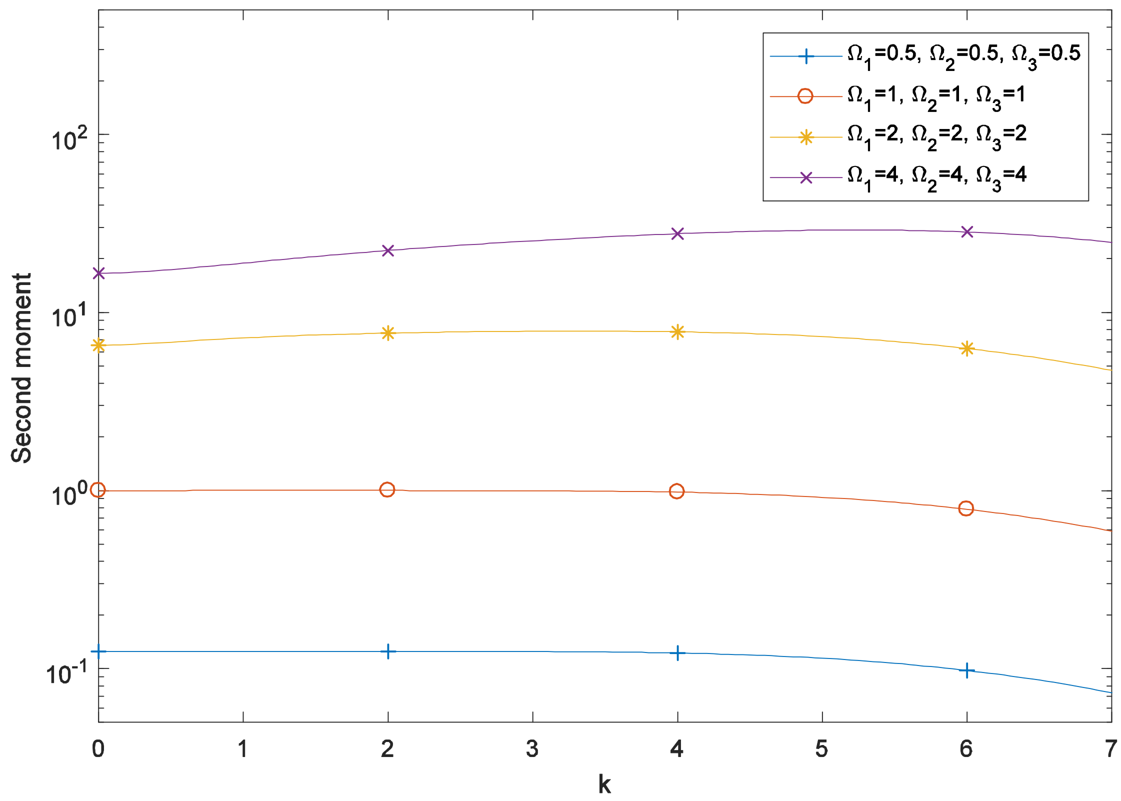
- In wireless communication we are speaking about signal’s average power

❖ The second moment m_2 of the product of three Rician RVs is:

$$m_2 = \overline{x^2} = \int_0^\infty dx x^2 p(x) =$$

$$= \frac{2(\kappa_1+1)}{\Omega_1 e^{\kappa_1}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_1+1)\kappa_1}{\Omega_1} \right)^{j_1} \frac{1}{(j_1!)^2} \cdot \frac{2(\kappa_2+1)}{\Omega_2 e^{\kappa_2}} \sum_{j_2=0}^{\infty} \left(\frac{(\kappa_2+1)\kappa_2}{\Omega_2} \right)^{j_2} \frac{1}{(j_2!)^2} \cdot \frac{2(\kappa_3+1)}{\Omega_3 e^{\kappa_3}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_3+1)\kappa_3}{\Omega_3} \right)^{j_1} \frac{1}{(j_3!)^2} \cdot$$

$$\cdot \frac{1}{2} \left(\frac{\Omega_1}{\kappa_1+1} \right)^{j_1+2} \Gamma(j_1+2) \cdot \frac{1}{2} \left(\frac{\Omega_2}{\kappa_2+1} \right)^{j_2+2} \Gamma(j_2+2) \cdot \frac{1}{2} \left(\frac{\Omega_3}{\kappa_3+1} \right)^{j_3+2} \Gamma(j_3+2)$$



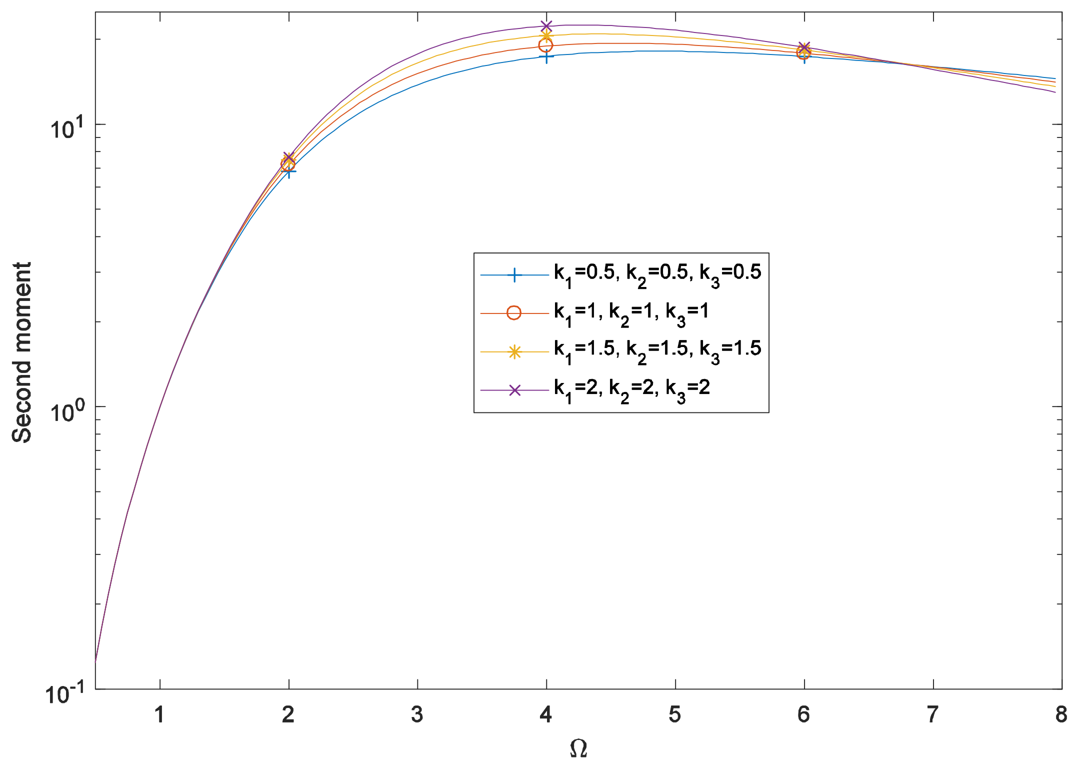
Ω1=0.5, Ω2=0.5, Ω3=0.5
Ω1=1, Ω2=1, Ω3=1
Ω1=2, Ω2=2, Ω3=2
Ω1=4, Ω2=4, Ω3=4
10^2
10^1
10^0
10^-1
Second moment
0
1
2
3
4
5
6
7
k

❖The influence of parameters of fading distribution to the second moment of product of three Rician RVs can be noticed from next figures

❖One can see from these figures that the second moment, variance, enlarges with increasing of power Ω

❖For bigger Ω and Rician factor κ, m_2 decreases

m_2 of product of three Rician RVs depending on signal power Ω for different values of Rician factor κ_i



❖The second moment achieve maximal value with increasing of Ω and then start to decline

n-th moment of a RV x is defined as: $\quad m_n = \overline{x^n} = \int_0^\infty dx x^n p(x) =$

$$= \frac{2(\kappa_1+1)}{\Omega_1 e^{\kappa_1}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_1+1)\kappa_1}{\Omega_1} \right)^{j_1} \frac{1}{(j_1!)^2} \cdot \frac{2(\kappa_2+1)}{\Omega_2 e^{\kappa_2}} \sum_{j_2=0}^{\infty} \left(\frac{(\kappa_2+1)\kappa_2}{\Omega_2} \right)^{j_2} \frac{1}{(j_2!)^2} \cdot$$

$$\cdot \frac{2(\kappa_3+1)}{\Omega_3 e^{\kappa_3}} \sum_{j_1=0}^{\infty} \left(\frac{(\kappa_3+1)\kappa_3}{\Omega_3} \right)^{j_1} \frac{1}{(j_3!)^2} \cdot$$

$$\cdot \frac{1}{2} \left(\frac{\Omega_1}{\kappa_1+1} \right)^{j_1+2} \Gamma(j_1+1+n/2) \cdot \frac{1}{2} \left(\frac{\Omega_2}{\kappa_2+1} \right)^{j_2+2} \Gamma(j_2+1+n/2) \cdot \frac{1}{2} \left(\frac{\Omega_3}{\kappa_3+1} \right)^{j_3+2} \Gamma(j_3+1+n/2)$$

AoF of Product of Three Rician RVs

❖The amount of fading (AoF) is a measure of severity of fading for observed channel

❖For defined distribution of power of a received signal, AoF is a ratio of the variance of the received energy to the square of the mean of the received energy

$$AoF = Var\left\{x^2\right\} / \left(E\left\{x^2\right\}\right)^2$$

- E{·} is statistical average value
- Var{·} denoting the variance

- To calculate AoF, the moments of distribution are used

- Because of that, this is simple and effective manner to quantify fading

$$AoF = m_2 / m_1^2 - 1$$

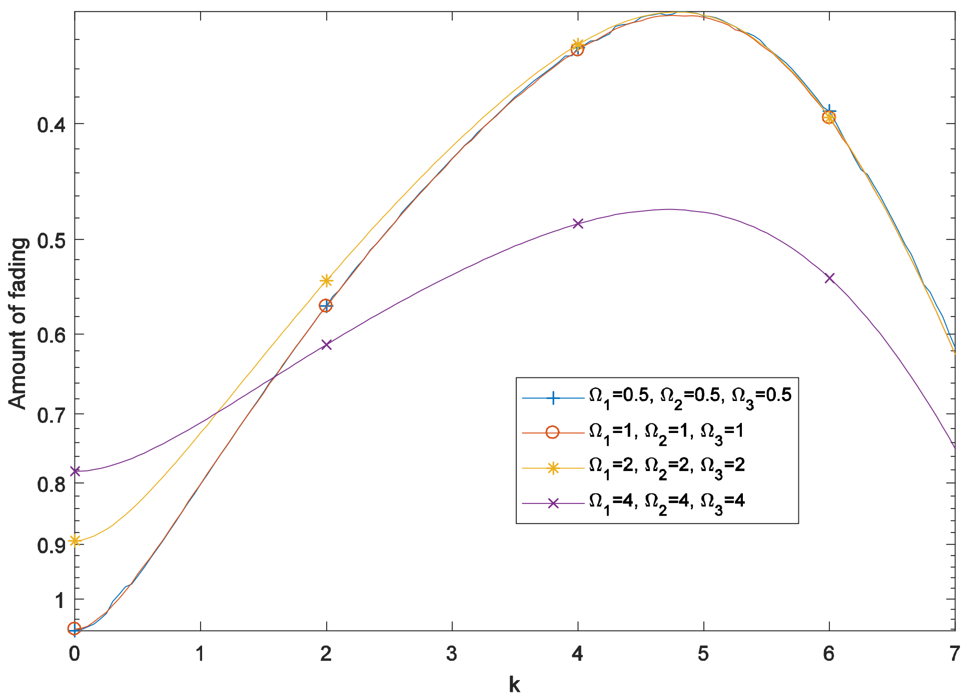
- The range of values of AoF is given in interval [0, 2]

- AoF=0 corresponds a situation of “no fading”

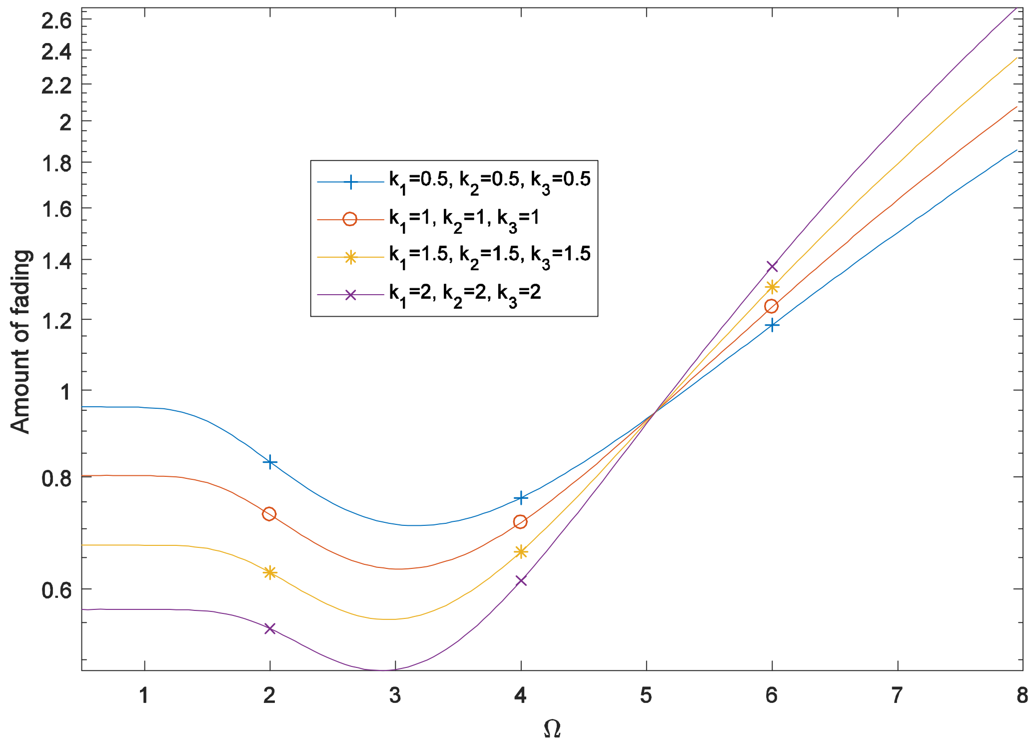
- AoF=1 corresponds to a single Rayleigh fading channel

- AoF= 2 refers to one-sided Gaussian distribution; this is the severest fading

AoF of product of three Rician RVs versus Rician factor κ for different Ω_i



AoF of product of three Rician RVs depending on mean powers Ω for more values of Rician factor κ_i



- It is visible that AoF increases with increasing of Ω for bigger values of Ω

- For higher values of κ_i and small Ω, AoF has smaller values, but the situation is reversed for bigger Ω where AoF has higher values for higher Rician factor κ

THE SECOND ORDER PERFORMANCE OF THE PRODUCT OF THREE RICIAN RANDOM VARIABLES

- Level crossing rate (LCR) and average fade duration (AFD) of the signal envelope are two important second-order statistics of wireless channel
- They give useful information about the dynamic temporal behavior of multipath wireless fading channels

A) LCR of Product of Three Rician RVs

- Level crossing rate is one of the most important second-order performance measures of wireless communication system, which has already found application in modelling and design of communication system but also in the design of error correcting codes, optimization of interleave size and throughput analysis

- The envelope LCR is defined as the expected rate (in crossings per second) at which a fading signal envelope crosses the given level in the downward direction
- The LCR of RV tells how often the envelope crosses a certain threshold x

- We should determine the joint probability density function (JPDF) between *x* and $\dot{x}$, $p_{x\dot{x}}(x\dot{x})$ first, then apply the Rice's formula to finally calculate the LCR
- LCR is defined as:

$$N_x = \int_0^{\infty} d\dot{x}\,\dot{x}\,p_{x\dot{x}}(x\dot{x})$$

- LCR of product of three Rician RVs is:

$$\cdot\left(\frac{\kappa_3\left(\kappa_3+1\right)}{\Omega_3}\right)^{i_3}\frac{1}{\left(i_3!\right)^2}x^{2i_1+1}$$

$$N_x=\frac{1}{\sqrt{2\pi}}\pi f_m\frac{\Omega_1^{1/2}}{\left(\kappa_1+1\right)^{1/2}}\cdot\frac{2\left(\kappa_1+1\right)}{\Omega_1}\cdot\frac{2\left(\kappa_2+1\right)}{\Omega_2}\cdot\frac{2\left(\kappa_3+1\right)}{\Omega_3}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\left(\frac{\kappa_1\left(\kappa_1+1\right)}{\Omega_1}\right)^{i_1}\frac{1}{\left(i_1!\right)^2}\left(\frac{\kappa_2\left(\kappa_2+1\right)}{\Omega_2}\right)^{i_2}\frac{1}{\left(i_2!\right)^2}$$

$$\cdot\int_0^{\infty}dx_2\int_0^{\infty}dx_3\left(1+\frac{x^2}{x_2^4x_3^2}\frac{\Omega_2}{\kappa_2+1}\frac{\kappa_1+1}{\Omega_1}+\frac{x^2}{x_2^2x_3^4}\frac{\Omega_3}{\kappa_3+1}\frac{\kappa_1+1}{\Omega_1}\right)^{1/2}\cdot$$

$$\cdot x_2^{-2i_1-1+2i_2+1}x_3^{-2i_1-1+2i_3+1}e^{-\frac{\kappa_1+1}{\Omega_1}\frac{x^2}{x_2^2x_3^2}-\frac{\kappa_2+1}{\Omega_2}x_2^2-\frac{\kappa_3+1}{\Omega_3}x_3^2}\qquad(**)$$

- Last integral can be solved by using Laplace approximation theorem for solution the two-fold integrals solved through:

$$\int_0^\infty dx_2 \int_0^\infty dx_3 g\left(x_2, x_3\right) e^{\lambda f\left(x_2, x_3\right)} = \frac{\pi}{\lambda} g\left(x_{20}, x_{30}\right) e^{\lambda f\left(x_{20}, x_{30}\right)} \frac{1}{\left(B\left(x_{20}, x_{30}\right)\right)^{1/2}}$$

- We give in this subsection some new graphs for normalized LCR of product of three Rician RVs depending on this product *x* with Rician factor κ_i and average power Ω_i as parameters of curves in Figs. 4 and 5

Fig. 4 LCR normalized by f_m depending on the signal envelope *x* for various values of Rician factor κ_i and signal power Ω=1

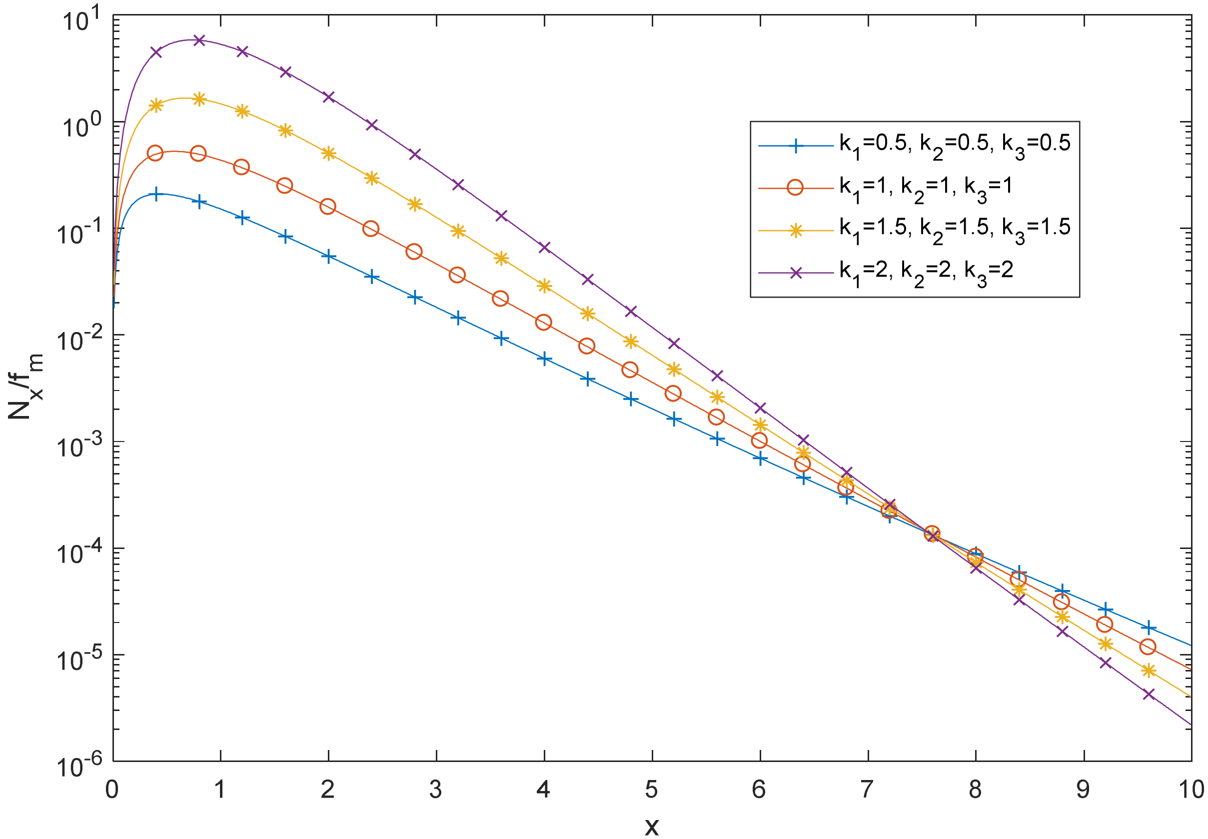
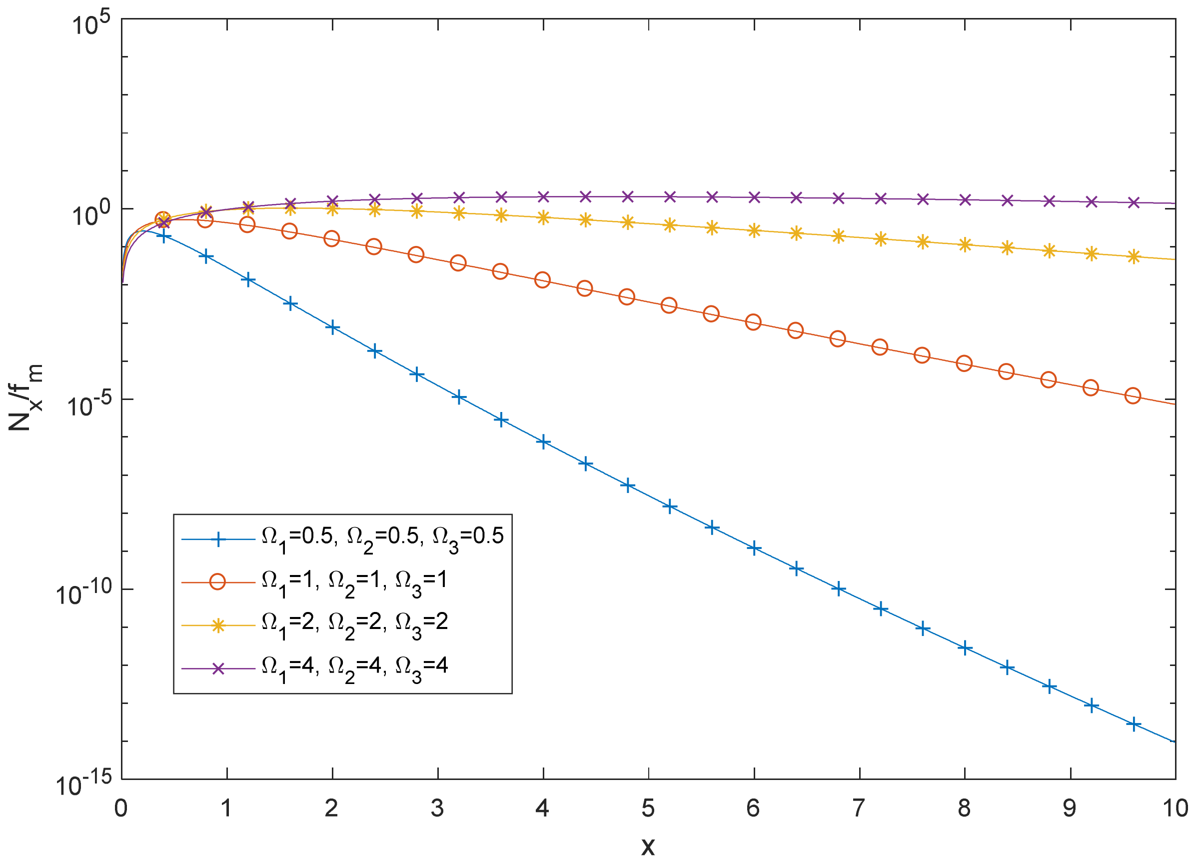


Fig. 5 LCR normalized by f_m versus signal envelope x for various values of signal powers Ω_i



- LCR grows as Rician signal power increases
- The impact of signal envelope power on the LCR is higher for bigger values of Rician factor κ_l
- LCR increases with increasing of Ω_i for all values of signal envelope
- The impact of signal envelope on the LCR is larger for higher values of the signal envelope when Ω_i changes
- It is important bring to mind that system has better performance for lower values of the LCR

B) AFD of Product of Three Rician RVs

- Average fade duration measures how long a signal's envelope or power stays below a given target threshold derived from the LCR
- According to that, AFD is:

$$T_x(x) = \frac{P(x \leq X)}{N_x(x)} = \frac{\int_0^X p_x(x)dx}{N_x(x)}$$

- The numerator is the cumulative distribution function of *x* from Eq. (*), and $N_x(x)$ is LCR obtained by solving (**)

- The normalized AFD ($T_x f_m$) of product of three Rician RVs is plotted in Figs. 6 and 7 versus signal envelope *x*
- One can see that for higher values of κ_i and lower *x*, AFD has smaller values
- Also, it is visible from Fig. 7 that AFD increases for all signal envelopes and lower Ω_i
- The impact of Ω_i is bigger at higher envelopes

Fig. 6 AFD normalized by f_m versus signal envelope *x* for different values of Rician factor κ_i and signal powers Ω_i=1

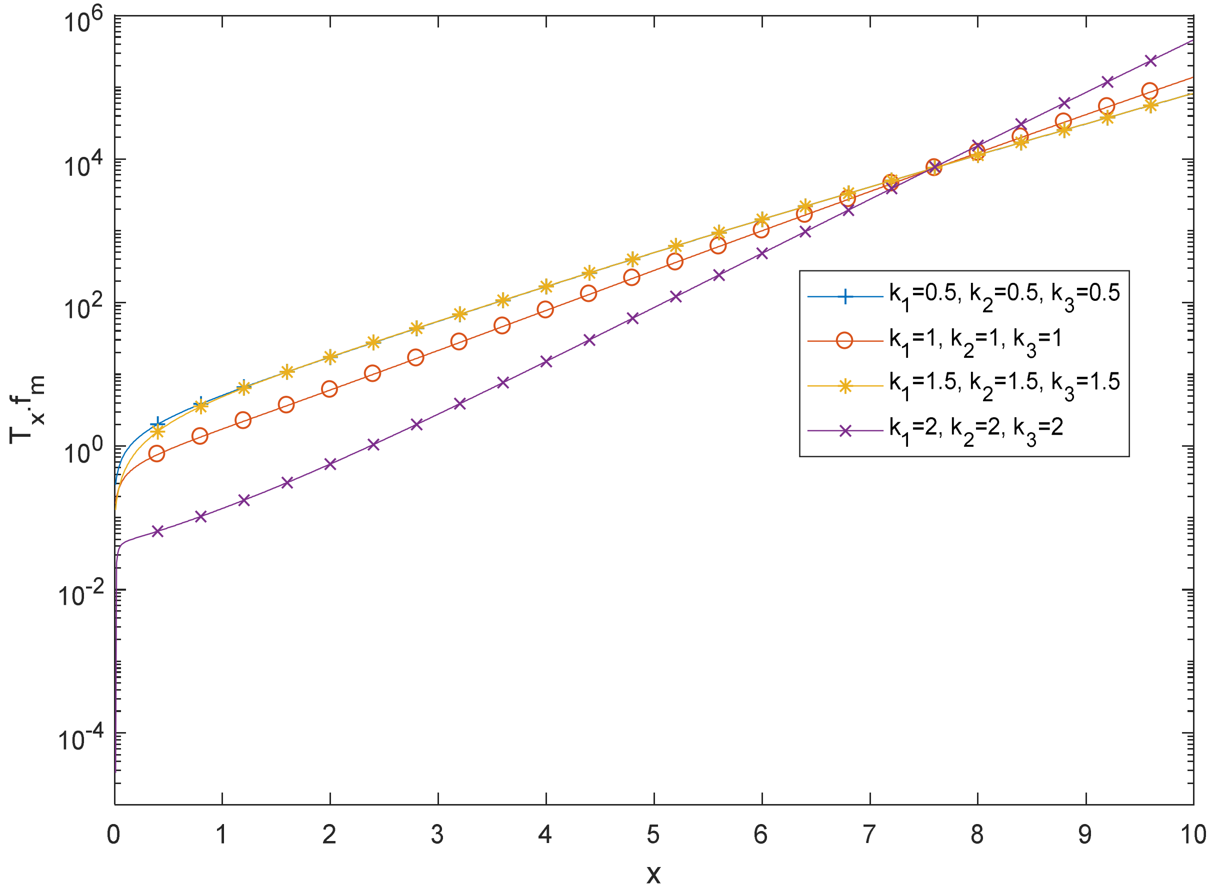
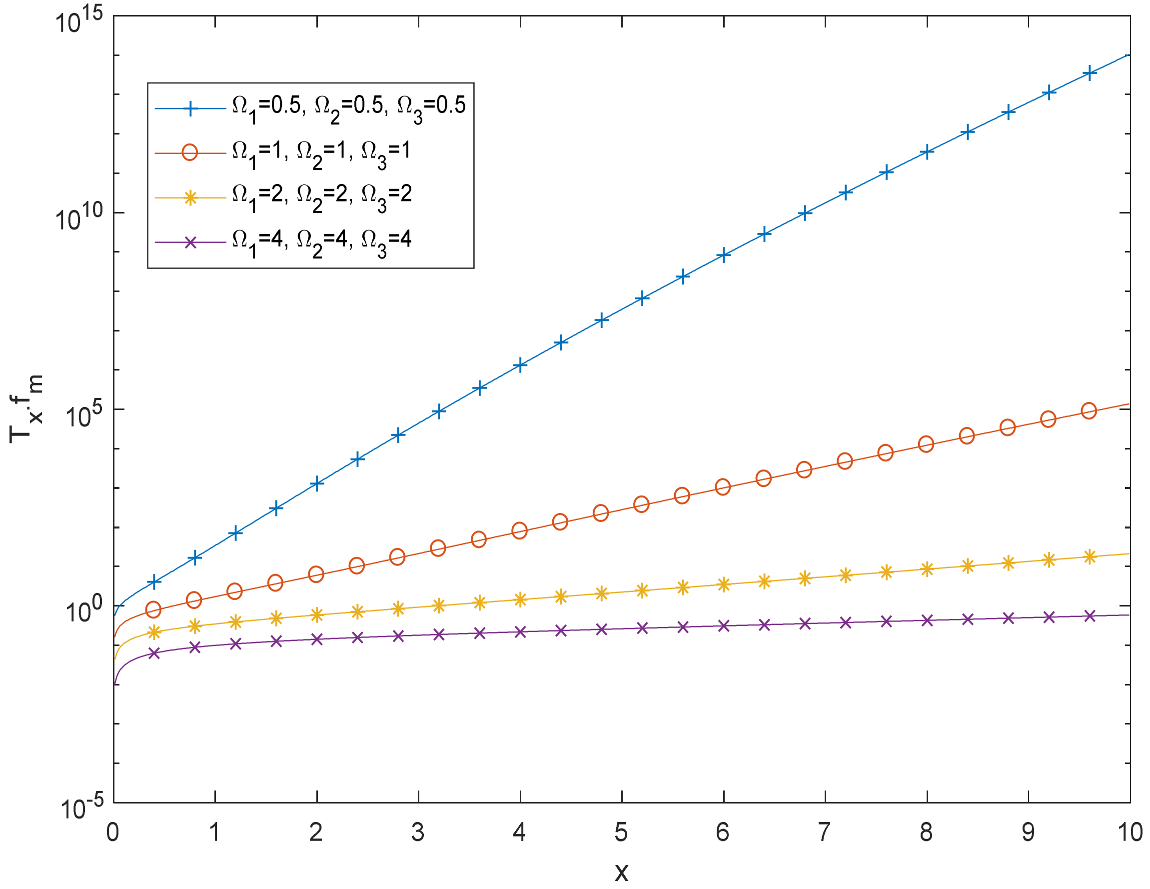


Fig. 7 AFD normalized by f_m depending on signal envelope *x* for κ=1 and different values of signal powers Ω_i



Conclusion 3

- In this part, formulas for the PDF, CDF, Pout, LCR and AFD of the three-hop wireless relay system in the presence of Rician fading are derived
- This system output signal is product of three Rician RVs

- I. Ghareeb, D. Tashman,

“Statistical Analysis of Cascaded Rician Fading Channels”,

International Journal of Electronics Letters,

ISSN: 2168-1724 (Print) 2168-1732 (Online),
2018, doi:10.1080/21681724.2018.1545925

- Exact-form expressions for the PDF and the CDF for product of n-Rician independent and not necessarily identically distributed RVs are obtained in that paper
- Moreover, expressions for the PDF and CDF of the instantaneous signal-to-noise ratio (SNR) in slow frequency nonselective independent and not necessarily identically distributed cascaded Rician fading channels are introduced and analyzed

- Exact-form expressions for the outage probability and average channel capacity are also derived
- The average bit error probability (BEP) expression for phase-shift keying (PSK) signals operating in additive white Gaussian noise (AWGN) channel as well as in independent but not necessarily identically distributed cascaded Rician fading channels are derived

Outage Probability of Wireless Relay Communication System with Three Sections in the Presence of Nakagami-m Short Term Fading

Danijela Aleksić, Dragana Krstić,

Zoran Popović, Ivana Dinić, Mihajlo Stefanović

- The wireless relay communication system with three sections operating over Nakagami-m multipath fading channel is the topic of this part
- The outage probability of proposed relay system is calculated again for two cases
- In the first case, the outage probability is evaluated when it is defined as probability that signal envelope falls below the specified threshold at any section using cumulative distribution function of minimum of three Nakagami-m random variables

- In the second case, the outage probability is calculated when it is defined as probability that output signal envelope is lower than predetermined threshold by using the cumulative distribution function of product of three Nakagami-m random variables
- Numerical expressions for the outage probability of relay system are presented graphically and the influence of Nakagami-m parameter from each section on the outage probability is estimated

- Nakagami-m distribution has some advantages versus the other models, such as that this is a generalized distribution which can model different fading environments
- It has greater flexibility and accuracy in matching some experimental data than the Rayleigh, lognormal or Rice distributions
- Rayleigh and one-sided Gaussian distribution are special cases of Nakagami-m model

- So the Nakagami-m channel model is of more general applicability in practical fading channels
- Nakagami-*m* statistical model describes signal envelope in non line of sight (LOS), linear multipath fading channel where signal propagates with one, two or more clusters
- Nakagami-*m* distribution has severity parameter *m* and signal envelope average power Ω

- The parameter *m* is is greater than 0.5
- When parameter *m* is equal to one, Nakagami-*m* distribution reduces to Rayleigh distribution;
- when parameter *m* tends to 0.5, Nakagami-*m* statistical model turn into one sided Gaussian statistical model and
- when parameter *m* goes to infinity, Nakagami-*m* multipath fading channel becomes no fading channel

- Here, considered wireless relay system has three sections and Nakagami-m fading is present in channel’s sections
- This channel can be denoted as Nakagami-Nakagami- Nakagami channel
- It has three parameters which are denoted with m_1, m_2 and m_3
- Also, Nakagami- Nakagami- Nakagami relay channel is general channel and several channels can be derived from this channel

- For m_1=1, Nakagami- Nakagami- Nakagami channel becomes Rayleigh- Nakagami- Nakagami channel;
- for m_1=1 and m_2=1, Nakagami- Nakagami- Nakagami channel becomes Rayleigh - Rayleigh - Nakagami channel, and
- for m_1=1, m_2=1 and m_3=1, Nakagami- Nakagami- Nakagami channel becomes Rayleigh- Rayleigh – Rayleigh channel

- For m_1=0.5, Nakagami- Nakagami- Nakagami channel becomes One sided Gaussian- Nakagami- Nakagami channel;
- for m_1=0.5 and m_2=0.5, Nakagami- Nakagami- Nakagami channel becomes One sided Gaussian- One sided Gaussian- Nakagami channel, and
- for m_1=1/2, m_2=1/2 and m_3=1/2, Nakagami- Nakagami- Nakagami channel becomes One sided Gaussian- One sided Gaussian- One sided Gaussian relay channel

- Also, for m_1 goes to infinity, Nakagami- Nakagami- Nakagami relay channel becomes no fading- Nakagami- Nakagami channel
- for m_1 goes to infinity and m_2 goes to infinity, Nakagami- Nakagami- Nakagami relay channel becomes no fading- no fading - Nakagami relay channel, and
- for m_1 goes to infinity, m_2 goes to infinity and m_3 goes to infinity, Nakagami- Nakagami- Nakagami relay channel becomes no fading- no fading - no fading channel

Statistics of Minimum of Three Nakagami Random Variables

- Random variables x_1, x_2 and x_3 follow Nakagami-m distribution:

$$p_{x_1}(x_1) = \frac{2}{\Gamma(m_1)} \left(\frac{m_1}{\Omega_1} \right)^{m_1} x_1^{2m_1 - 1} e^{-\frac{m_1}{\Omega_1} x_1^2}, \; x_1 \geq 0$$

$$p_{x_2}(x_2) = \frac{2}{\Gamma(m_2)} \left(\frac{m_2}{\Omega_2} \right)^{m_2} x_2^{2m_2 - 1} e^{-\frac{m_2}{\Omega_2} x_2^2}, \; x_2 \geq 0$$

$$p_{x_3}(x_3) = \frac{2}{\Gamma(m_3)} \left(\frac{m_3}{\Omega_3} \right)^{m_1} x_3^{2m_3 - 1} e^{-\frac{m_3}{\Omega_3} x_3^2}, \; x_3 \geq 0$$

- $\Gamma(.)$ is the Gamma function, Ω_i is the average signal power
- m_i represents the inverse normalized variance y^2, which must satisfy $m_1 \geq 1/2$, describing the fading severity

- Cumulative distribution functions (CDF) of x_1, x_2 and x_3 are:

$$F_{x_1}(x_1) = \frac{1}{\Gamma(m_1)} \gamma\left(m_1, \frac{m_1}{\Omega_1} x_1^2\right), \; x_1 \geq 0$$

$$F_{x_2}(x_2) = \frac{1}{\Gamma(m_2)} \gamma\left(m_2, \frac{m_2}{\Omega_2} x_2^2\right), \; x_2 \geq 0$$

$$F_{x_3}(x_3) = \frac{1}{\Gamma(m_3)} \gamma\left(m_3, \frac{m_3}{\Omega_3} x_3^2\right), \; x_3 \geq 0$$

- Minimum of x_1, x_2 and x_3 is:

$$x = \min(x_1, x_2, x_3)$$

- Probability density function (PDF) of *x* is:

$$p_x(x) = p_{x_1}(x) F_{x_2}(x) F_{x_3}(x) +$$

$$+ p_{x_2}(x) F_{x_1}(x) F_{x_3}(x) + p_{x_3}(x) F_{x_1}(x) F_{x_2}(x)$$

$$
\begin{aligned}
p_x(x) = {} & \frac{2}{\Gamma(m_1)}\left(\frac{m_1}{\Omega_1}\right)^{m_1} x_1^{2m_1-1} e^{-\frac{m_1}{\Omega_1}x^2} \frac{1}{\Gamma(m_2)}\gamma\left(m_2, \frac{m_2}{\Omega_2}x^2\right)\cdot \\
& \cdot\frac{1}{\Gamma(m_3)}\gamma\left(m_3, \frac{m_3}{\Omega_3}x^2\right) + \frac{2}{\Gamma(m_2)}\left(\frac{m_2}{\Omega_2}\right)^{m_2} x^{2m_2-1} e^{-\frac{m_2}{\Omega_2}x^2}\cdot \\
& \cdot\frac{1}{\Gamma(m_1)}\gamma\left(m_1, \frac{m_1}{\Omega_1}x^2\right)\cdot\frac{1}{\Gamma(m_3)}\gamma\left(m_3, \frac{m_3}{\Omega_3}x^2\right) + \\
& + \frac{2}{\Gamma(m_3)}\left(\frac{m_3}{\Omega_3}\right)^{m_1} x^{2m_3-1} e^{-\frac{m_3}{\Omega_3}x^2} \cdot\frac{1}{\Gamma(m_1)}\gamma\left(m_1, \frac{m_1}{\Omega_1}x^2\right)\cdot \\
& \cdot\frac{1}{\Gamma(m_2)}\gamma\left(m_2, \frac{m_2}{\Omega_2}x^2\right),\ x \ge 0
\end{aligned}
$$

- Cumulative distribution function of minimum of three Nakagami-*m* random variables is:

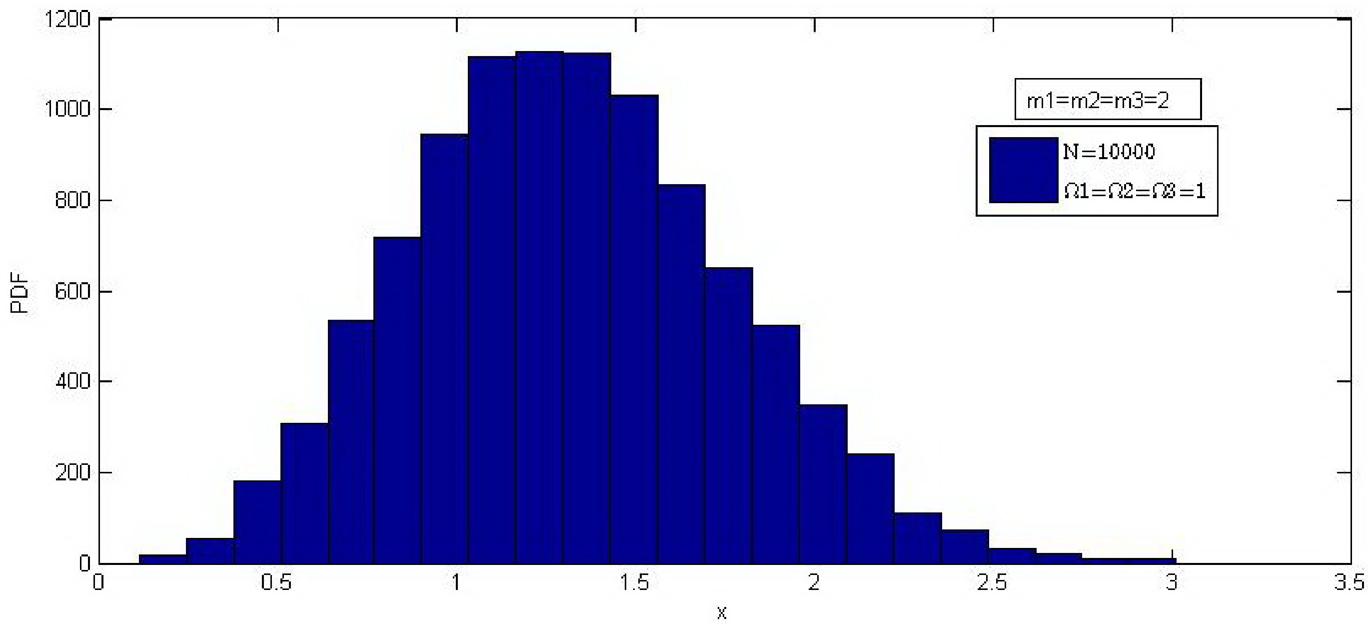
$$F_X(x)=\int_0^x dt\ p_X(t)=\left(1-\left(1-F_{X_1}(x)\right)\cdot\left(1-F_{X_2}(x)\right)\cdot\left(1-F_{X_3}(x)\right)\right)=$$

$$=1-\left(1-\frac{1}{\Gamma(m_1)}\gamma\left(m_1,\frac{m_1}{\Omega_1}x^2\right)\right)\cdot\left(1-\frac{1}{\Gamma(m_2)}\gamma\left(m_2,\frac{m_2}{\Omega_2}x^2\right)\right)\cdot$$

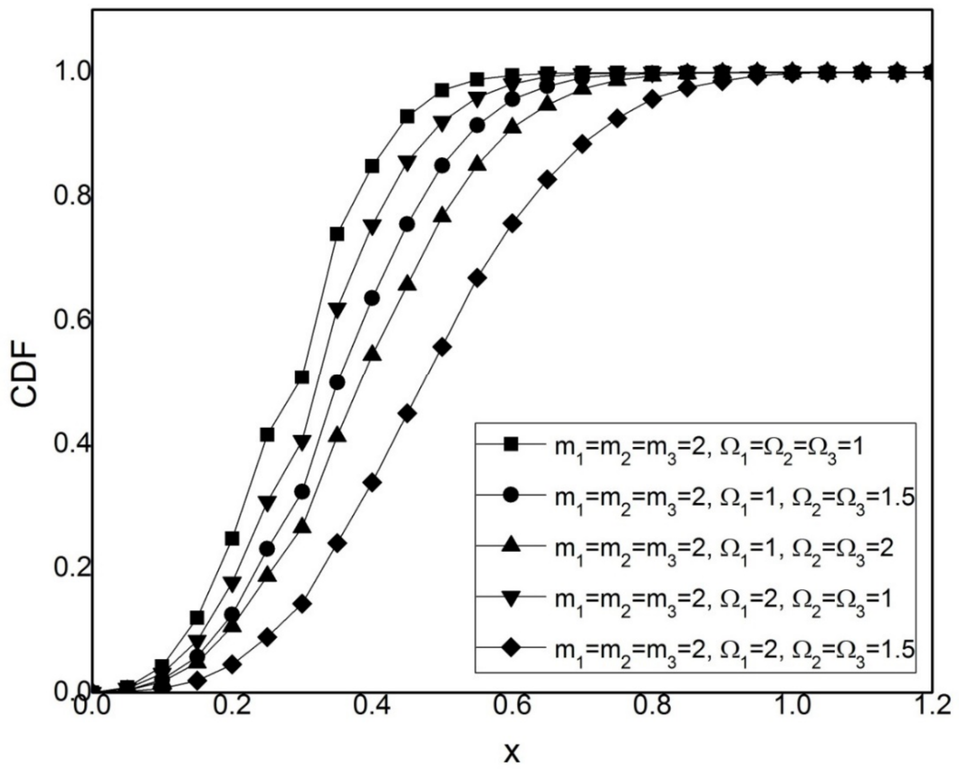
$$\cdot\left(1-\frac{1}{\Gamma(m_3)}\gamma\left(m_3,\frac{m_3}{\Omega_3}x^2\right)\right),\ x\geq 0$$

- In the previous expressions, parameter m_1 is severity parameter of Nakagami-m fading in the first section, m_2 is severity parameter of Nakagami-m fading in the second section and m_3 is the severity parameter of Nakagami-m fading in the third section
- The Ω_1 is signal envelope average power in the first section, Ω_2 is signal envelope average power in the second section and Ω_3 is signal envelope average power in the third section

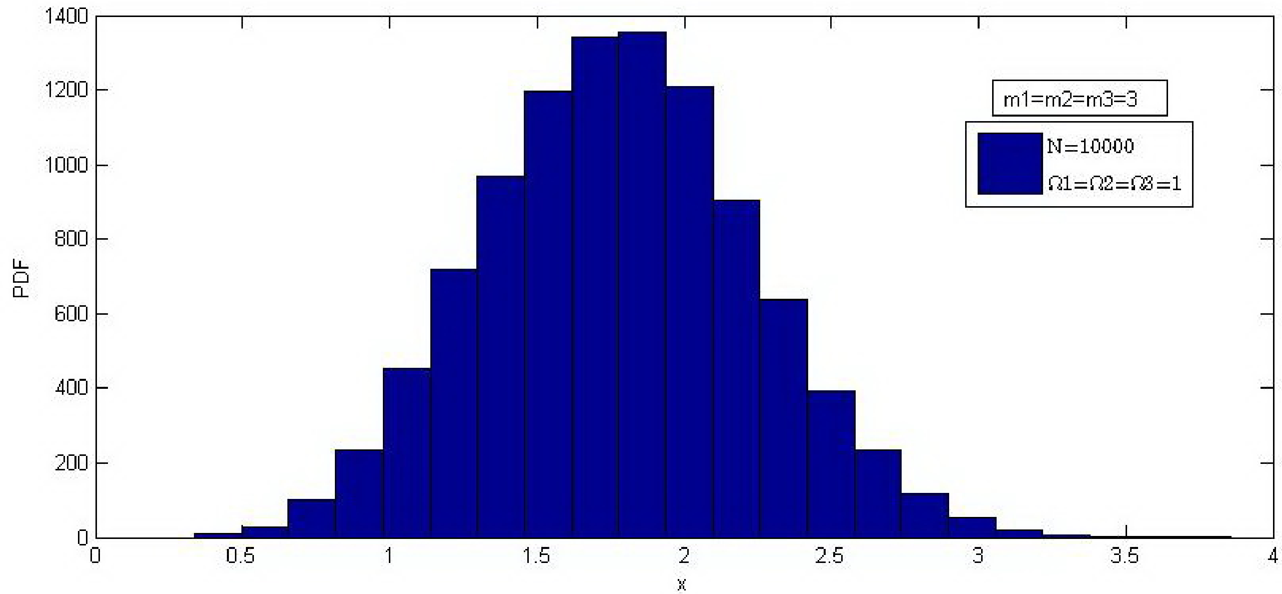
PDF of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 2$



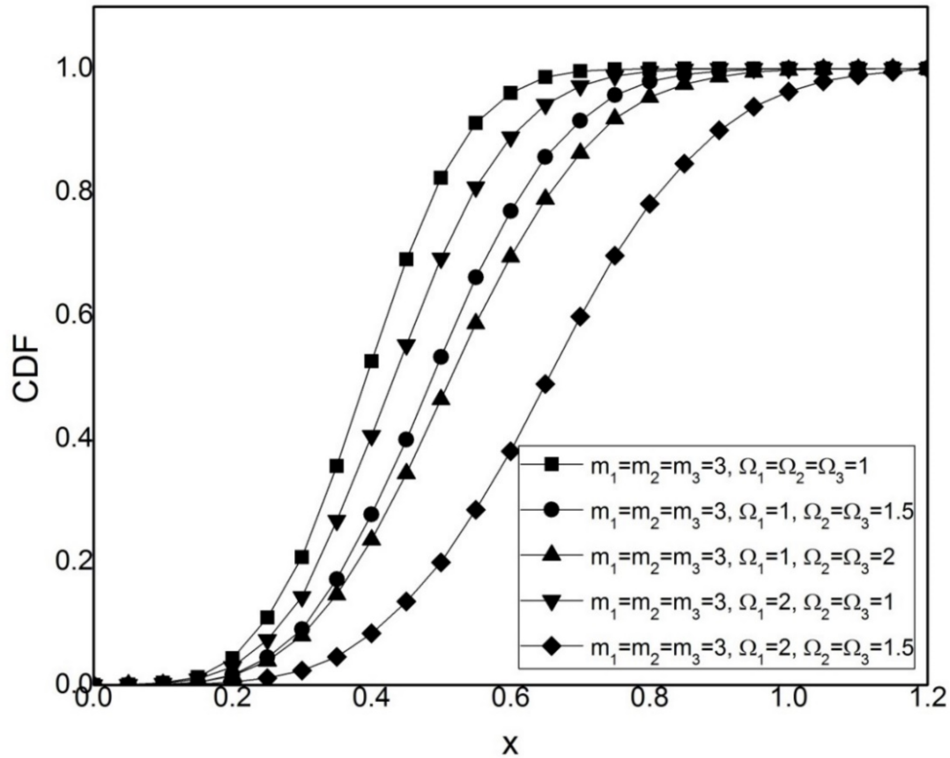
The outage probability of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 2$



PDF of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 3$



The outage probability of minimum of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 3$



- Probability density functions of x are shown in Figs. 1. and 3 versus of minimum of three Nakagami-m random variables
- Severity parameters of Nakagami-m fading are $m_1 = m_2 = m_3 = 2$ in Fig. 1 and $m_1 = m_2 = m_3 = 3$ in Fig. 2
- Signal envelope average powers are $\Omega_1 = \Omega_2 = \Omega_3 = 1$ in both figures

- In Figs. 2 and 4, the outage probability in terms of minimum of three Nakagami-m random variables are shown for several values of severity Nakagami parameters and several values of signal envelopes average powers in sections

- The outage probability decreases when Nakagami severity parameter m_1 in the first section increases, Nakagami severity parameter m_2 in the second section increases, and Nakagami severity parameter m_3 in the third section increases

Statistics of Product of Three Nakagami Random Variables

- Product of three Nakagami-m random variables is:

$$x = x_1 \cdot x_2 \cdot x_3 \ , \ x_1 = \frac{x}{x_2 \cdot x_3}$$

- Conditional probability density function of *x* is:

$$p_x\left(x / x_2 x_3\right) = \left|\frac{dx_1}{dx}\right| p_{x_1}\left(\frac{x}{x_2 x_3}\right)$$

- After substituting and averaging, probability density function of *x* is derived as

$$p_x(x) = \int_0^\infty dx_2 \int_0^\infty dx_3 \frac{1}{x_2 x_3} p_{x_1}\left(\frac{x}{x_2 x_3}\right) p_{x_2}(x_2)\, p_{x_3}(x_3)$$

- After solving we obtain:

$$p_x(x) = \frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_3}{\Omega_3}\right)^{m_3} \cdot$$

$$\cdot x^{2m_1-1+2m_3-2m_1} \cdot \left(\frac{m_1\Omega_3}{\Omega_1 m_3}\right)^{m_3-m_1} \cdot \int_0^\infty dx_2 \; x_2^{2m_2-2m_3-1} \; e^{-\frac{m_2}{\Omega_2}x_2^2} \cdot K_{2m_3-2m_1}\left(2\sqrt{\frac{m_1 x^2 m_3}{\Omega_1\Omega_3 x_2^2}}\right)$$

- $K_n(x)$ is the modified Bessel function of the second kind

- Cumulative distribution function of *x* is:

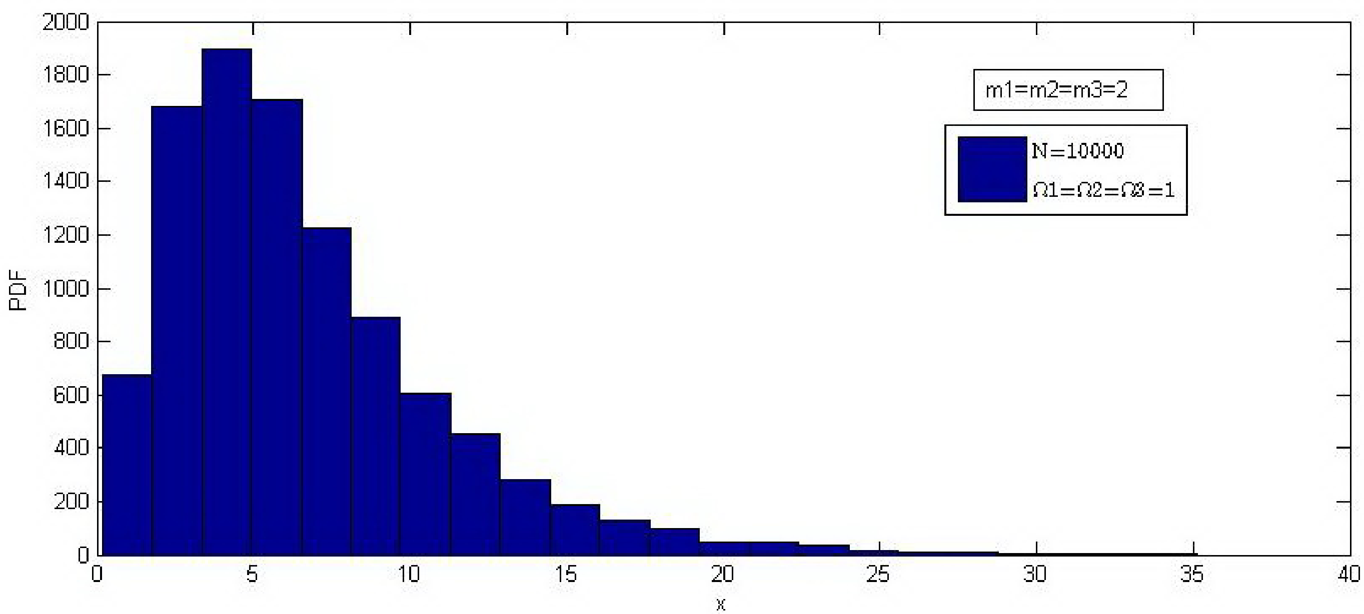
$$F_x(x) = \frac{8}{\Gamma(m_1)\Gamma(m_2)\Gamma(m_3)} \left(\frac{m_1}{\Omega_1}\right)^{m_1} \left(\frac{m_2}{\Omega_2}\right)^{m_2} \left(\frac{m_3}{\Omega_3}\right)^{m_3} \cdot$$

$$\cdot \frac{1}{m_1} x^{2m_1} \cdot \sum_{j_1=0}^{\infty} \frac{1}{(m_1+1)(j_1)} \frac{m_1^{j_1} x^{2j_1}}{\Omega_1^{j_1}} \cdot$$

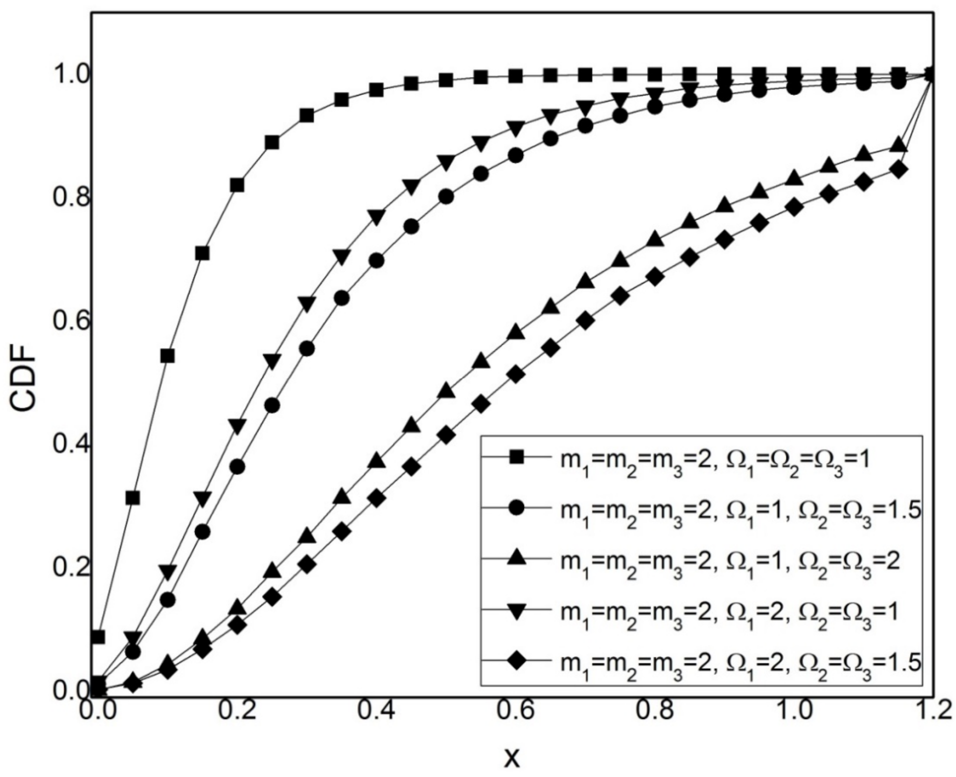
$$\cdot \int_0^{\infty} dx_2 \quad x_2^{2m_2-2m_3-1} \, e^{-\frac{m_2}{\Omega_2} x_2^2} \cdot$$

$$\cdot \left(\frac{m_1 x^2 \Omega_3}{\Omega_1 m_3}\right)^{m_3-m_1-j_1} \cdot K_{2m_3-2m_1-2j_1}\left(2\sqrt{\frac{m_1 m_3 x^2}{\Omega_1 \Omega_2 x_2^2}}\right)$$

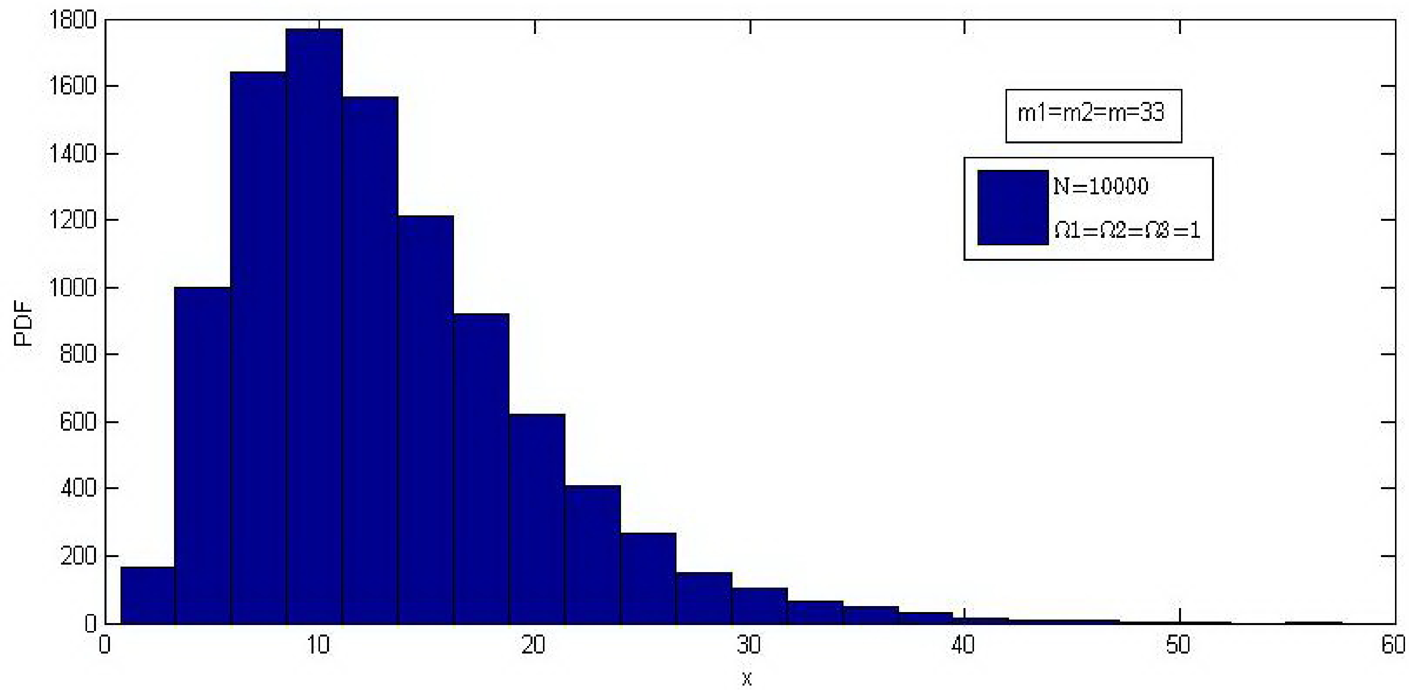
PDF of product of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 2$



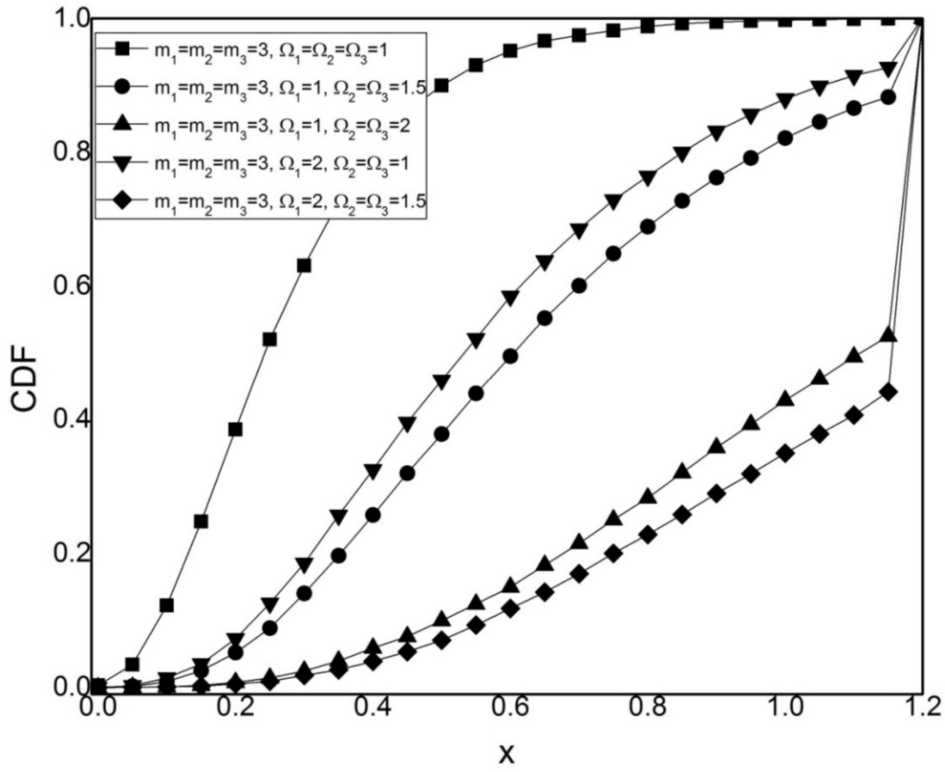
The outage probability of product of three Nakagami-*m* random variables for $m_1=m_2=m_3=2$



PDF of product of three Nakagami-m random variables for $m_1 = m_2 = m_3 = 3$.



The outage probability of product of three Nakagami-m random variables for m_1=m_2=m_3 =3



- Probability density functions of x are shown in Figs. 5. and 7 versus of product of three Nakagami-m random variables
- Severity parameters of Nakagami-m fading are $m_1 = m_2 = m_3 = 2$ in Fig. 5 and $m_1 = m_2 = m_3 = 3$ in Fig. 7
- Signal envelope average powers are $\Omega_1 = \Omega_2 = \Omega_3 = 1$ in both figures

- In Figs. 6 and 8, the outage probability depending of product of three Nakagami-m random variables is shown for several values of Nakagami severity parameters and several values of signal envelopes average powers in sections

- The outage probability decreases when severity Nakagami parameter m_1 in the first section increases, severity Nakagami parameter m_2 in the second section increases, and severity Nakagami parameter m_3 in the third section increases

Conclusion 4

- In this part of Lecture, wireless mobile relay radio communication system with three sections operating over Nakagami-m small scale fading channel is considered
- Nakagami- Nakagami- Nakagami relay channel is defined
- For proposed relay system, the outage probability is determined

- In this work, probability density functions and cumulative distribution functions of minimum of three Nakagami random variables and product of three Nakagami random variables are evaluated
- Cumulative distribution function of minimum of three Nakagami random variables is derived in the closed form
- Cumulative distribution function of product of three Nakagami random variables is obtained as expression with one integral

- For both cases, the outage probability decreases when severity parameters of Nakagami fading increase at any sections
- These results are useful for designing of wireless mobile relay radio communication system with more sections in the presence of gading

Outline

- **THE FIRST ORDER PERFORMANCE OF PRODUCT OF THREE RICIAN RANDOM VARIABLES**
  - ***PDF of Product of Three Rician RVs***
  - ***CDF of Product of Three Rician RVs***
  - ***Outage probability of Product of Three Rician RVs***
- **THE SECOND ORDER PERFORMANCE OF THE PRODUCT OF THREE RICIAN RANDOM VARIABLES**
  - ***LCR of Product of Three Rician RVs***
  - ***AFD of Product of Three Rician RVs***

G. K. Karagiannidis, N. C. Sagias, P. T. Mathiopoulos,

“The N * Nakagami Fading Channel Model”

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- A generic distribution, referred as N*Nakagami, obtained as the product of *N* statistically independent, but not necessarily identically distributed, Nakagami-*m* random variables is introduced
- The proposed distribution is a convenient tool for analyzing the performance of digital communication systems over generalized fading channels

- The moments-generating function (MGF), PDF, CDF, and moments of the N*Nakagami distribution are derived in closed-form
- Then, closed form expressions for the outage probability, amount of fading, and average symbol error probability for several binary and multilevel modulation signals of digital communication systems operating over the N*Nakagami fading channel model are presented

Level Crossing Rate of Wireless Relay System with Three Sections Output Signal Envelope in the Presence of Multipath k-µ Fading

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- In this part of work, the wireless relay system with three sections in the presence of multipath k-μ fading is presented
- The useful closed form expression for the average Level Crossing Rate (LCR) is calculated
- The resulting integrals are solved by using the Laplace approximating formula for two random variables

- Product of three k-μ random variables x_1, x_2 and x_3 is:

$$z = x_1 \cdot x_2 \cdot x_3$$

- x_1, x_2 and x_3 are independent
- Then it is valid:

$$x_1 = \frac{z}{x_2 \cdot x_3}$$

- The first derivative of product of three *k-μ* random variables x_1, x_2 and x_3 is:

$$\dot{z} = x_2 x_3 \dot{x}_1 + x_1 x_3 \dot{x}_2 + x_1 x_2 \dot{x}_3$$

- The first derivative of *k-μ* random process is Gaussian distributed
- The linear combination of Gaussian random processes is Gaussian random process.

- Therefore, the first derivative of product of three *k-μ* random variables has conditional Gaussian distribution
- The main of $\dot{z}$ is zero
- The variance of $\dot{z}$ is:

$$\sigma_{\dot{z}}^2 = x_2^2 x_3^2 \sigma_{\dot{x}_1}^2 + x_1^2 x_3^2 \sigma_{\dot{x}_2}^2 + x_1^2 x_2^2 \sigma_{\dot{x}_3}^2$$

- Here:

$$\sigma_{\dot{x}_i}^2 = \pi^2 f_m^2 \frac{\Omega_i}{\mu\left(k+1\right)}$$

- f_m is maximal Dopler frequency
- Ω_i, i=1, 2, 3, are power of *k*-*μ* random variables and
- *μ* is severity of *k*-*μ* fading

- The joint probability density function of $z, \dot{z}$, x_2 and x_3 is:

$$p_{z\dot{z}x_2x_3}\left(z\dot{z}x_2x_3\right) = p_{\dot{z}}\left(\dot{z}/z\,\dot{z}x_2x_3\right)\cdot p_{zx_2x_3}\left(z\,x_2x_3\right) =$$

$$= p_{\dot{z}}\left(\dot{z}/z\,\dot{z}x_2x_3\right)\cdot p_{x_2}\left(x_2\right)\cdot p_{x_3}\left(x_3\right)\cdot p_z\left(z/x_2x_3\right)$$

- and:

$$p_z\left(z/x_2x_3\right) = \left|\frac{dx_1}{dz}\right| p_{x_1}\left(\frac{z}{x_2x_3}\right)$$

- The joint probability density function of z and $\dot{z}$ can be calculated by integrating:

$$p_{z\dot{z}}\left(z\dot{z}\right)=\int_0^\infty dx_2\int_0^\infty dx_3\,\frac{1}{x_2x_3}\,p_{x_1}\left(z/x_2x_3\right)p_{x_2}\left(x_2\right)p_{x_3}\left(x_3\right)p_{\dot{z}}\left(\dot{z}/z\,\dot{z}x_2x_3\right)$$

- The level crossing rate of product of three *k-μ* random processes can be calculated as average value of the first derivative of product of three *k-μ* random processes:

$$N_z = \int_0^\infty d\dot{z}\, \dot{z}\, p_{z\dot{z}}\left(z\dot{z}\right) = \int_0^\infty dx_2 \int_0^\infty dx_3 \frac{1}{x_2 x_3} p_{x_1}\left(z / x_2 x_3\right) p_{x_2}\left(x_2\right) p_{x_3}\left(x_3\right) \cdot$$

$$\cdot \int_0^\infty d\dot{z}\, \dot{z}\, p_{\dot{z}}\left(\dot{z} / z x_2 x_3\right) = \int_0^\infty dx_2 \int_0^\infty dx_3 \frac{1}{x_2 x_3} p_{x_1}\left(z / x_2 x_3\right) p_{x_2}\left(x_2\right) p_{x_3}\left(x_3\right) \cdot \frac{\sigma_{\dot{z}}}{\sqrt{2\pi}}$$

- We applied:

$$\int_0^{\infty} d\dot{z}\,\dot{z}\,\frac{1}{\sqrt{2\pi}\sigma_{\dot{z}}} e^{-\frac{\dot{z}^2}{2\sigma_{\dot{z}}^2}} = \frac{\sigma_{\dot{z}}}{\sqrt{2\pi}}$$

- The probability density functions of *k-μ* random variables x_1, x_2 and x_3 are given by *k-μ* distribution earlier

- The expression for average level crossing rate of product of three *k-μ* random processes becomes:

$$N_z = \frac{2\mu_1 (k_1+1)^{\frac{\mu_1+1}{2}}}{k_1^{\frac{\mu_1-1}{2}} e^{\mu_1 k_1} \Omega_1^{\frac{\mu_1+1}{2}}} \cdot \sum_{j_1=0}^{\infty} \left(\mu_1 \sqrt{\frac{k_1(k_1+1)}{\Omega_1}} \right)^{2j_1+\mu_1-1} \frac{1}{j_1!\Gamma(j_1+1)} \cdot$$

$$\cdot \frac{2\mu_2 (k_2+1)^{\frac{\mu_2+1}{2}}}{k_2^{\frac{\mu_2-1}{2}} e^{\mu_2 k_2} \Omega_2^{\frac{\mu_2+1}{2}}} \cdot \sum_{j_2=0}^{\infty} \left(\mu_2 \sqrt{\frac{k_2(k_2+1)}{\Omega_2}} \right)^{2j_2+\mu_2-1} \frac{1}{j_2!\Gamma(j_2+1)} \cdot$$

$$\cdot \frac{2\mu_3 (k_3+1)^{\frac{\mu_3+1}{2}}}{k_3^{\frac{\mu_3-1}{2}} e^{\mu_3 k_3} \Omega_3^{\frac{\mu_3+1}{2}}} \cdot \sum_{j_3=0}^{\infty} \left(\mu_3 \sqrt{\frac{k_3(k_3+1)}{\Omega_3}} \right)^{2j_3+\mu_3-1} \frac{1}{j_3!\Gamma(j_3+1)} \cdot$$

$$\cdot \pi f_m \sqrt{\frac{\Omega_1}{\mu(k+1)}} \cdot z^{2j_1+2\mu-1} \cdot \int_0^{\infty} dx_2 \int_0^{\infty} dx_3 \frac{1}{x_2 x_3} x_2 x_3 \frac{1}{(x_2 x_3)^{2j_1+2\mu-1}} \cdot$$

$$\cdot x_2^{2j_2+2\mu-1} \cdot x_3^{2j_3+2\mu-1} \cdot \sqrt{1 + \frac{z^2}{x_2^4 x_3^2} \frac{\Omega_2}{\Omega_1} + \frac{z^2}{x_2^2 x_3^4} \frac{\Omega_3}{\Omega_1}} \cdot e^{-\frac{\mu(k+1)}{\Omega_1} \frac{z^2}{x_2^2 x_3^2} - \frac{\mu(k+1)}{\Omega_2} x_2^2 - \frac{\mu(k+1)}{\Omega_3} x_3^2}$$

- The integral J is:

$$J=\int_0^\infty dx_2\int_0^\infty dx_3\, x_2^{2j_2-2j_1}x_3^{2j_3-2j_1}\sqrt{1+\frac{z^2}{x_2^4x_3^2}\frac{\Omega_2}{\Omega_1}+\frac{z^2}{x_2^2x_3^4}\frac{\Omega_3}{\Omega_1}}\cdot$$

$$\cdot e^{-\frac{\mu(k+1)}{\Omega_1}\frac{z^2}{x_2^2x_3^2}-\frac{\mu(k+1)}{\Omega_2}x_2^2-\frac{\mu(k+1)}{\Omega_3}x_3^2}=$$

$$=\int_0^\infty dx_2\int_0^\infty dx_3\sqrt{1+\frac{z^2}{x_2^4x_3^2}\frac{\Omega_2}{\Omega_1}+\frac{z^2}{x_2^2x_3^4}\frac{\Omega_3}{\Omega_1}}\cdot$$

$$\cdot e^{-\frac{\mu(k+1)}{\Omega_1}\frac{z^2}{x_2^2x_3^2}-\frac{\mu(k+1)}{\Omega_2}x_2^2-\frac{\mu(k+1)}{\Omega_3}x_3^2+(2j_2-2j_1)\ln x_2+(2j_3-2j_1)\ln x_3}$$

- Previous integral is solved by usage of the Laplace approximation formula:

$$\int_0^{\infty} dx_2 \int_0^{\infty} dx_3\, g\left(x_2, x_3\right) \cdot e^{-f\left(x_2, x_3\right)} = \left(\frac{\sqrt{2\pi}}{\lambda}\right) \frac{g\left(x_{2m}, x_{3m}\right)}{\sqrt{\det B}} \cdot e^{-\lambda f\left(x_{2m}, x_{3m}\right)}$$

- with B defined by dint of:

$$B = \begin{vmatrix} \dfrac{\partial^2 f\left(x_{2m}, x_{3m}\right)}{\partial x_{2m}^2} & \dfrac{\partial^2 f\left(x_{2m}, x_{3m}\right)}{\partial x_{2m} \partial x_{3m}} \\ \dfrac{\partial^2 f\left(x_{2m}, x_{3m}\right)}{\partial x_{3m} \partial x_{2m}} & \dfrac{\partial^2 f\left(x_{2m}, x_{3m}\right)}{\partial x_{3m}^2} \end{vmatrix}$$

- The functions *g(x₂, x₃)* and *f(x₂, x₃)* are:

$$g(x_2, x_3) = \sqrt{1 + \frac{z^2}{x_2^4 x_3^2} \frac{\Omega_2}{\Omega_1} + \frac{z^2}{x_2^2 x_3^4} \frac{\Omega_3}{\Omega_1}}$$

$$f(x_2, x_3) = \frac{\mu(k+1)}{\Omega_1} \frac{z^2}{x_2^2 x_3^2} + \frac{\mu(k+1)}{\Omega_2} x_2^2 + \frac{\mu(k+1)}{\Omega_3} x_3^2 - 2(j_2 - j_1)\ln x_2 - 2(j_3 - j_1)\ln x_3$$

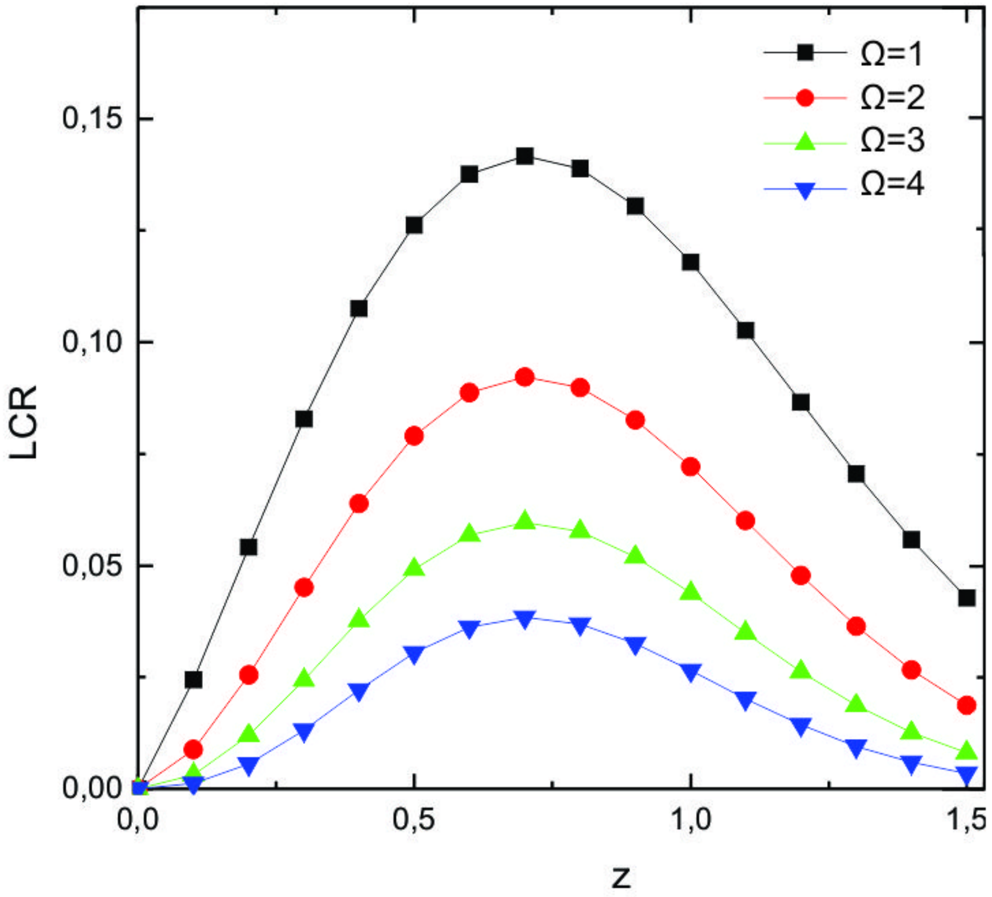
- x_{2m} and x_{3m} are determined from the next equations:

$$-\frac{\mu(k+1)}{\Omega_1}\frac{z^2}{x_2^2 x_3^2}+\frac{\mu(k+1)}{\Omega_2}x_2-\frac{(j_2-j_1)}{x_2}=0$$

$$-\frac{\mu(k+1)}{\Omega_1}\frac{z^2}{x_2^2 x_3^2}+\frac{\mu(k+1)}{\Omega_3}x_3-\frac{(j_3-j_1)}{x_3}=0$$

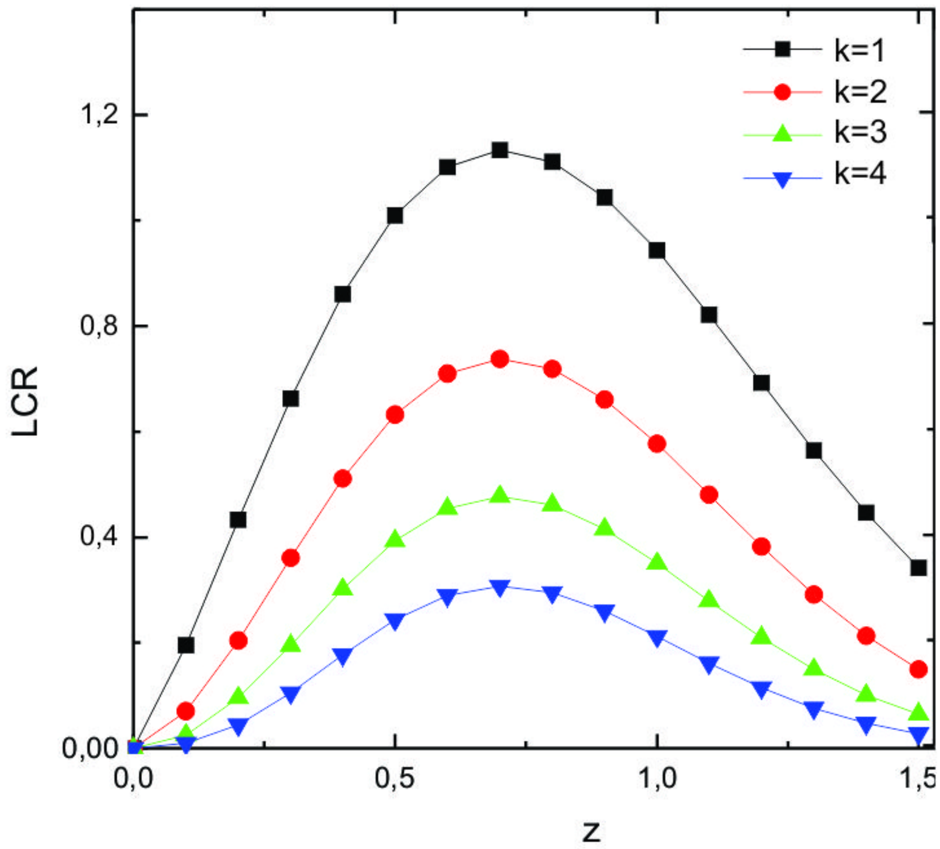
- In Fig. 1, the normalized average level crossing rate of product of three *k-μ* random processes is presented
- The family of curves is shown for several values of Rician factor, *k-μ* fading parameter *μ* and average signal envelope powers at sections, Ω
- Rician factor *k* and fading parameter *μ* are equal to 1 in all cases

Average level crossing rate (LCR) versus output signal envelope for *μ=1* and *k*=1



- Signal envelope powers are variable
- It is visible from this graph that average level crossing rate increases when average signal envelope power decreases
- The influence of average signal envelope power on average level crossing rate is higher for lower values of average signal envelope power

Average level crossing rate (LCR) versus output signal envelope for Ω=1 and μ=1



- In this Figure, the normalized average level crossing rate of product of three *k-μ* random processes is shown for several values of Rician factor k, while parameter *μ* and average signal envelope power Ω have constant values: *μ=1* and Ω=1 for all curves
- It can be seen from this figure that average LCR increases with decreasing of Rician factor
- The influence of Rician factor on average LCR is higher for lower values of Rician factor *k*

Conclusion 5

- By this result, average level crossing rate of product of three Nakagami-*m* random processes or average level crossing rate of product of three Rician random processes can be evaluated
- Average level crossing rate can be used for evaluation the average fade duration (AFD) of relay system with three sections in the presence of *k*-*μ* multipath fading

- The system performance is better for lower values of average level crossing rate
- The average level crossing rate decreases when Rician factor increases, dominant component power increases and scattering components power decreases
- For higher values of parameter μ average level crossing rate has lower values and outage probability decreases

- When average power at sections increases, average level crossing rate decreases
- The numerical expressions show the influence of Rician factor, *k-μ* multipath fading parameter *μ* and average powers on average LCRof product of three *k-μ* random processes
- The expression for the LCR can be used for calculating the average fade duration of the proposed relay system

Outage Probability of Two Relay Systems with Two Sections on Selection Combining in the Presence of κ-μ Short Term Fading

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- Two wireless relay communication systems with two sections on selection combining (SC) are considered in this part of Lecture
- Received signal in sections experiences κ-μ small scale fading
- Signal envelope at output of relay systems can be evaluated as product of signal envelopes in sections

- The probability density function (PDF) and cumulative distribution function (CDF) at output of relay system are calculated
- SC receiver selects relay system with higher signal envelope at its inputs
- Thus, PDF and CDF of SC receiver signal envelope are calculated by solving integrals in the closed forms by using sums and Bessel function of the second kind

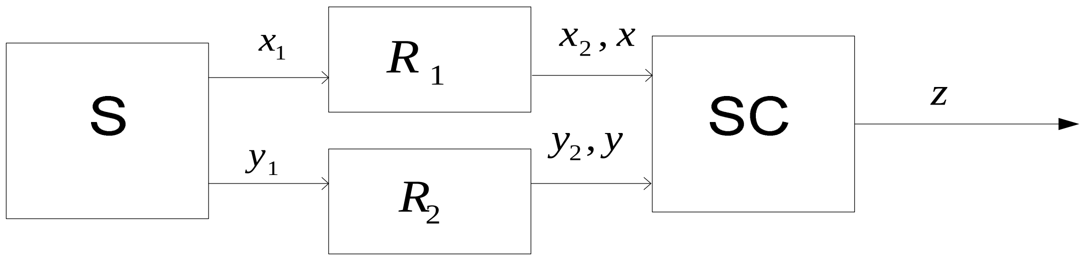
- Then, they are graphically presented
- The influence of Rician factors of κ-μ short term fading in sections and κ-μ short term fading severity parameters on the outage probability of considered relay system is analyzed and discussed
- These results serve to designers of wireless systems to choose optimal system parameters in appropriate fading environment

Introduction

- Here, two relay systems on selection combining (SC) receiver are considered
- Such relay system has two sections
- Received signal in each section is subjected to κ-μ short term fading
- Signal envelope at output of proposed wireless relay system can be evaluated as maximum of two signal envelopes at outputs of relay systems

- In this part, two wireless relay communication systems with two sections and with SC receiver in the presence of κ-μ short term fading in sections are studied
- Signal envelopes at output of relay systems can be written as product of two κ-μ random variables
- Therefore, probability density function and cumulative distribution function of product of two κ-μ random variables with different parameters are evaluated

System model



System model

System model

- In this paper, the wireless relay communication mobile radio system with SC receiver is considered
- Model of proposed system is shown in previous Figure

- The signal envelopes x_1 and x_2 follow κ-μ distribution:

$$p_{x_i}(x_i) = \frac{2\mu_{1i}(k_{1i}+1)^{\frac{\mu_{1i}+1}{2}} x_i^{\mu_{1i}}}{k_{1i}^{\frac{\mu_{1i}-1}{2}} e^{k_{1i}\mu_{1i}} \Omega_{1i}^{\frac{\mu_{1i}+1}{2}}} \cdot$$

$$\cdot e^{-\frac{\mu_{1i}(k_{1i}+1)}{\Omega_{1i}} x_i^2} I_{\mu_{1i}-1}\left(2\mu_{1i}\sqrt{\frac{k_{1i}(k_{1i}+1)}{\Omega_{1i}}} x_{1i}\right)$$

- By solving we have:

$$p_{x_i}(x_i) = \frac{2\mu_{1i}\left(k_{1i}+1\right)^{\frac{\mu_{1i}+1}{2}}}{k_{1i}^{\frac{\mu_{1i}-1}{2}} e^{k_{1i}\mu_{1i}} \Omega_{1i}^{\frac{\mu_{1i}+1}{2}}} \cdot$$

$$\cdot \sum_{k_1=0}^{\infty} \left(\mu_{1i} \sqrt{\frac{k_{1i}\left(k_{1i}+1\right)}{\Omega_{1i}}} \right)^{2k_1+\mu_{1i}-1} \cdot \frac{1}{k_1!\,\Gamma\left(k_1+\mu_{1i}\right)} \cdot$$

$$x_i^{2k_1+2\mu_{1i}-1} e^{-\frac{\mu_{1i}\left(k_{1i}+1\right)}{\Omega_{1i}} x_2^2}, \; x_i \geq 0, \;\; i = 1,2$$

- Random variables y_1 and y_2, also, follow κ-μ distribution:

$$p_{y_i}\left(y_i\right)=\frac{2\mu_{2i}\left(k_{2_i}+1\right)^{\frac{\mu_{2i}+1}{2}}}{k_{2i}^{\frac{\mu_{2i}-1}{2}}\, e^{k_{2i}\mu_{2i}}\,\Omega_{2i}^{\frac{\mu_{2i}+1}{2}}}\cdot$$

$$\cdot\sum_{k_2=0}^{\infty}\left(\mu_{2i}\sqrt{\frac{k_{2i}\left(k_{2i}+1\right)}{\Omega_{2i}}}\right)^{2k_2+\mu_{2i}-1}\cdot\frac{1}{k_2!\,\Gamma\left(k_2+\mu_{2i}\right)}\cdot$$

$$y_i^{2k_2+2\mu_{2i}-1}e^{-\frac{\mu_{2i}\left(k_{2i}+1\right)}{\Omega_{2i}}x_2^2},\ y_i\geq 0,\ \ i=1,2$$

- Random variable *x* can be evaluated as product of two κ-μ random variables:

$$x = x_1 \cdot x_2, \quad x_1 = \frac{x}{x_2}$$

- PDF of *x* is:

$$p_x(x) = \int_0^\infty dx_2 \cdot \frac{1}{x_2} p_{x_1}\left(\frac{x}{x_2}\right) p_{x_2}(x_2)$$

- After solving we have:

$$p_x(x) = \frac{2\mu_{11}(k_{11}+1)^{\frac{\mu_{11}+1}{2}}}{k_{11}^{\frac{\mu_{11}-1}{2}} e^{k_{11}\mu_{11}} \Omega_{11}^{\frac{\mu_{11}+1}{2}}} \cdot$$

$$\cdot \sum_{k_1=0}^{\infty} \left(\mu_{11} \sqrt{\frac{k_{11}(k_{11}+1)}{\Omega_{11}}} \right)^{2k_1+\mu_{11}-1} \cdot \frac{1}{k_1!\Gamma(k_1+\mu_{11})} \cdot x^{2\mu_{11}+2k_1-1} \cdot \frac{2\mu_{12}(k_{12}+1)^{\frac{\mu_{12}+1}{2}}}{k_{12}^{\frac{\mu_{12}-1}{2}} e^{k_{12}\mu_{12}} \Omega_{12}^{\frac{\mu_{12}+1}{2}}} \cdot$$

$$\cdot \frac{1}{2} \left(\frac{\mu_{11}(k_{11}+1)x^2}{\Omega_{11}} \frac{\Omega_{12}}{\mu_{12}(k_{12}+1)} \right)^{\frac{\mu_{12}}{2}+\frac{k_2}{2}-\frac{\mu_{11}}{2}-\frac{k_1}{2}} K_{\mu_{12}+k_2-\mu_{11}-k_1} \left(2\sqrt{\frac{\mu_{11}(k_{11}+1)x^2\mu_{12}(k_{12}+1)}{\Omega_{11}\Omega_{12}}} \right)$$

- Here, $K_n(x)$ is the modified Bessel function of the second kind with argument x and order n

- CDF of *x* is:

$$F_x(x)=\int_0^x dt\cdot p_x(t)=\frac{2\mu_{11}(k_{11}+1)^{\frac{\mu_{11}+1}{2}}}{k_{11}^{\frac{\mu_{11}-1}{2}}e^{k_{11}\mu_{11}}\Omega_{11}^{\frac{\mu_{11}+1}{2}}}\cdot\sum_{k_1=0}^{\infty}\left(\mu_{11}\sqrt{\frac{k_{11}(k_{11}+1)}{\Omega_{11}}}\right)^{2k_1+\mu_{11}-1}\cdot\frac{1}{k_1!\Gamma(k_1+\mu_{11})}\cdot$$

$$\cdot\frac{2\mu_{12}(k_{12}+1)^{\frac{\mu_{12}+1}{2}}}{k_{12}^{\frac{\mu_{12}-1}{2}}e^{k_{12}\mu_{12}}\Omega_{12}^{\frac{\mu_{12}+1}{2}}}\cdot\sum_{k_2=0}^{\infty}\left(\mu_{12}\sqrt{\frac{k_{12}(k_{12}+1)}{\Omega_{12}}}\right)^{2k_2+\mu_{12}-1}\cdot\frac{1}{k_2!\Gamma(k_2+\mu_{12})}\cdot$$

$$\cdot x^{2(\mu_{11}+k_1)}\frac{1}{\mu_{11}+k_1}\cdot\sum_{j_1=0}^{\infty}\frac{1}{(\mu_{11}+k_{11}+1)(j_1)}\left(\mu_{11}(k_{11}+1)\right)^{j_1}$$

$$\cdot\frac{1}{2}\left(\frac{\mu_{11}(k_{11}+1)\Omega_{12}x^2}{\mu_{12}(k_{12}+1)\Omega_{11}}\right)^{\frac{\mu_{12}}{2}+\frac{k_2}{2}-\frac{\mu_{11}}{2}-\frac{k_1}{2}-\frac{j_1}{2}}K_{\mu_{12}+k_2-\mu_{11}-k_1-j_1}\left(2\sqrt{\frac{\mu_{11}(k_{11}+1)x^2\mu_{12}(k_{12}+1)}{\Omega_{11}\Omega_{12}}}\right)$$

- Similarly, the PDF of *y* is:

$$p_y(y)=\frac{2\mu_{21}\left(k_{21}+1\right)^{\frac{\mu_{21}+1}{2}}}{k_{21}^{\frac{\mu_{21}-1}{2}}e^{k_{21}\mu_{21}}\Omega_{21}^{\frac{\mu_{21}+1}{2}}}\cdot\sum_{k_3=0}^{\infty}\left(\mu_{21}\sqrt{\frac{k_{21}\left(k_{21}+1\right)}{\Omega_{21}}}\right)^{2k_3+\mu_{21}-1}\cdot\frac{1}{k_3!\Gamma\left(k_3+\mu_{21}\right)}\cdot$$

$$\cdot\frac{2\mu_{22}\left(k_{22}+1\right)^{\frac{\mu_{22}+1}{2}}}{k_{22}^{\frac{\mu_{22}-1}{2}}e^{k_{22}\mu_{22}}\Omega_{22}^{\frac{\mu_{22}+1}{2}}}\cdot\sum_{k_4=0}^{\infty}\left(\mu_{22}\sqrt{\frac{k_{22}\left(k_{22}+1\right)}{\Omega_{22}}}\right)^{2k_4+\mu_{22}-1}\cdot\frac{1}{k_4!\Gamma\left(k_4+\mu_{22}\right)}\cdot y^{2k_3+2\mu_{21}-1}\cdot$$

$$\cdot\frac{1}{2}\left(\frac{\mu_{21}\left(k_{21}+1\right)y^2\Omega_{22}}{\Omega_{21}\mu_{22}\left(k_{22}+1\right)}\right)^{\frac{\mu_{22}}{2}+\frac{k_4}{2}-\frac{\mu_{21}}{2}-\frac{k_3}{2}}K_{\mu_{22}+k_4-\mu_{21}-k_3}\left(2\sqrt{\frac{\mu_{21}\left(k_{21}+1\right)y^2\mu_{22}\left(k_{22}+1\right)}{\Omega_{21}\Omega_{22}}}\right)$$

- The CDF of *y* is:

$$F_y(y)=\frac{2\mu_{21}(k_{21}+1)^{\frac{\mu_{21}+1}{2}}}{k_{21}^{\frac{\mu_{21}-1}{2}}e^{k_{21}\mu_{21}}\Omega_{21}^{\frac{\mu_{21}+1}{2}}}\cdot\sum_{k_3=0}^{\infty}\left(\mu_{21}\sqrt{\frac{k_{21}(k_{21}+1)}{\Omega_{21}}}\right)^{2k_3+\mu_{21}-1}\cdot\frac{1}{k_3!\Gamma(k_3+\mu_{21})}\cdot$$

$$\cdot\frac{2\mu_{22}(k_{22}+1)^{\frac{\mu_{22}+1}{2}}}{k_{22}^{\frac{\mu_{22}-1}{2}}e^{k_{22}\mu_{22}}\Omega_{22}^{\frac{\mu_{22}+1}{2}}}\cdot\sum_{k_4=0}^{\infty}\left(\mu_{22}\sqrt{\frac{k_{22}(k_{22}+1)}{\Omega_{22}}}\right)^{2k_4+\mu_{22}-1}\cdot\frac{1}{k_4!\Gamma(k_4+\mu_{22})}\cdot$$

$$y^{2(k_3+\mu_{21})}\frac{1}{\mu_{21}+k_3}\sum_{j_2=0}^{\infty}\frac{1}{(\mu_{21}+k_{21}+1)(j_2)}(\mu_{21}(k_{21}+1))^{j_2}$$

$$\cdot\frac{1}{2}\left(\frac{\mu_{21}(k_{21}+1)\Omega_{22}y^2}{\mu_{22}(k_{22}+1)\Omega_{21}}\right)^{\frac{\mu_{22}}{2}+\frac{k_4}{2}-\frac{\mu_{21}}{2}-\frac{k_3}{2}}K_{\mu_{22}+k_4-\mu_{21}-k_3}\left(2\sqrt{\frac{\mu_{21}(k_{21}+1)y^2\mu_{22}(k_{22}+1)}{\Omega_{21}\Omega_{22}}}\right)$$

- The outage probability is probability that communication relay system output signal envelope drops below the defined threshold
- The outage probability for this case is equal to the CDF of product of signal envelopes at sections

The CDF of SC Receiver Output Signal

- The SC receiver output signal is:

$$z = max(x, y)$$

- The CDF of *z* is:

$$F_z(z) = F_x(z) \cdot F_y(z)$$

- After solving it is valid:

$$F_z(z)=\frac{2\mu_{11}(k_{11}+1)^{\frac{\mu_{11}+1}{2}}}{k_{11}^{\frac{\mu_{11}-1}{2}}e^{k_{11}\mu_{11}}\Omega_{11}^{\frac{\mu_{11}+1}{2}}}\cdot\sum_{k_1=0}^{\infty}\left(\mu_{11}\sqrt{\frac{k_{11}(k_{11}+1)}{\Omega_{11}}}\right)^{2k_1+\mu_{11}-1}\cdot\frac{1}{k_1!\Gamma(k_1+\mu_{11})}\cdot$$

$$\cdot\frac{2\mu_{12}(k_{12}+1)^{\frac{\mu_{12}+1}{2}}}{k_{12}^{\frac{\mu_{12}-1}{2}}e^{k_{12}\mu_{12}}\Omega_{12}^{\frac{\mu_{12}+1}{2}}}\cdot\sum_{k_2=0}^{\infty}\left(\mu_{12}\sqrt{\frac{k_{12}(k_{12}+1)}{\Omega_{12}}}\right)^{2k_2+\mu_{12}-1}\cdot\frac{1}{k_2!\Gamma(k_2+\mu_{12})}\cdot$$

$$\cdot z^{2(\mu_{11}+k_1)}\frac{1}{\mu_{11}+k_1}\cdot\sum_{j_1=0}^{\infty}\frac{1}{(\mu_{11}+k_{11}+1)(j_1)}\left(\mu_{11}(k_{11}+1)\right)^{j_1}$$

$$\cdot\frac{1}{2}\left(\frac{\mu_{11}(k_{11}+1)\Omega_{12}z^2}{\mu_{12}(k_{12}+1)\Omega_{11}}\right)^{\frac{\mu_{12}}{2}+\frac{k_2}{2}-\frac{\mu_{11}}{2}-\frac{k_1}{2}}K_{\mu_{12}+k_2-\mu_{11}-k_1}\left(2\sqrt{\frac{\mu_{11}(k_{11}+1)z^2\mu_{12}(k_{12}+1)}{\Omega_{11}\Omega_{12}}}\right).$$

$$\cdot \frac{2\mu_{21}\left(k_{21}+1\right)^{\frac{\mu_{21}+1}{2}}}{k_{21}^{\frac{\mu_{21}-1}{2}} e^{k_{21}\mu_{21}} \Omega_{21}^{\frac{\mu_{21}+1}{2}}} \cdot \sum_{k_3=0}^{\infty} \left(\mu_{21} \sqrt{\frac{k_{21}\left(k_{21}+1\right)}{\Omega_{21}}} \right)^{2k_3+\mu_{21}-1} \cdot \frac{1}{k_3!\Gamma\left(k_3+\mu_{21}\right)} \cdot$$

$$\cdot \frac{2\mu_{22}\left(k_{22}+1\right)^{\frac{\mu_{22}+1}{2}}}{k_{22}^{\frac{\mu_{22}-1}{2}} e^{k_{22}\mu_{22}} \Omega_{22}^{\frac{\mu_{22}+1}{2}}} \cdot \sum_{k_4=0}^{\infty} \left(\mu_{22} \sqrt{\frac{k_{22}\left(k_{22}+1\right)}{\Omega_{22}}} \right)^{2k_4+\mu_{22}-1} \cdot \frac{1}{k_4!\Gamma\left(k_4+\mu_{22}\right)} \cdot$$

$$z^{2(k_3+\mu_{21})} \frac{1}{\mu_{21}+k_3} \sum_{j_2=0}^{\infty} \frac{1}{\left(\mu_{21}+k_{21}+1\right)\left(j_2\right)} \left(\mu_{21}\left(k_{21}+1\right)\right)^{j_2}$$

$$\cdot \frac{1}{2} \left(\frac{\mu_{21}\left(k_{21}+1\right)\Omega_{22} z^2}{\mu_{22}\left(k_{22}+1\right)\Omega_{21}} \right)^{\frac{\mu_{22}}{2}+\frac{k_4}{2}-\frac{\mu_{21}}{2}-\frac{k_3}{2}} K_{\mu_{22}+k_4-\mu_{21}-k_3} \left(2\sqrt{\frac{\mu_{21}\left(k_{21}+1\right) z^2 \mu_{22}\left(k_{22}+1\right)}{\Omega_{21}\Omega_{22}}} \right)$$

Numerical results

- The probability density function of SC receiver output signal envelope is plotted in next Figures for some values of fading severity parameter μ and Rician factors
- In the first Figure, fading severity parameter $\mu=2$ and Rician factors are $k_{ij}=1$; $i, j = 1, 2$
- In the second one, fading severity parameter $\mu=3$ and Rician factors are $k_{ij}=2$; $i, j = 1, 2$

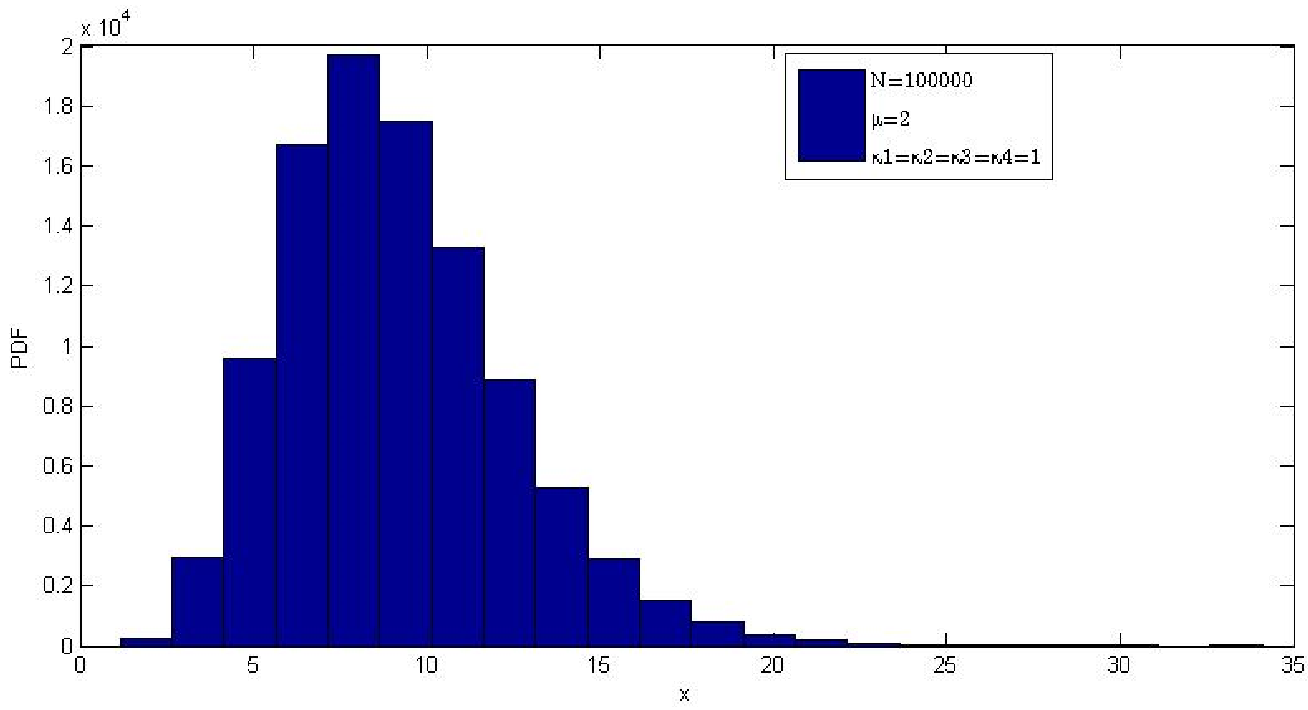


Fig. 2. PDF of *x* for μ=2 and k_{ij}=1

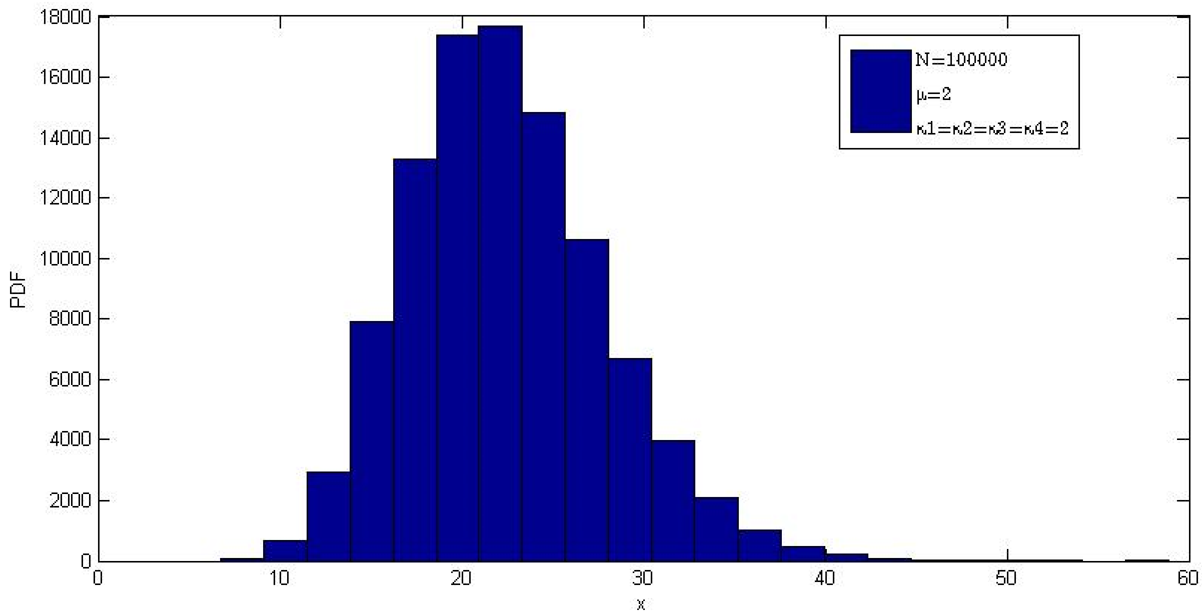
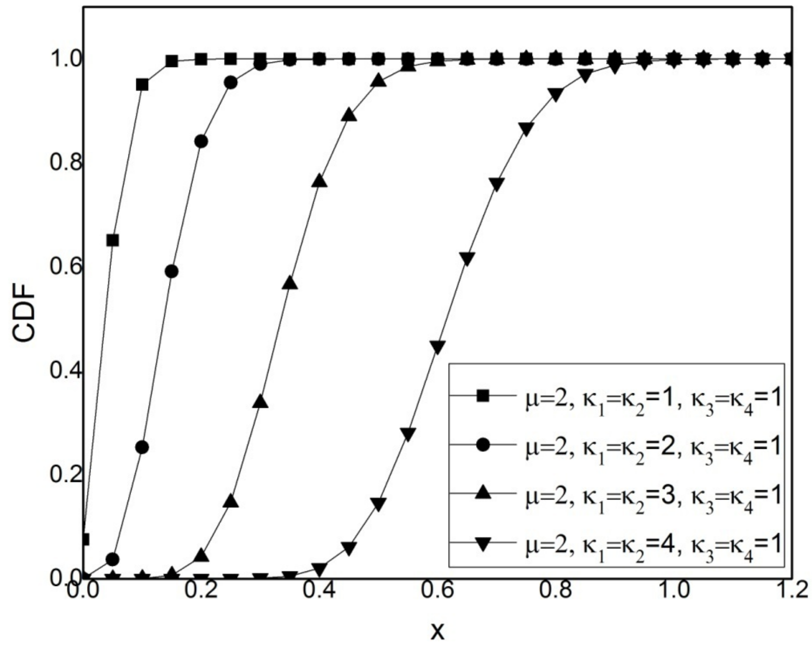
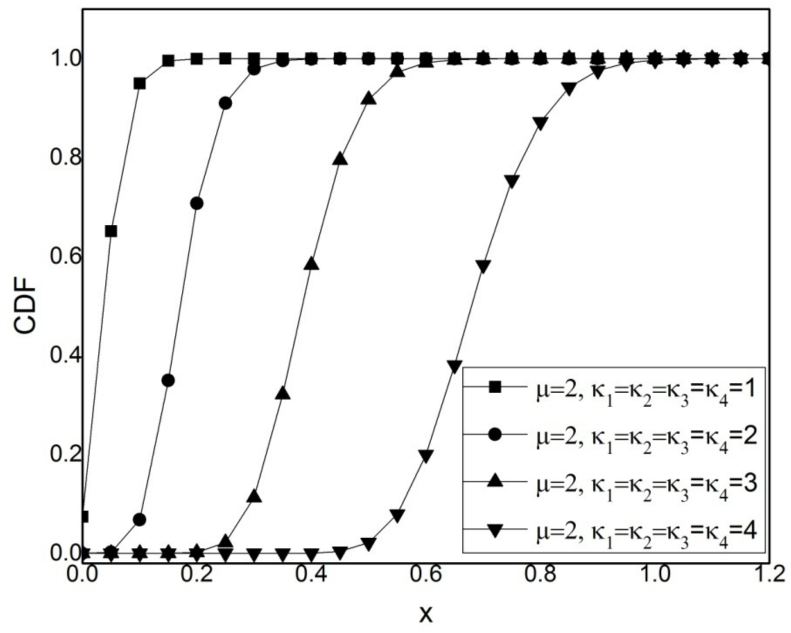


Fig. 3. PDF of *x* for μ=3 and k_{ij}=2

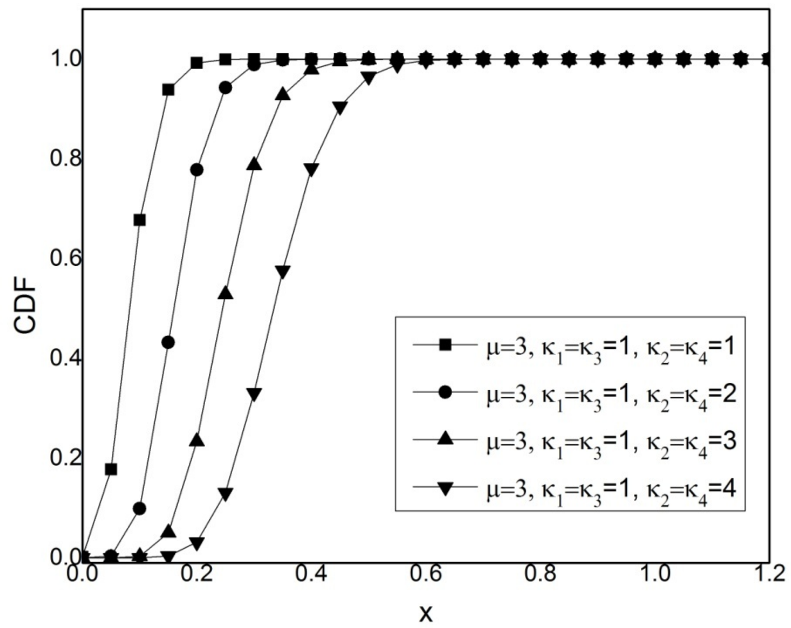
- The cumulative distribution functions of SC receiver output signal envelope are plotted in the next few Figures for different quantities of fading severity parameter μ and Rician factors
- In first some figures, parameter μ=2, and in last two, fading severity parameter μ=3
- The CDF is plotted for variable parameters κ



The cumulative distribution function of SC receiver output signal envelope



The CDF of SC receiver output signal envelope



The cumulative distribution function of SC receiver output signal envelope

- It is visible that CDF increases with increasing of the signal envelope
- The cumulative distribution function decreases for larger values of Rician factor κ_{ij}
- Also, one can see from these figures that CDF is smaller for bigger values of fading severity parameter μ
- System performances are better for lower values of the outage probability

Conclusion 6

- In this article, wireless system with two relay communication systems, both with two sections, whose outputs are inputs in SC receiver, in the presence of κ-μ short term fading in sections, is studied
- Signal envelopes at output of relay systems are products of two κ-μ random variables
- The probability density function and cumulative distribution function of products of two κ-μ random variables with different parameters are evaluated

- The signal envelope at output of proposed system is presented as maximum of signal envelopes at outputs of relay systems
- Then, probability density function, cumulative distribution function and outage probability of considered system are determined and the influence of Rician factors at sections on outage probability is analyzed and discussed

More references:

A. Panajotović, N. Sekulović, A. Cvetković, D. Milović, "*System performance analysis of cooperative multihop relaying network applying approximation to dual-hop relaying network*", *International Journal of Communication Systems,* 2020, *e4476.* doi:10.1002/dac.4476


Thank You

Any questions or comments please email me:

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