Direct Adaptive Control and Infinite Dimensional Quantum Systems

Mark's Autonomous Control Laboratory

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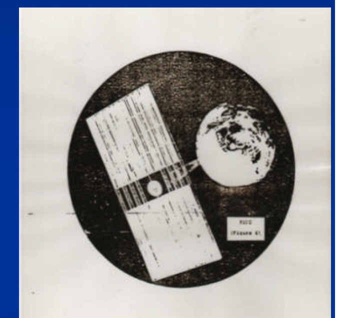
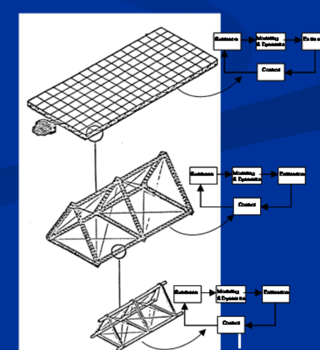
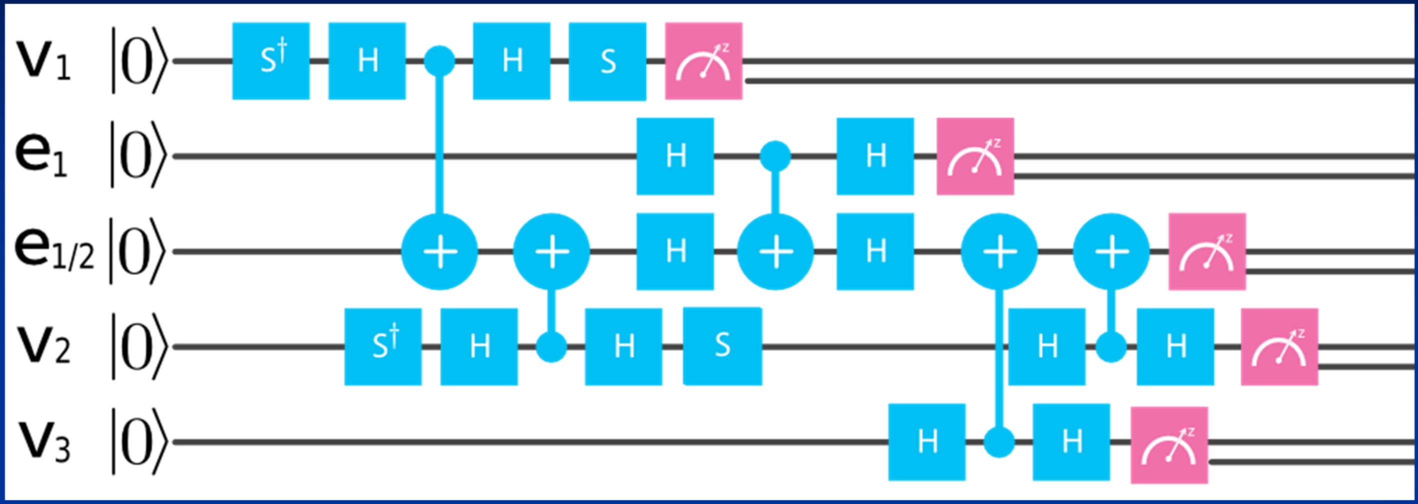




Practical Quantum Computers

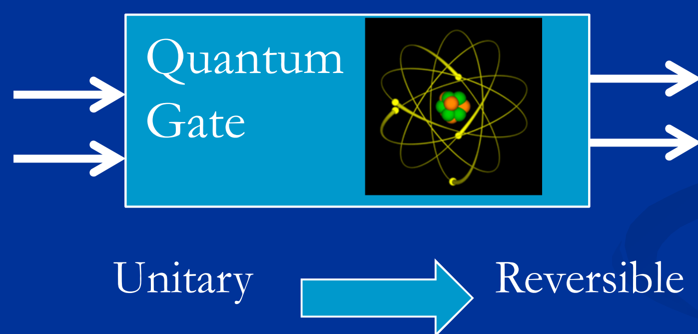


Requirements for a practical quantum computer:

- scalable physically to increase the number of qubits;
- qubits that can be initialized to arbitrary values;
- quantum gates that are faster than decoherence time;
- universal gate set;
- qubits that can be read easily.

Quantum Computing

"A Quantum computer will operate differently from a Classical one. It will be involved w physical systems on an atomic scale, eg atoms, photons, trapped ions, or nuclear magnetic moments"
… R. Feynman 40 years ago

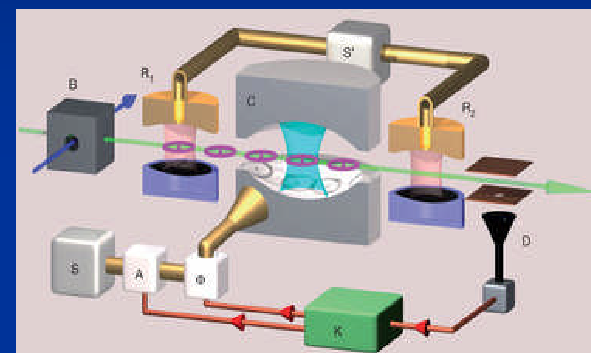


Decoherence is the loss of information from a system into the environment. Entanglements are generated between the system and environment, which have the effect of sharing quantum information with—or transferring it to—the surroundings

Reduced with Infinite Dimensional Direct Adaptive Control
(And Quantum Error Correction)

Small Quantum Systems

- We can begin to experiment with just one electron, atom or small molecule



- Need:

Precise control

Isolation from the environment

Simple small systems : single particles or small groups of particles

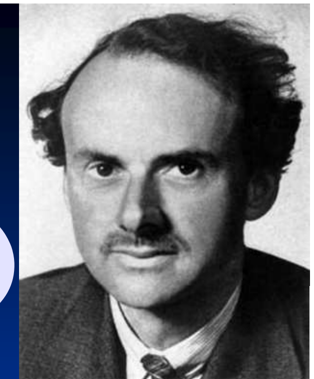
…… David Wineland NIST

Physics Nobel Prize 2012

S. Haroche & D. Wineland

What really happens

Quantum Basics (Dirac & Von Neumann)



Observable $A : X \xrightarrow[]{\substack{bounded/unbounded \\ self-adjoint \\ compact}} X$

Orthonormal Eigen –Basis for X

$$Ax = \sum_{k=1}^{\infty} \lambda_k \underbrace{(x, \phi_k)\phi_k}_{P_k x} = \sum_{k=1}^{\infty} \lambda_k P_k x \ \& \ \sigma(A) \equiv \left\{ \underbrace{\lambda_1, \lambda_2, \lambda_3, \ldots}_{\substack{Observed \\ Values\ of\ A}} \right\}$$

<u>Pure States</u>: ϕ_k eigenfunctions of A

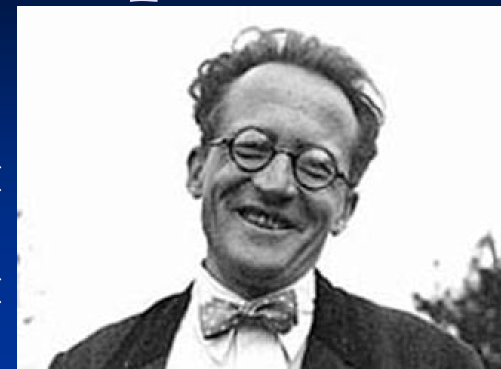
State $\phi \in X$ <u>complex</u> infinite-dimensional separable Hilbert Space:

$$(\phi, \phi) = 1 \text{ or } \|\phi\| = 1 \Rightarrow \phi = \sum_{k=1}^{\infty} c_k \phi_k \ \& \ \|\phi\|^2 = \sum_{k=1}^{\infty} |c_k|^2 = 1$$

$\therefore$ "A (mixed) state is a linear combination of pure states"

Special Case: Quantum SPIN Systems are FINITE Dimensional

Dynamics: Schrodinger Wave Equation



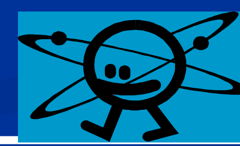
$\phi \in X$ complex Hilbert Space

$$i\hbar \frac{\partial \phi}{\partial t} = \underbrace{H_0}_{\text{Hamiltonian Energy Operator}} \phi \qquad \text{Discrete Spectrum } \sigma(H_0) = \{\lambda_k\}_{k=1}^{\infty}$$

$$\Rightarrow \phi(t) = \underbrace{U_0(t)}_{\text{Unitary Group}} \phi(0) = e^{-\frac{i}{\hbar}H_0 t}\phi(0) = \sum_{k=1}^{\infty} e^{-\frac{i\lambda_k}{\hbar}t}(\phi(0),\phi_k)\phi_k \text{ with } (\phi_k,\phi_l) = \delta_{kl}$$

$\therefore \|\phi(t)\|^2$ = Probability Distribution for the Energy

in the Quantum State ϕ(t) $\Rightarrow \|\phi(t)\| = \|\phi(0)\|$



$-\infty$

Marginally Stable

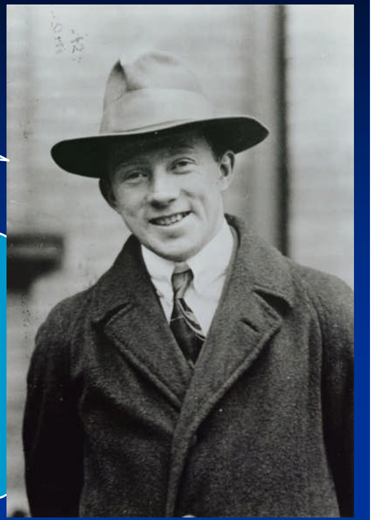
$\Rightarrow \therefore \|\phi(t)\|^2$ = Probability Distribution for the Energy

in the Quantum State ϕ(t)

$\Rightarrow \|\phi(t)\| = \|\phi(0)\|$

QuantumMeasurement

The interpretation of Quantum Measurement is still a controversial part of Quantum Theory



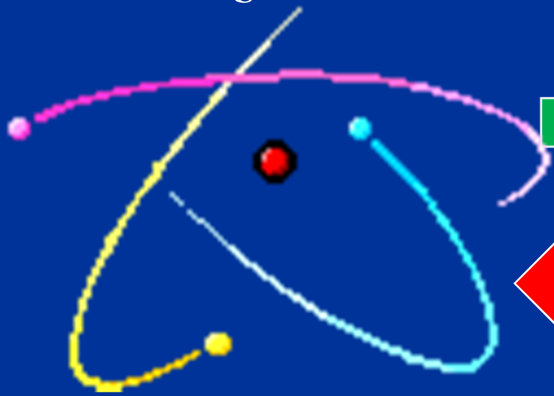
Entanglement

A quantum measurement is an entanglement with the environment (measuring device)

$$X = X_S \otimes X_M$$

$$\phi = \sum_{k,l} \alpha_{kl} (\phi_k^S \otimes \phi_l^M) \neq h \otimes w$$

S

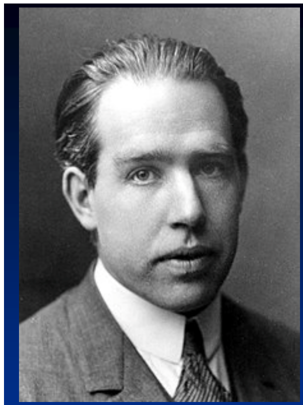


M

Back Action

Heisenberg Uncertainty Principle

$$(\Delta z)^2 (\Delta p)^2 \geq \left| (\underbrace{[z, p]}_{\frac{i\hbar}{2}} \phi, \phi) \right| = (\frac{\hbar}{2})^2 ; \hbar \approx 10^{-34}$$



Neils Bohr

Quantum Collapse:

Ontology vs Epistemology

Observable $A : X \xrightarrow[\text{self-adjoint}]{\text{bounded/unbounded}} X$

$$Ax = \sum_{k=1}^{\infty} \lambda_k \underbrace{(x, \phi_k)\phi_k}_{P_k x}$$

<u>Pure States</u>: ϕ_k eigenfunctions of A

Max Born

An <u>observation/measurement</u> of the observable A produces

a <u>collapse</u> of the wave function for a mixed state $\phi = \sum_{k=1}^{\infty} c_k \phi_k$

into one of the pure eigenstates ϕ_k $(A\phi_k = \lambda_k \phi_k)$with probability $|c_k|^2$

Sally Shrapnel, Fabio Costa, & Gerard Milburn , "Updating the Born Rule", New Journal of Physics, 20, 2018 (a linear quantum probability rule)

Control of Quantum Master Equation

A **density operator** describes a quantum system in a *mixed state*, a statistical ensemble of several quantum states

$\rho \geq 0$, symmetric (Hermitian) operator with Trace$\rho = 1$

$$\rho = \sum_{k=1}^{\infty} \rho_k \underbrace{\phi_k(\phi_k, \square)}_{P_k}; \quad Trace(\rho) \equiv \sum_{k=1}^{\infty} \underbrace{\rho_k}_{\geq 0} = 1 \text{ (convex combination of pure observables)}$$

Commutator

$$\frac{\partial}{\partial t}\rho = -\frac{i}{\hbar}[\rho, H] \equiv -\frac{i}{\hbar}(\rho H - H\rho) = \frac{i}{\hbar}[H, \rho]$$

$\rho \equiv$ density operator

$$H = H_0 + H_{environment} + H_{interactions} + H_{control}$$

Master Equation and Expectation

Master Equation

$$\frac{\partial}{\partial t}\rho = -\frac{i}{\hbar}[\rho, H_0]$$

$$\Rightarrow \rho(t) = U_0^*(t)\rho(t)U_0(t) \text{ where } U_0(t) \equiv e^{-\frac{i}{\hbar}H_0 t} \text{unitary group}$$

Expectation of $\rho = \langle \rho \rangle \equiv (\phi, \rho\phi)$

$$\Rightarrow \frac{\partial}{\partial t}\langle \rho \rangle = \frac{\partial}{\partial t}(\phi, \rho\phi) = -\frac{i}{\hbar}(\phi, [H_0, \rho]\phi) = -\frac{i}{\hbar}\langle [H_0, \rho] \rangle$$

$$\therefore \langle \rho(t) \rangle = (\phi, U_0^*(t)\rho(0)U_0(t)\phi) = (U_0(t)\phi, \rho(0)U_0(t)\phi) = \langle \rho(0) \rangle$$

"Simplicity" via Infinite Dimensional Spaces

$$\begin{cases} \dfrac{\partial x}{\partial t} = Ax + Bu = Ax + \sum_{i=1}^{m} b_i u_i ;\ A \text{ generates a } C_0 \text{ semigroup} \\ x(0) = x_0 \in D(A) \subset X \\ y = Cx = \left[(c_1, x) \quad (c_2, x) \quad \ldots \quad (c_m, x) \right]^* ; b_i, c_j \in D(A) \end{cases}$$

$$\Rightarrow x(t, w_0) = \underbrace{U(t)x_0}_{\substack{Evolution \\ in\ X}} ; \forall t \geq 0$$

Eliminate all the special properties of $\Re^N$

C_0 – Semigroup of Bounded Operators $U(t)$:

$$\begin{cases} U(t+s) = U(t)U(s) \ \text{(semigroup property)} \\ \dfrac{d}{dt} U(t) = AU(t) = U(t)A \ (A \text{ generates } U(t)) \\ U(t)x_0 \xrightarrow[t \to 0]{} x_0 \ (\text{continuous at t} = 0) \end{cases}$$

J. Wen & M.Balas, "Robust Adaptive Control in Hilbert Space ", J. Mathematical. Analysis and Applications, Vol 143, pp 1-26,1989.

J. Wen & M.Balas ,"Direct Model Reference Adaptive Control in Infinite-Dimensional Hilbert Space," Chapter in Applications of Adaptive Control Theory, Vol.11, K. S. Narendra, Ed., Academic Press, 1987

Semigroups

Closed Linear Operator

Solve $\begin{cases} \dfrac{\partial x}{\partial t} = Ax \\ x(0) = x_0 \in D(A) \end{cases} \Rightarrow x(t) = U(t)x_0$

$\dim X < \infty \Rightarrow U(t) = e^{At} \equiv \sum_{k=0}^{\infty} A^k \frac{t^k}{k!}$

$\underline{C_0 - \text{Semigroup}}$

$U(t): X \to X$ bounded operators $t \geq 0$

$\underline{\text{Generator}}$: $Ax = \lim_{t \to 0+} \frac{U(t)x - x}{t}$ with $D(A) \equiv \{x / \lim_{t \to 0+} \text{ exists }\}$ dense in X

LaPlace Transform $\begin{cases} L(U(t)) = (\lambda I - A)^{-1} \equiv R(\lambda, A) \ \underline{\text{Resolvent Operator}} \\ L^{-1}(R(\lambda, A)) = U(t) \end{cases}$

Spectrum of A

Resolvent Set $\rho(A) \equiv \{\lambda / R(\lambda, A) : X \to X \text{ bounded linear op on } X\}$

Spectrum $\sigma(A) \equiv \rho(A)^{C} = \sigma_{\text{point}}(A) \cup \sigma_{cont}(A) \cup \sigma_{residual}(A)$

$\sigma_{point}(A) \equiv \{\lambda / \lambda I - A \text{ is NOT 1-1}\} = \{\lambda / \exists \phi \neq 0 \ni \lambda\phi = A\phi\}$

$\sigma_{cont}(A) \equiv \{\lambda / \lambda I - A \text{ is 1-1, but its range is only dense in } X\}$

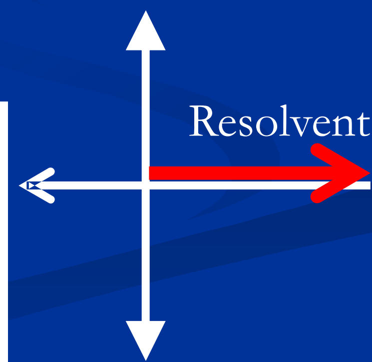
$\sigma_{residual}(A) \equiv \{\lambda / \lambda I - A \text{ is 1-1, but range is a proper subspace of } X\}$

Theorem (Gearhart,Pruss,&Greiner) :

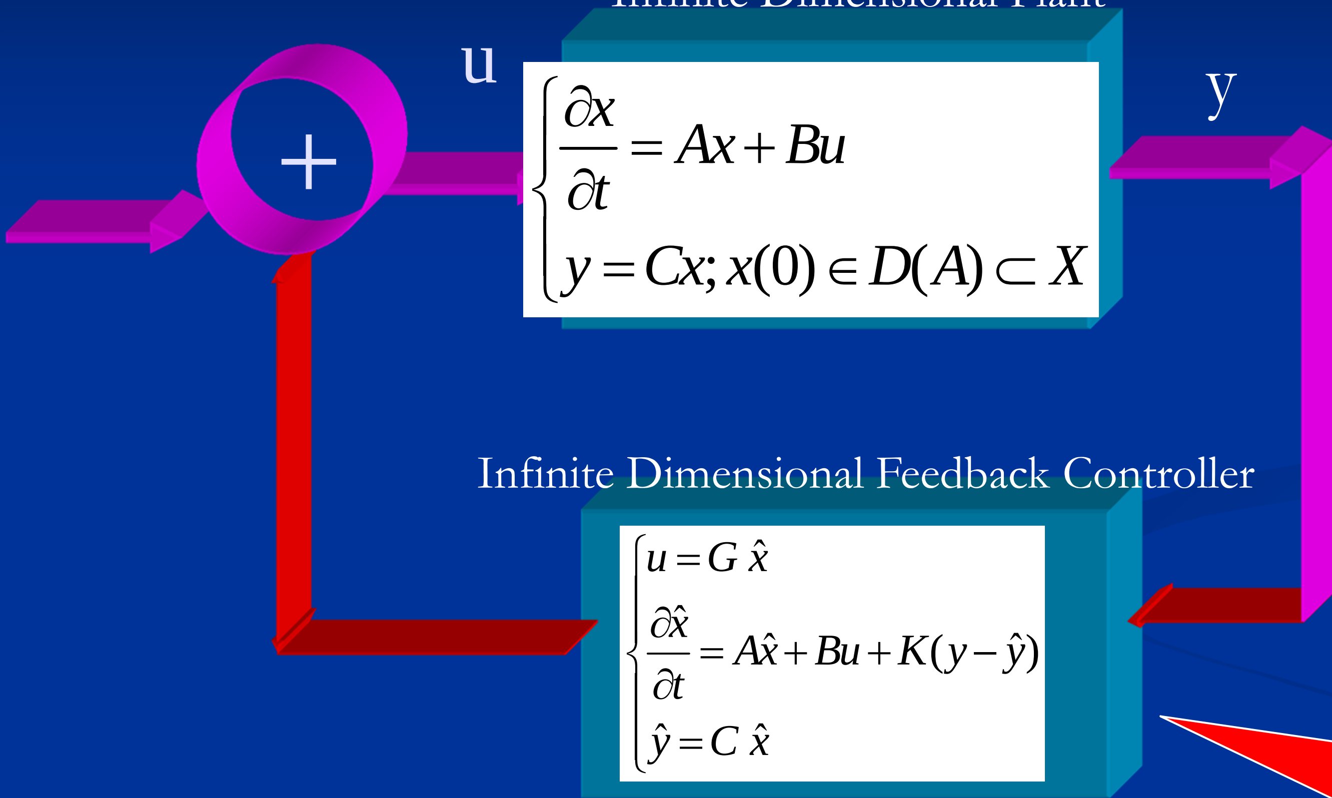
Assume A generates a C_0-semigp $U(t)$ on a Hilbert space X.

$U(t)$ is exponentially stable $\Leftrightarrow$ $\text{Re}\lambda > 0 \Rightarrow \lambda \in \rho(A)$ and

$\|R(\lambda, A)\| \leq M < \infty$, for all such complex λ



State Estimator-Based Feedback Control of Infinite Dimensional Systems

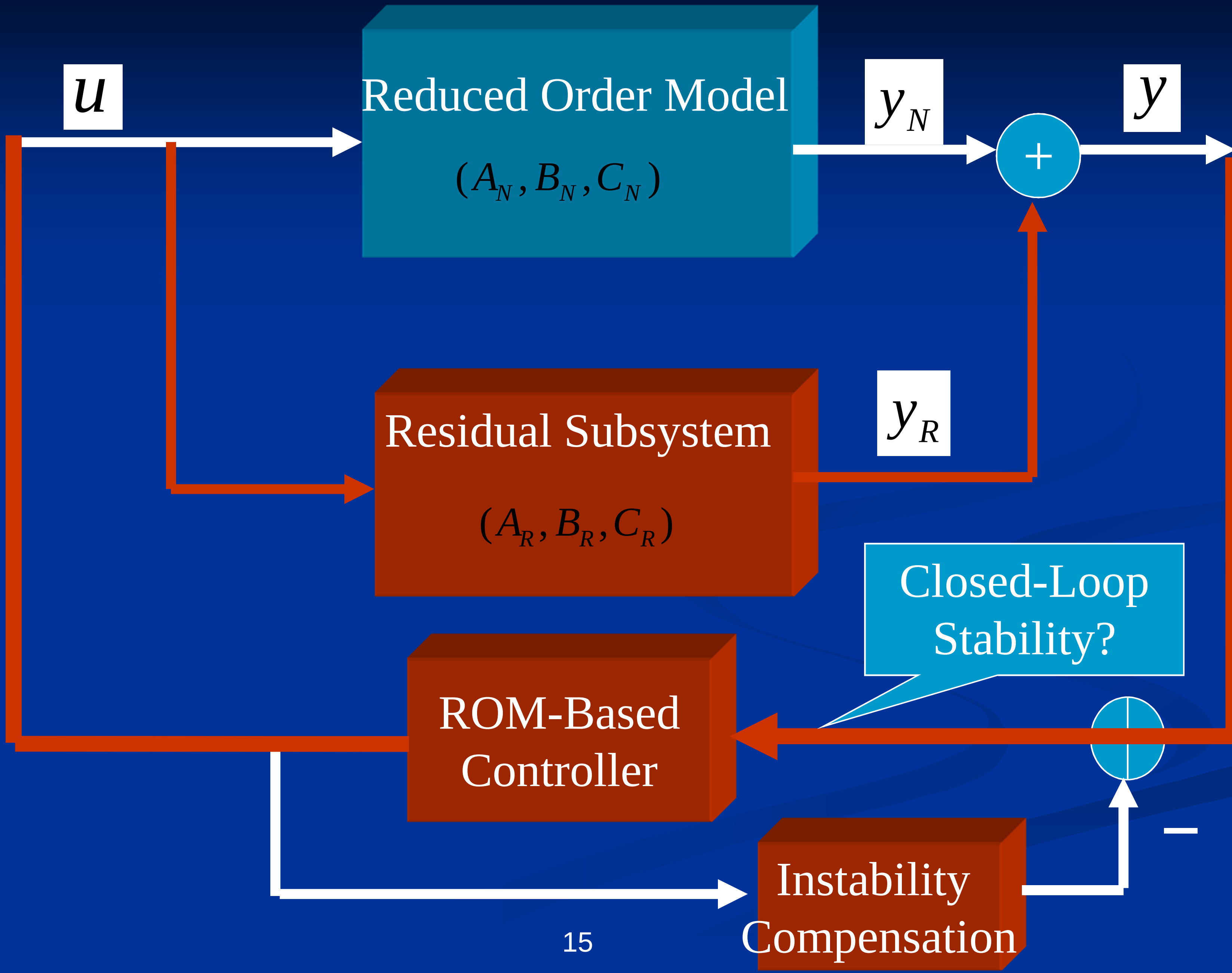


Practicality: Need a Realizable <u>Finite-dimensional</u> Controller

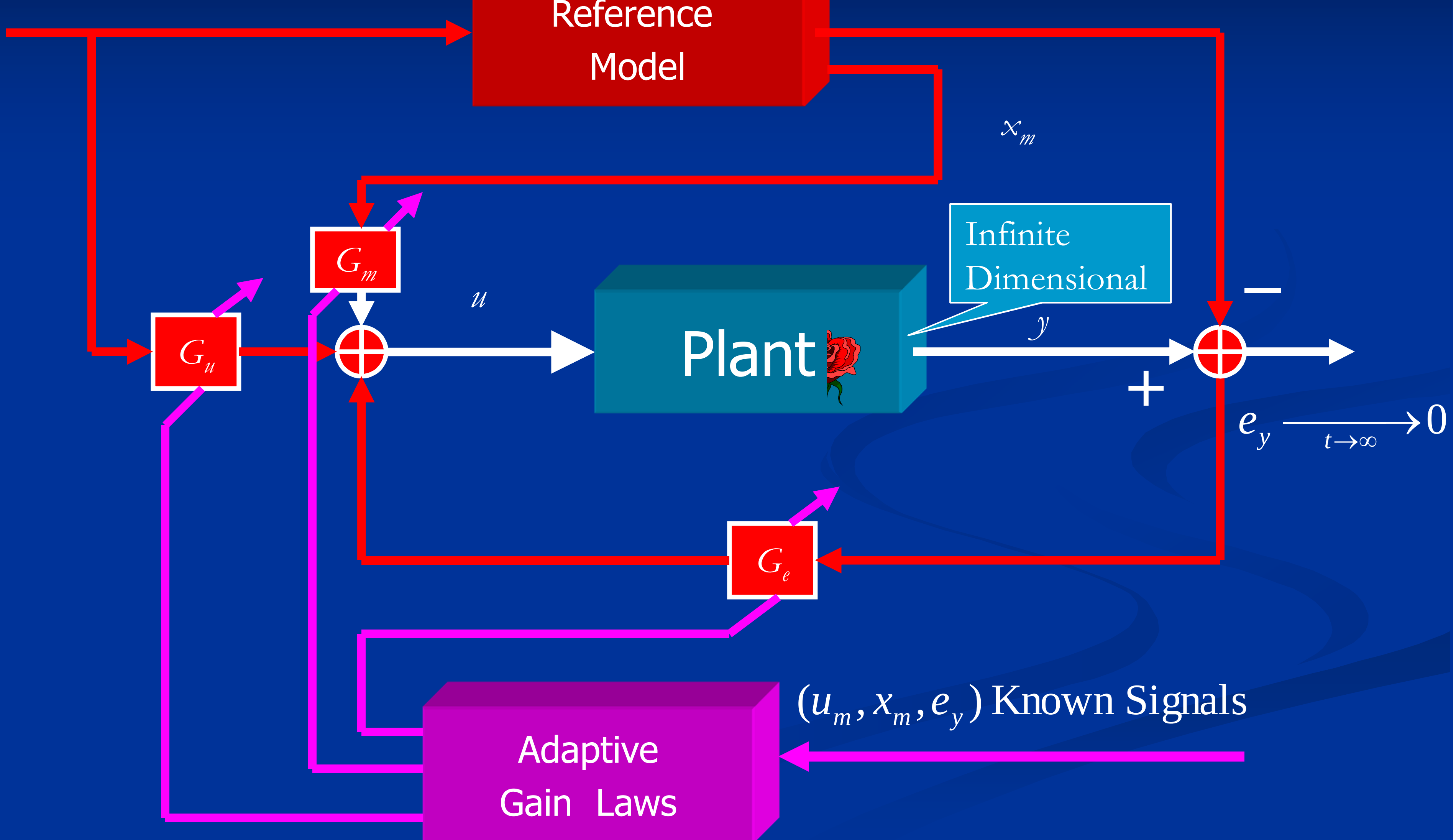
M. Balas, "Exponentially Stabilizing Finite Dimensional Controllers for Linear Distributed Parameter Systems: Galerkin Approximation of Infinite Dimensional Controllers', JMAA, Vol 117, 1986

L. Arccardi, Quantum Kalman Filters, <u>Mathematical System Theory</u>, Springer, 1991

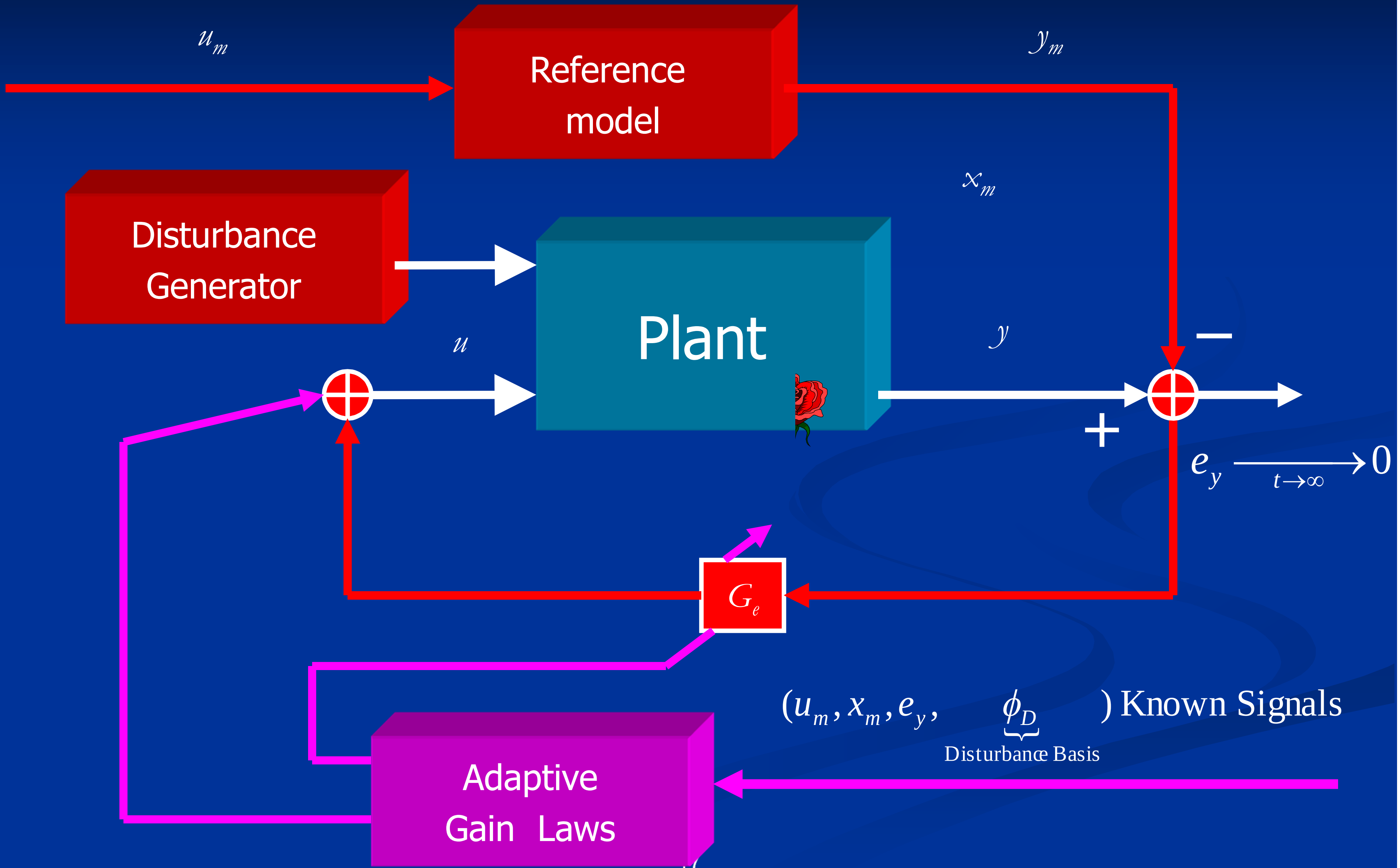
Model Reduction for Control



Direct Adaptive Model Following Control
(Wen-Balas 1989)



Direct Adaptive Persistent Disturbance Rejection (Fuentes-Balas 2000)



Linear System Strict Dissipativity

<u>Energy Storage Function</u> : $\begin{cases} V(x) \equiv (x, Px) > 0; \forall x \neq 0 \\ V(0) = 0 \end{cases}$

A <u>Linear</u> Dynamic Infinite-Dimensional System is STRICTLY DISSIPATIVE when

$$\exists P : X \xrightarrow[\text{Coercive}]{\substack{\text{Bounded Linear Op} \\ \text{Self-Adjoint}}} X \quad \text{HilbertSpace}$$

$$p_{\min} \|x\|^2 \leq V(x) \equiv (Px, x) \leq p_{\max} \|x\|^2 \ni$$

$$\begin{cases} \operatorname{Re}(PAx, x) \equiv \frac{1}{2}[(PAx, x) + (x, PAx)] \leq \underbrace{-\alpha \|x\|^2}_{W(x)}; \forall x \in D(A) \\ PB = C^* \end{cases}$$

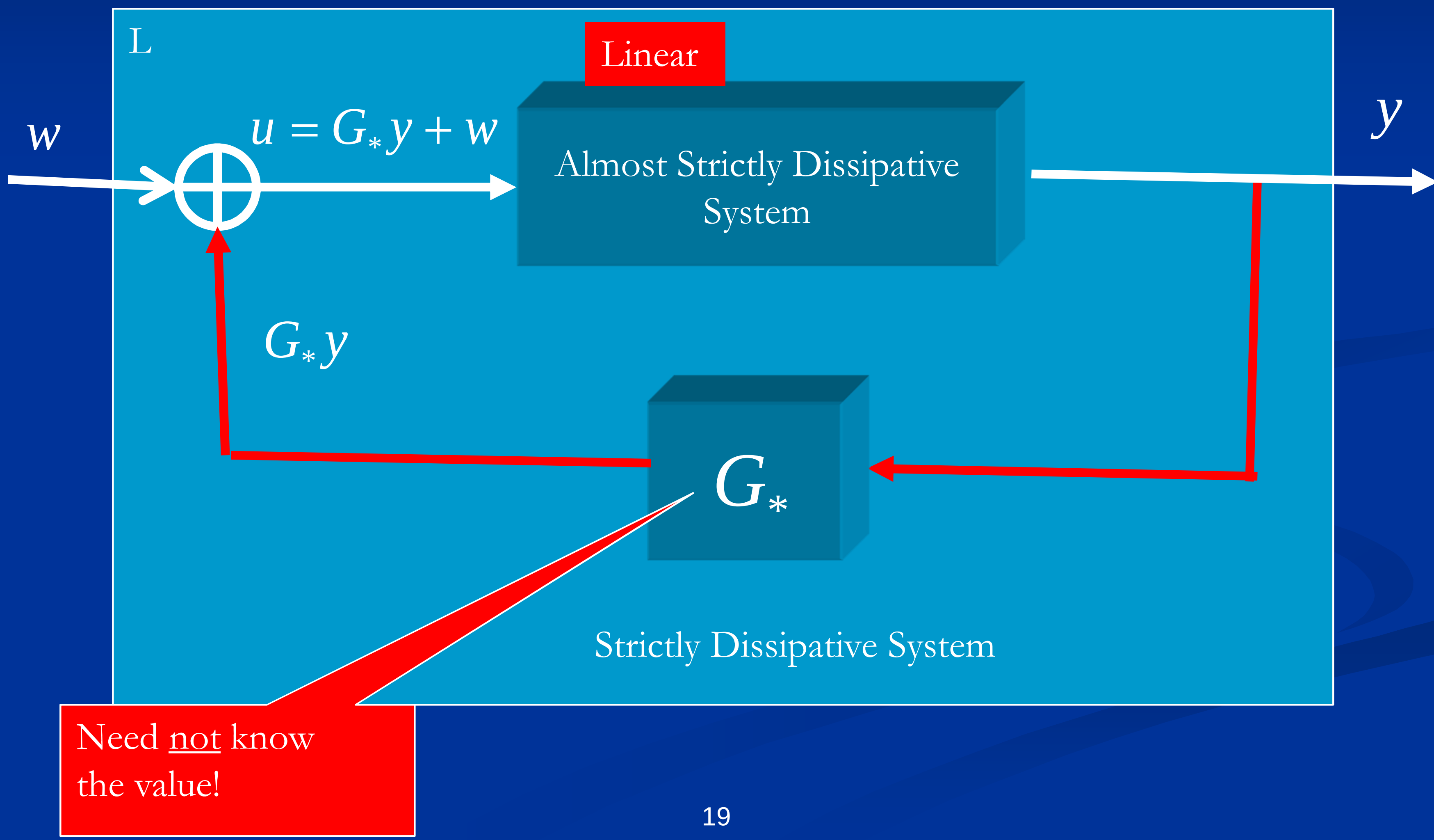
DISSIPATIVE when α=0

$$\Rightarrow \frac{1}{2} \underbrace{\frac{dV}{dt}}_{\substack{\text{Energy} \\ \text{Storage} \\ \text{Rate}}} = \underbrace{\operatorname{Re}(Px, x)}_{\leq -\alpha \|x\|^2} + \underbrace{(x, PBu)}_{(y,u)} \leq \underbrace{(y, u)}_{\substack{\text{External} \\ \text{Power}}} - \underbrace{\alpha \|x\|^2}_{\substack{\text{Internally} \\ \text{Dissipated} \\ \text{Power}}}$$

Almost Strictly Dissipative (ASD) Systems

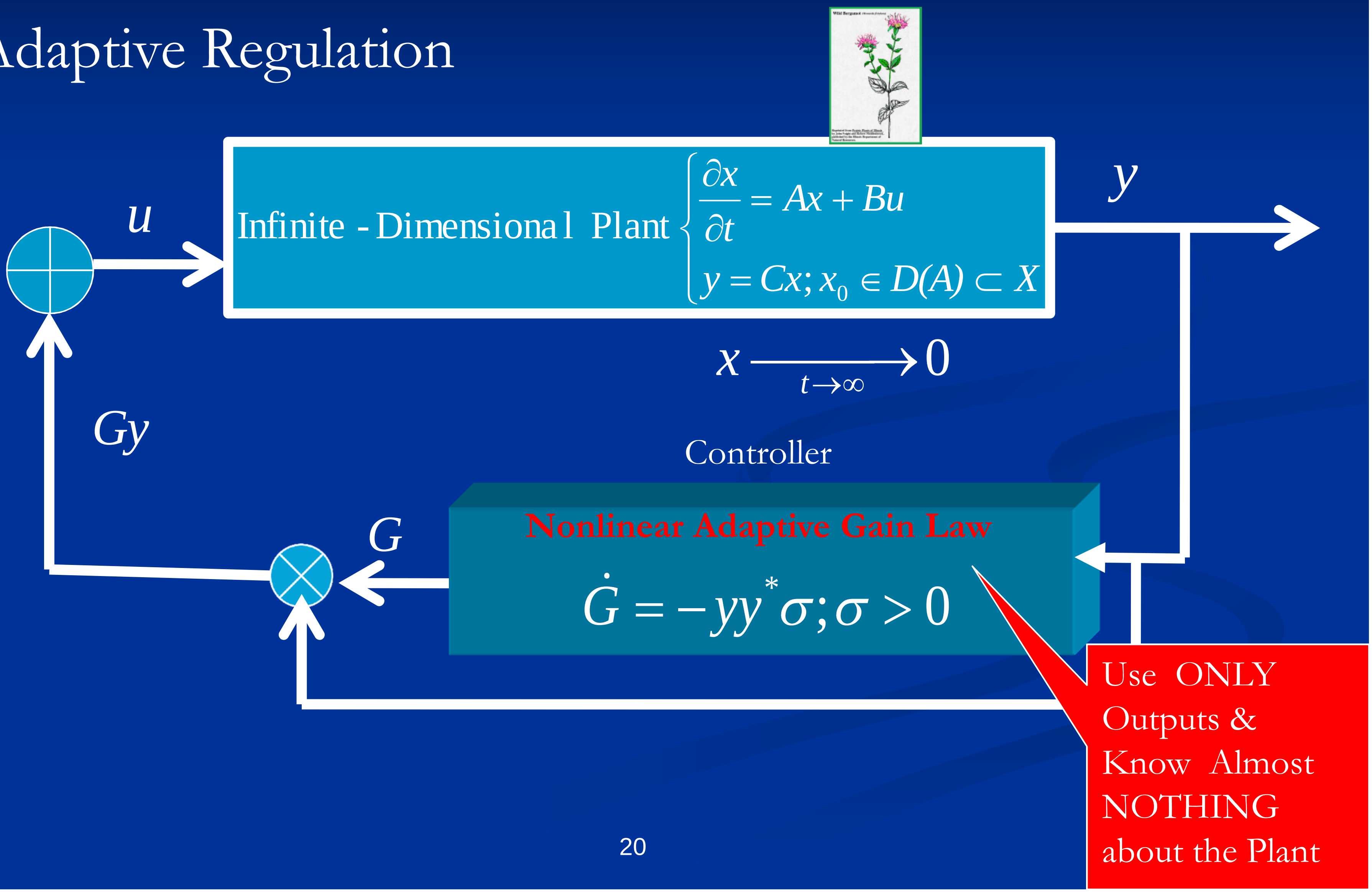
(A, B, C) ASD means

$\exists G_* \ni (A_C \equiv A + BG_*C, B, C)$ Strictly Dissipative



Direct Adaptive Control Theory Is Not Complicated !

Adaptive Regulation



For Finite & Infinite Dimensions
All Roads Lead To Rome

Control Porno

$$\begin{cases} \dfrac{\partial x}{\partial t} = Ax + Bu = Ax + \sum_{i=1}^{m} b_i u_i \\ x(0) = x_0 \in D(A) \subset X \\ y = Cx = \left[(c_1, x) \quad (c_2, x) \quad \ldots \quad (c_m, x) \right]^T \end{cases}$$

with (A, B, C) Almost Strictly Dissipative (ASD)

$\Rightarrow$ Adaptive Controller $\begin{cases} u = G(t)y \\ \dot{G}(t) = -yy^{*}\sigma; \sigma > 0 \end{cases}$

produces $x(t) \xrightarrow[t\to\infty]{} 0$

with bounded adaptive gains $G(t)$

Finite- Dimensional LINEAR ASD: Two Simple Open-Loop Properties

High Frequency Gain is Sign-Definite (CB>0)

Open-Loop Transfer Function is Minimum Phase (i.e. Transmission Zeros are all stable)

⟺ Almost Strictly Dissipative

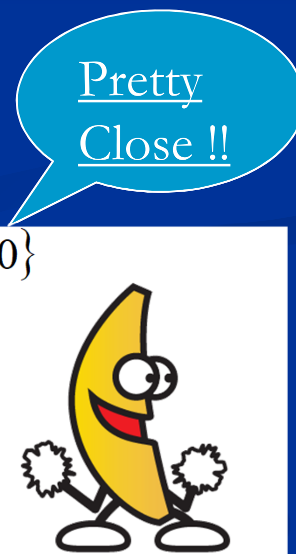
⟹ Adaptive Regulation $\begin{cases} u = Gy \\ \dot{G} = -yy^{*}\sigma; \sigma > 0 \end{cases}$

produces $x(t) \xrightarrow[t \to \infty]{} 0$

with bounded adaptive gains $G(t)$

Our Infinite-Dimensional Version of the "Two Simple Open Loop Properties" Theorem

$$\begin{cases} \dfrac{\partial x}{\partial t} = Ax + Bu = Ax + \sum_{i=1}^{m} b_i u_i \,;\, A \text{ generates a } C_0 \text{ semigroup} \\ x(0) = x_0 \in D(A) \subset X \\ y = Cx = \left[(c_1, x) \quad (c_2, x) \quad \ldots \quad (c_m, x) \right]^* ; b_i, c_j \in D(A) \end{cases}$$



Theorem : Def : $\lambda_* \in C$ is a transmission zero of (A,B,C) when $N(H(\lambda_*)) \neq \{0\}$

where $H(\lambda) \equiv \begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix} : D(A) x \Re^M \to X x \Re^M$ closed linear operator

(A,B,C) is Almost Strictly Dissipative if and only if

$CB = \left[(c_j, b_i) \right]_{mxm} > 0$ and Transmission Zeros$(A,B,C) \equiv \{ \lambda / N(H(\lambda)) \neq \{0\} \} = \sigma_p(\overline{A}_{22})$ "*stable*"

(i.e., $\overline{A}_{22}$ generates exponentially stable semigroup)

Mark Balas and Susan Frost, "Robust Adaptive Model Tracking for Distributed Parameter Control of Linear Infinite-dimensional Systems in Hilbert Space",IEEE/CAA JOURNAL OF AUTOMATICA SINICA, VOL. 1, NO. 3, JULY 2014.

Adaptive Model Tracking in the Presence of Persistent Disturbances

Reference Model

$$\begin{cases} \dot{x}_m = A_m x_m + B_m u_m \\ y_m = C_m x_m \end{cases}$$

y_m

$u_D = \theta L \phi_D$

$$\begin{cases} \dfrac{\partial}{\partial t} x = Ax + Bu + \Gamma u_D \\ y = Cx \end{cases}$$

u

y

–

$e_y \xrightarrow[t \to \infty]{} 0$

$\Delta x \equiv x - x_* \xrightarrow[t \to \infty]{} 0$

Controller

Adaptive Gain Law

$$\dot{G} = \left[\dot{G}_e \quad \dot{G}_u \quad \dot{G}_m \quad \dot{G}_D \right] = -h(e_y, u_m, y_m, \phi_D)$$

Can also be Infinite Dimensional

Adaptive Control Law

$$u = \underbrace{G_u u_m + G_m x_m}_{\text{Model Tracking}} + \underbrace{G_D \phi_D}_{\text{Disturbance Rejection}} + \underbrace{G_e e_y}_{\text{Stabilization}}$$

where

$$\begin{cases} \dot{G}_u = -e_y u_m^* \sigma_u ; \sigma_u > 0 \\ \dot{G}_m = -e_y x_m^* \sigma_m ; \sigma_m > 0 \\ \dot{G}_D = -e_y \phi_D^* \sigma_D ; \sigma_D > 0 \\ \dot{G}_e = -e_y e_y^* \sigma_e ; \sigma_e > 0 \end{cases}$$

Gain Adaptation Laws

Existence of Ideal Trajectories

Find Bounded Linear Operators S_1 & S_2 ∍

$$\begin{cases} x_* = S_{11}^* x_m + S_{12}^* u_m + S_{13}^* z_D = S_1 z \\ u_* = S_{21}^* x_m + S_{22}^* u_m + S_{23}^* z_D = S_2 z \end{cases} \text{ with } z \equiv \begin{bmatrix} x_m \\ u_m \\ z_D \end{bmatrix}$$

satisfying <u>Matching Conditions</u> $\begin{cases} AS_1 + BS_2 = S_1 L_m + H_1 \\ CS_1 = H_2 \end{cases}$

$$\Rightarrow \begin{cases} \dfrac{\partial}{\partial t} x_* = Ax_* + Bu_* + \Gamma u_D \\ y_* = Cx_* = y_m \end{cases}$$

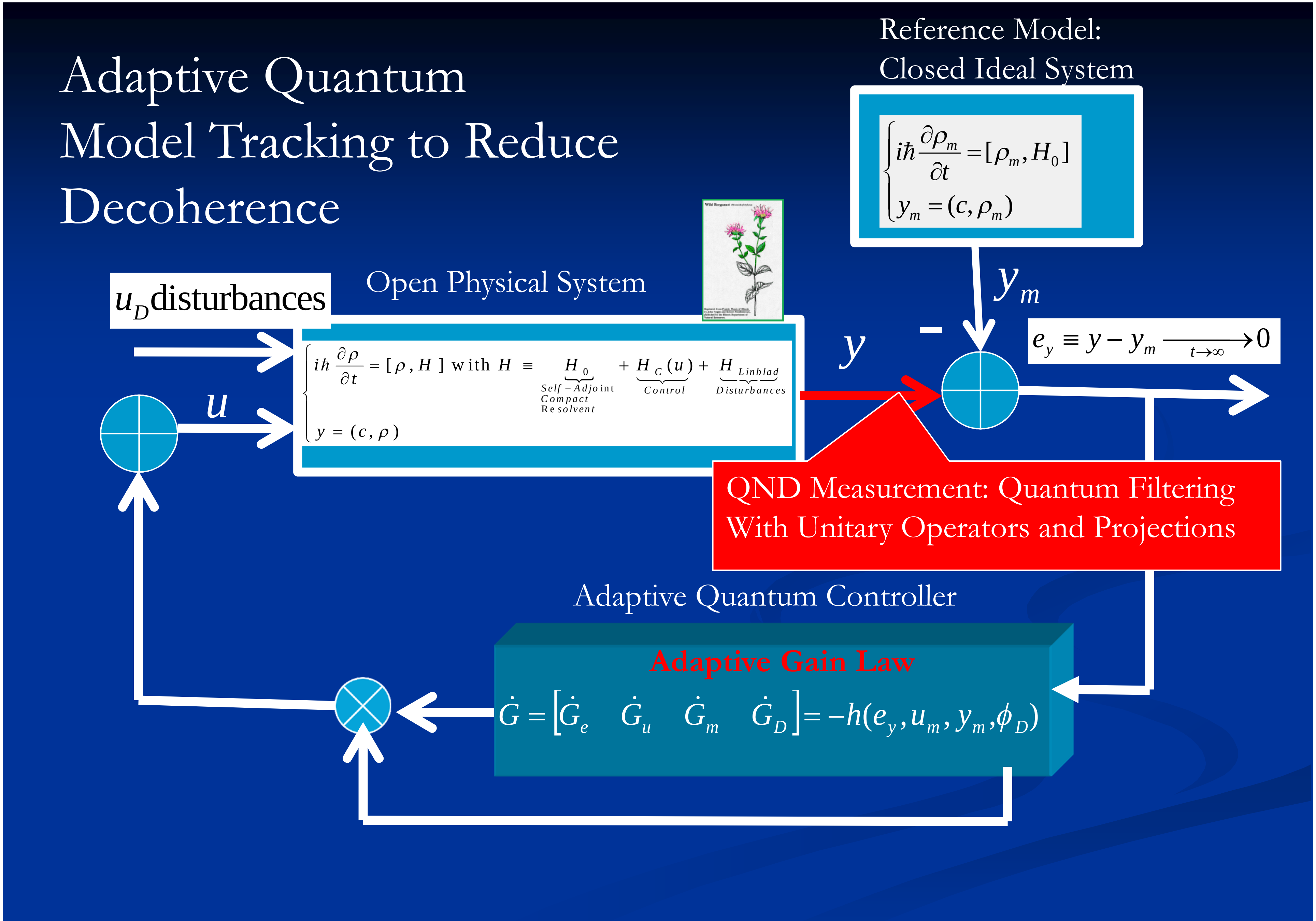
<u>Theorem</u>: Assume CB nonsingular.

$\exists(S_1, S_2)$ satisfying the Matching Conditions

$\Leftrightarrow$ The Spectrum of the Reference Model & Disturbance Generator shares no common points with

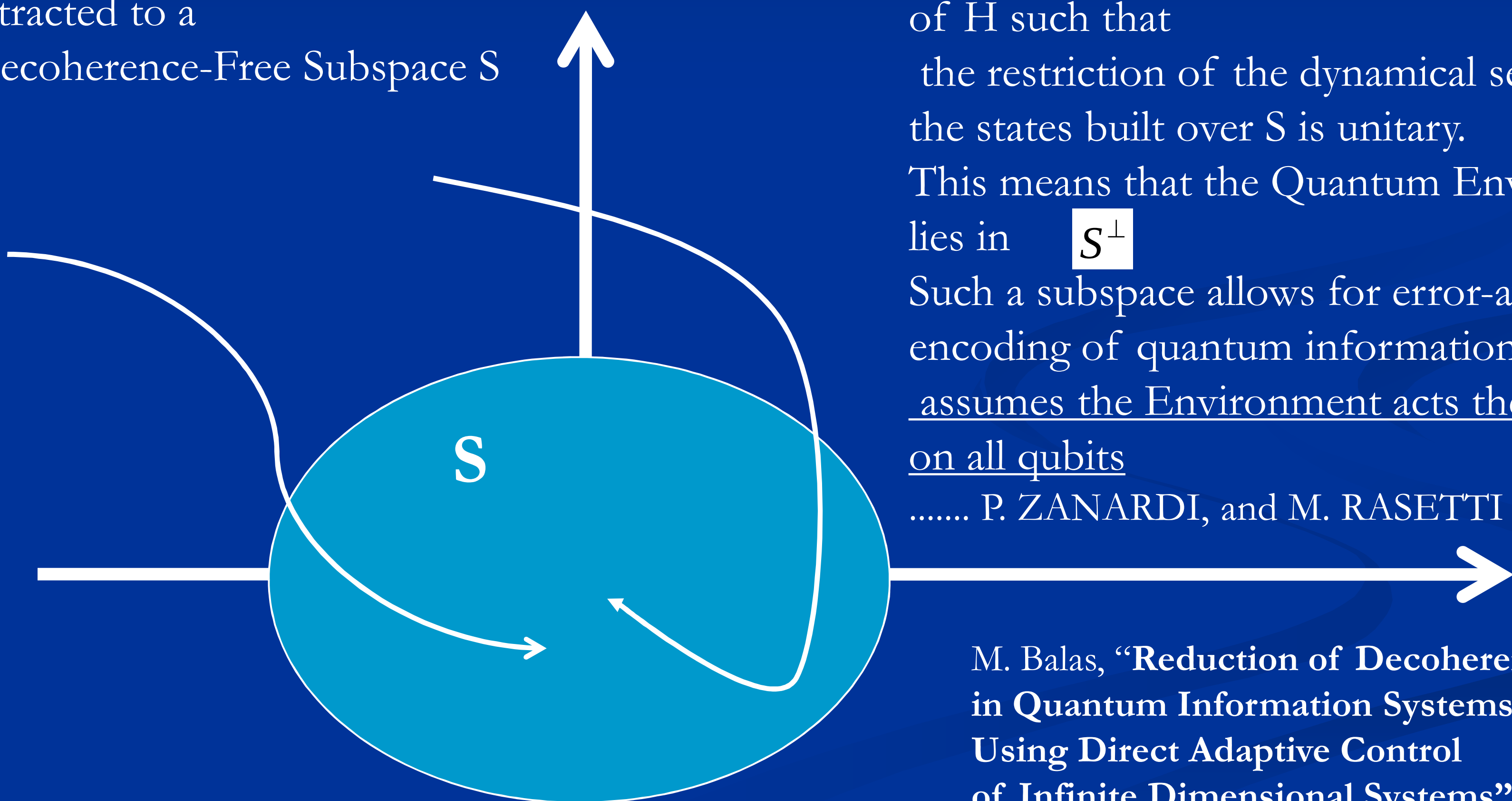
the Transmission Zeros of $(A, B, C) : \sigma(\mathrm{L}_m) \cap Z(A, B, C) = \phi$

Mark Balas and Susan Frost, “Robust Adaptive Model Tracking for Distributed Parameter Control of Linear Infinite-dimensional Systems in Hilbert Space”,IEEE/CAA JOURNAL OF AUTOMATICA SINICA, VOL. 1, NO. 3, JULY 2014.


Adaptive Quantum Model Tracking to Reduce Decoherence
Reference Model: Closed Ideal System
$i\hbar \frac{\partial \rho_m}{\partial t} = [\rho_m, H_0]$
$y_m = (c, \rho_m)$
u_D disturbances
Open Physical System
$i\hbar \frac{\partial \rho}{\partial t} = [\rho, H]$ with $H \equiv \underbrace{H_0}_{\text{Self-Adjoint Compact Resolvent}} + \underbrace{H_C(u)}_{\text{Control}} + \underbrace{H_{Linblad}}_{\text{Disturbances}}$
$y = (c, \rho)$
u
y
y_m
$-$
$e_y \equiv y - y_m \xrightarrow[t\to\infty]{} 0$
QND Measurement: Quantum Filtering With Unitary Operators and Projections
Adaptive Quantum Controller
Adaptive Gain Law
$\dot{G} = [\dot{G}_e \quad \dot{G}_u \quad \dot{G}_m \quad \dot{G}_D] = -h(e_y, u_m, y_m, \phi_D)$

Robust Adaptive Control : Convergence to a Decoherence-Free Subspace

All quantum trajectories attracted to a Decoherence-Free Subspace S



$$d(\varphi(t), S) \equiv \inf_{x \in S} \|\varphi(t) - x\| \xrightarrow[t \to \infty]{} 0$$

A class of open quantum systems that admits a linear subspace S of H such that the restriction of the dynamical semigroup to the states built over S is unitary.

This means that the Quantum Environment lies in $S^{\perp}$

Such a subspace allows for error-avoiding encoding of quantum information, <u>but assumes the Environment acts the same on all qubits</u>

……. P. ZANARDI, and M. RASETTI 1997

M. Balas, "**Reduction of Decoherence in Quantum Information Systems Using Direct Adaptive Control of Infinite Dimensional Systems**", ICAS 2020

Quantum Cognition

WTF ?

<u>QuantumProbability:</u>

EventSpace : X <u>complex</u> (infinite-dimensional, separable) Hilbert Space

$X = \overline{span\{\phi_1, \phi_2, \phi_3, ...\}}$ orthonormal basis $(\phi_k, \phi_l) = \delta_{kl}$

Events $\equiv$ Closed Subspaces S of X (or their Projections)

$S_k \equiv span\{\phi_k\}$ basic subspace

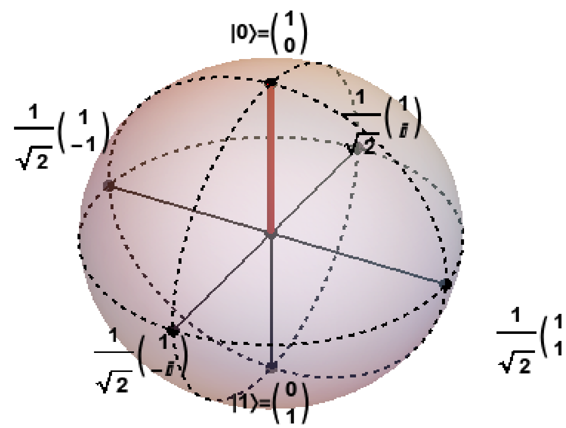
Mixed States: $x = \sum_{k=1}^{\infty} \underbrace{(x, \phi_k)\phi_k}_{P_k x}$ & $\|x\|^2 = 1$

Superposition of Projections

$$\sum_{k=1}^{\infty} P_k = I$$

<u>Quantum Probability:</u>

$p(x \in S_k) \equiv \|P_k x\|^2 = |(x, \phi_k)|^2 = |c_k|^2$



Model of
Human Decision-Making with
Non-commuting Projections

2019 NSF Proposal: A Quantum Approach to Human Cognition and the Autonomy Conundrum in Self Driving Vehicles, PI James Hubbard, Co PIs Theodora Chaspari, Mark Balas

"We don't know where we are stupid until we stick our necks out"
……………………..Richard Feynman