Deep Reinforcement Learning for Power Grid Operations

ENERGY 2020 Tutorial

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Motivation

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Why more AI in the Power Grid?

- Power prid operations increase in complexity
  - More DERs
  - New market concepts, e.g., local markets
  - Anciallary services also from DERs, also market-based
- AI technologies already widespread
  - Forecasting
  - Multi-Agent Systems (mostly rule-based)
  - Distributed heuristics (e.g., schedule planning)
- Resilience: Reaction for the “unknown unknowns”
- Bottom line: **Dynamic strategy development needed**; Deep Reinforcement Learning (DRL) is the next meta-level

A Gentle Introduction to Reinforcement Learning

About *Reinforcement Learning*

DRL in Relation to other Terms in Deep Learning

- **Model-based Learning:** ANN develops problem model (vs. *Instance-based Learning*)
- Supervised Learning
  - Classification
  - Regression
- Unsupervised Learning
  - Clustering
- Reinforcement Learning

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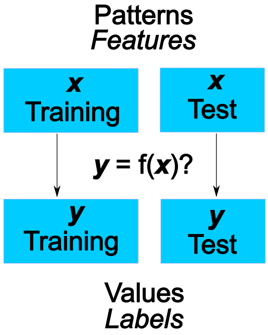

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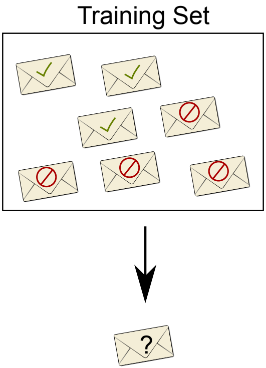

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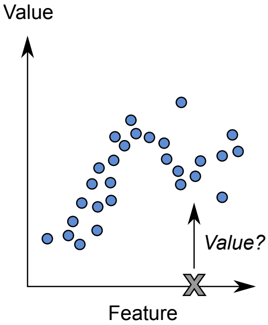

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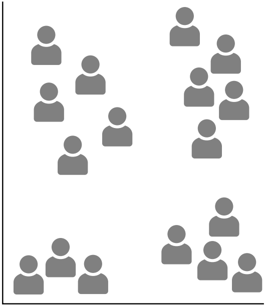

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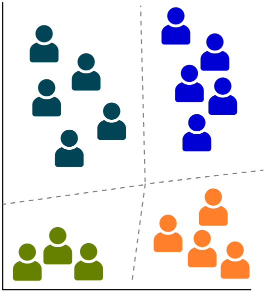

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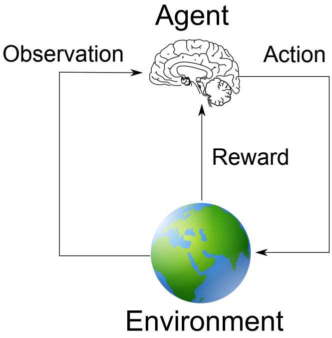

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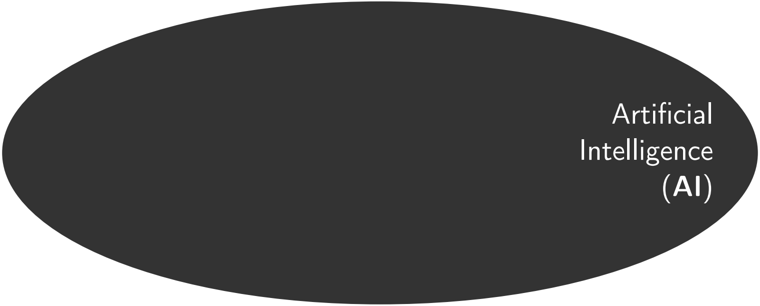

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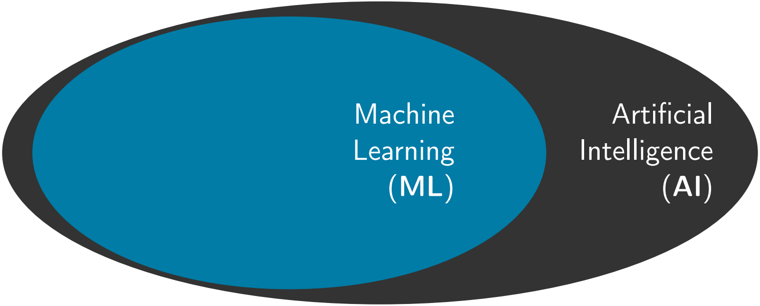

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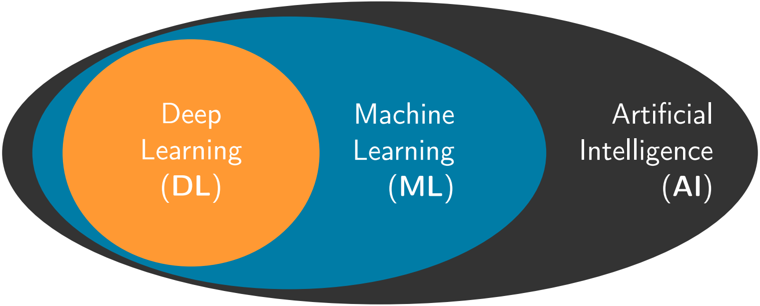

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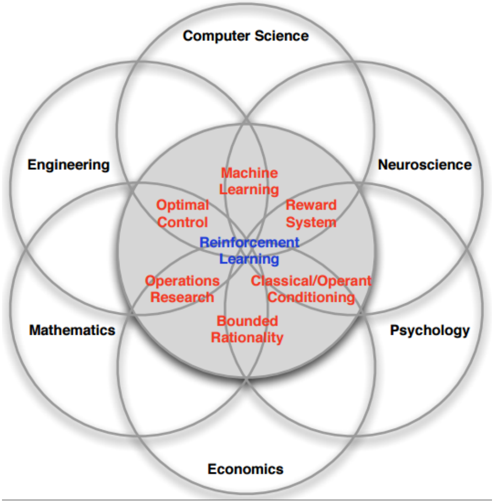
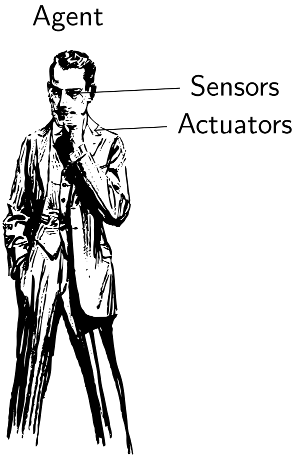


(David Silver)

Basic Terminology

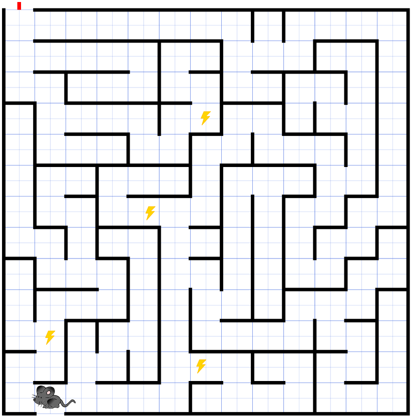
Agent, Sensors, Actuators



- **Agent:** Acting Entity
- Through **Sensors,** the Agent perceives its environment
- . . . which it acts upon with its **Actuators**.

Basic Terminology

Agent, Sensors, Actuators: An Example



- **Agent:** Mouse
- **Sensors:** Board (encoding?)
- **Actuators:** Forward, backward, turn $\pm 90°$

Basic Terminology

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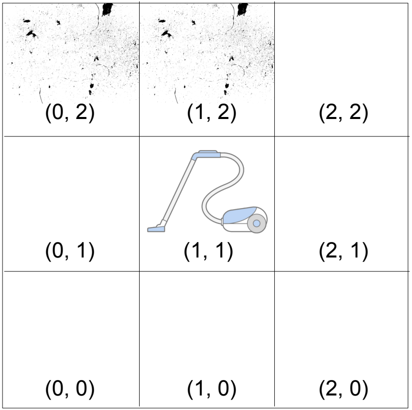

- **Agent:** Vacuum bot
- **Sensoren:** Area immediately in front of the bot
  - Encoding: *dirty* $\in \{yes, no\}$
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Basic Terminology

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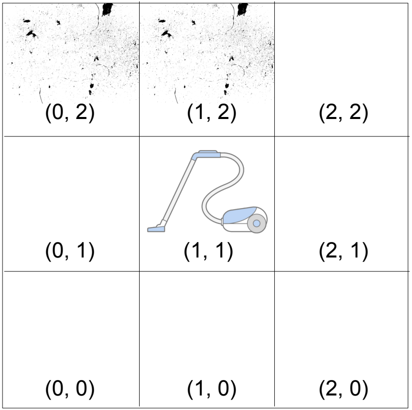

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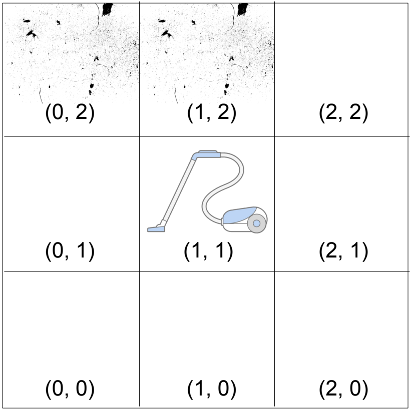

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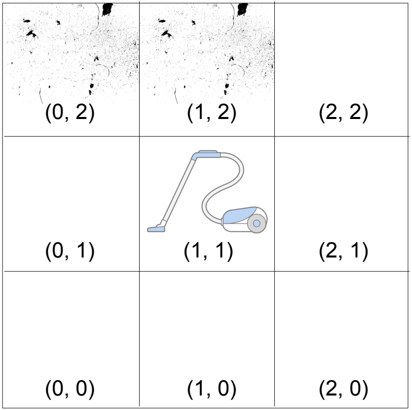

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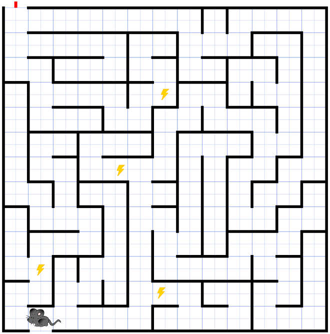
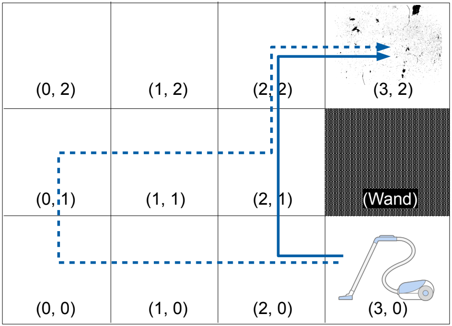
  - Slippage?

Reward

Feedback for the Agent

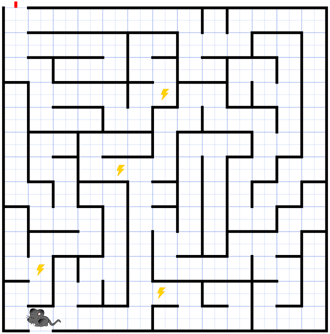


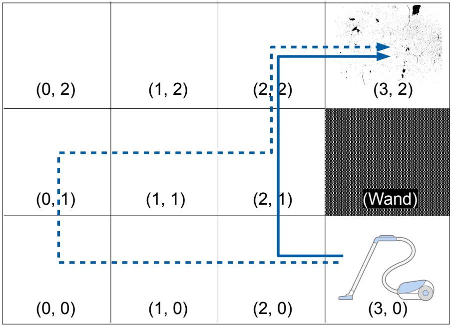


> What route do mouse and bot take?

Reward

Feedback for the Agent





- What route do mouse and bot take?
- . . . or, even more interisting: *Why* do mouse/bot take a particular route?

Reward

Feedback to the Agent

- **Reward:** Feedback from the environment about the agent's action regarding the agent's goal
- "Reward *reinforces* the agent to do the right thing."
- **Scalar:** Unitless, no futher form — big, small, positive, negative, ...
- No requirements to frequency; most common: per fixed t, per action
- **Local:** Rewards the immediate action
- *Training* based on reward (directly or indirectly)
  Problem: associating actions and rewards (e.g., bank robbery: high immediate reward, long-term: not so good)

Examples for Reward Values

Stock Trading: Profits/Losses

Chess: Values of a chess piece, value of a position, result of a game (ELO; or simply win: +1, draw: 0, loss: -1)

Dopamine Level: Biological reward: Joy

Vacuum Bot: Fill state of the dust tank

Arcade: +1 for every frame survived, +1 for every enemy overcome, . . .

Web Crawler: Information gain

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Power Grid Voltage band

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Web Crawler Information gain

Power Grid Voltage band, CO_2, MW from DER, line losses avoided , rel. self-supply, ...

Caution Agent maximizes reward — not always the same as succeeding at an objective

Markov Process

Model for Observable Systems

- System with N **states**
- **State Space**

$$\boldsymbol{S} = \{s_1, s_2, \dots, s_N\} \tag{1}$$

- **Markov Property:** Chain without memory
  - Let $Y = (X_t)_{t \in \mathbb{N}}$ be a space of random numbers, $X_t \in \boldsymbol{S}$
  - Y is a markov chain, iff:

$$P(X_{t+1} = s_{j_{t+1}} \mid X_t = s_{j_t}, X_{t-1} = s_{j_{t-1}}, \dots, X_0 = s_{j_0}) \tag{2}$$

$$= P(X_{t+1} = s_{j_{t+1}} \mid X_t = s_{j_t}). \tag{3}$$

- **Transition Probabilities:**

$$p_{ij}(t) := P(X_{t+1} = s_j \mid X_t = s_i), \quad i,j = 1, \dots, m \tag{4}$$

- **Transitions Matrix:**

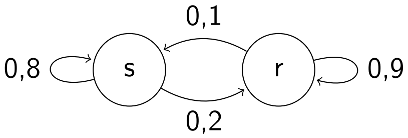
$$\mathbf{M}(t) = (p_{ij}(t))_{s_i, s_j \in S}, \ |\mathbf{M}| = N \times N \tag{5}$$

Weather Prediction

A simple Markov Process

- States: *sunny* or *rainy*: $\boldsymbol{S} = \{s, r\}$
- History: $[s, s, s, r, s, \ldots]$
- Probabilities calculated from history: **M**:

	s	**r**
s	0.8	0.2
r	0.1	0.9



Markov Chains I

Fun with texts

```
use strict;
use warnings;
use Algorithm::MarkovChain;
use Path::Class;
use autodie; # die if problem reading or writing a file

my @inputs = qw(king_james_bible.txt lovecraft_complete.txt);
my $dir = dir(".");
my $f = "";
my @symbols = ();
foreach $f (@inputs) {
    my $file = $dir->file($f);
    my $lcounter = 0;
    my $wcounter = 0;
    my $file_handle = $file->openr();
    while( my $line = $file_handle->getline() ) {
```

Markov Chains II

Fun with texts

```
        chomp ($line);
        my @words = split(' ', $line);
        push(@symbols, @words);
        $lcounter++;
        $wcounter += scalar(@words);
    }
    print "$lcounter lines, $wcounter words read from $f\n";
}
my $chain = Algorithm::MarkovChain::->new();
$chain->seed(symbols => \@symbols, longest => 6);
print "About to spew ...\n";
print "---\n\n";
foreach (1 .. 20) {
    my @newness = $chain->spew(length   => 40,
                               complete => [ qw( the ) ]);
    print join (" ", @newness), ".\n\n";
}
```

Markov Chains III

Fun with texts

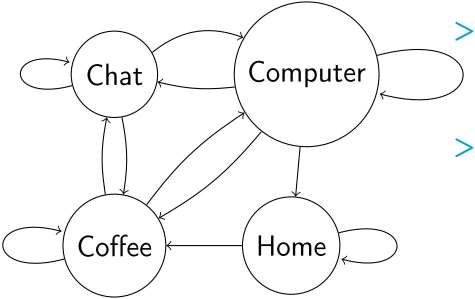
```
$ ./lovebible.pl 2> /dev/null
99820 lines, 821134 words read from king_james_bible.txt
16536 lines, 775603 words read from lovecraft_complete.txt
About to spew ...
---

    the backwoods folk -had glimpsed the battered mantel,
    rickety furniture, and ragged draperies. It spread
     ↪  over it a
    robber, a shedder of blood, when I listened with mad
    intentness. At last you know!At last to come to see
     ↪  me. Now
    Absalom.
```

(Charlie Stross — http://www.antipope.org/charlie/blog-static/2013/12/lovebiblepl.html)

More Complex Systems

Office Routine



(**lapan2018deep**)

- Transition probabilities from observation (count transitions, normalize)
- What motivates transitions?

Markov Reward Process

Where Transitions Come From

- Transition **Probabilities**: System Dynamic
- Transition **Values**: "Belohnung" for a transition
- **Return** of an episode:

$$G_t = \gamma^0 R_{t+1} + \gamma^1 R_{t+2} + \gamma^2 R_{t+2} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \quad (6)$$

G_t Overall Return
R_t Reward for a transition at t
γ Discount Factor (counters infinite loop)

Discount Factor γ

How far to look into the Future?

$$G_t = \gamma^0 R_{t+1} + \gamma^1 R_{t+2} + \gamma^2 R_{t+2} + \cdots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \qquad (7)$$

- For each t: Calculate return as sum of following rewards R_t:

$$\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \qquad (8)$$

- In eq. (8) $k \to \infty$: Stopping condition?
- Multiplication with $\gamma \in [0{,}9; 0{,}99]$: Agent's "foresight"

Return, Reward, Value

What is a State worth?

- **Reward** from transition
- **Return** at the end of a chain of transitions
- How does an agent choose an action in s_t?

Return, Reward, Value

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$$V(s) = \mathbb{E}\left[G | S_t = s\right] \tag{9}$$

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  - For each state s,
  - is the value of this state, $V(s)$,
  - is the mean (alias *expected*) return
  - that follows from the *Markov Reward Process*.

Return, Reward, Value

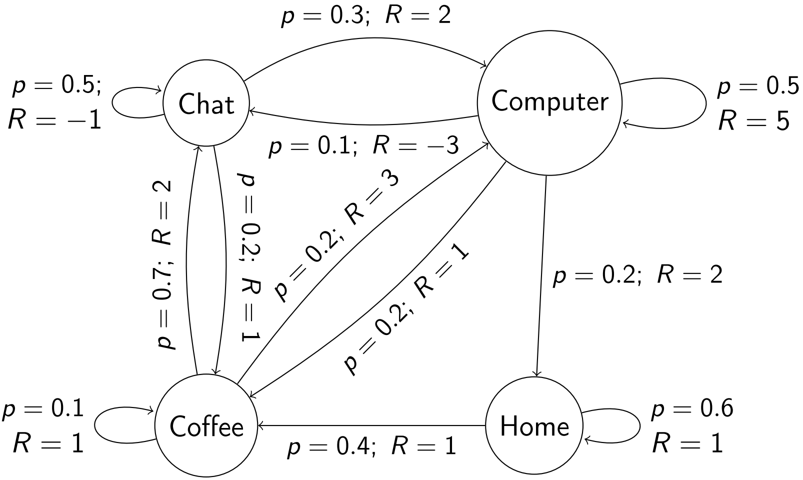
An Example: The *Dilbert Reward Process*

- *home* $\rightarrow$ *home* : 1 (It's good to be home.)
- *home* $\rightarrow$ *coffee* : 1 (Coffee first!)
- *computer* $\rightarrow$ *computer* : 5 (Hard work bears fruit.)
- *computer* $\rightarrow$ *chat* : -3 (Do not disturb!)
- *chat* $\rightarrow$ *computer* : 2 (Back to work.)
- *computer* $\rightarrow$ *coffee* : 1 (Coders are catalysts that turn coffee into code.)
- *coffee* $\rightarrow$ *computer* : 3 (...)
- *coffee* $\rightarrow$ *coffee* : 1 (Good coffee needs time.)
- *coffee* $\rightarrow$ *chat* : 2 (Some chat at the coffee maker.)
- *chat* $\rightarrow$ *coffee* : 1 (Cup already empty?)
- *chat* $\rightarrow$ *chat* : -1 (Long conversations become boring fast.)

(**lapan2018deep**)

Gewinn, Belohnung und Wert

Ein Beispiel: Der *Dilbert Reward Process*



Return, Reward, Value

Values of States in the *Dilbert Reward Process*

With $\gamma = 0$:

- $V(chat) = -1 \cdot 0.5 + 2 \cdot 0.3 + 1 \cdot 0.2 = 0.3$
- $V(coffee) = 2 \cdot 0.7 + 1 \cdot 0.1 + 3 \cdot 0.2 = 2.1$
- $V(home) = 1 \cdot 0.6 + 1 \cdot 0.4 = 1.0$
- $V(computer) = 5 \cdot 0.5 + (-3) \cdot 0.1 + 2 \cdot 0.2 = 2.6$

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Most valuable state?

Return, Reward, Value

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Most valuable state? *Computer*:

- $computer \rightarrow computer$: common
- $computer \rightarrow computer$: high reward
- $computer \rightarrow computer$: seldom interrupted

Value for $\gamma = 1$?

Return, Reward, Value

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With $\gamma = 0$:

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- $computer \to computer$: high reward
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Value for $\gamma = 1$? $V(s) = \infty$!

- No *Sink State*
- $V(s) > 0 \; \forall s$

Markov Decision Process

From Observation to Action

- **Markov Process:** States and transition probabilities (Markov Chains)
- **Markov Reward Process:** MP plus value of a state
- ... and now for the decision?!

Markov Decision Process

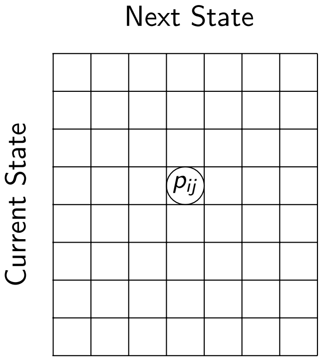
From Observation to Action

- **Markov Process:** States and transition probabilities (Markov Chains)
- **Markov Reward Process:** MP plus value of a state
- ... and now for the decision?! Right, that is still missing:
- **Markov Decision Process:** MRP plus Actions
- Action Space $\boldsymbol{A}$ (*action space*): set of actions
  $\boldsymbol{A} = \{a_1, a_2, \ldots, a_n\}$

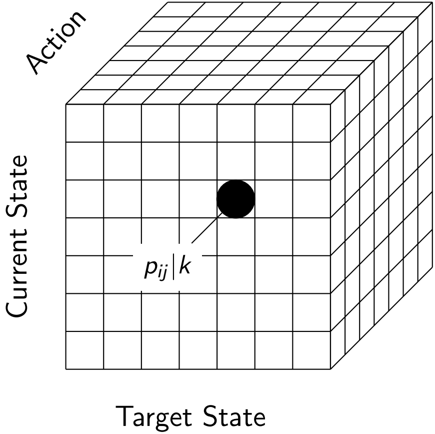
Erweiterung der Transitionsmatrix

Vom *Markov Reward Process* zum *Markov Decision Process*

Markov Reward Process

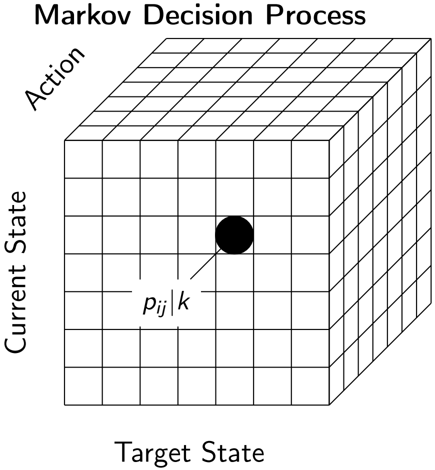


Markov Decision Process



Policy

Action Indicator



- $p_{ij}|k$ probability for $i \to j$, if k chosen as action
- k aus **Policy:**

$$\pi(a|s) = P\left[A_t = a|S_t = s\right] \tag{10}$$

- Formal: Probability distribution over all actions in a given state
- This definition includes random actions during exploration

The Cross-Entropy Method

Based on *Sampling Theorem*

Choosing an Action as Probability Distribution

Sampling Theorem:

$$\mathbb{E}_{x\sim p(x)}\Big[H(x)\Big] = \int_x p(x)H(x)\,\mathrm{d}x \tag{11}$$

$H(x)$ Reward from a Policy *Policy* $x \Leftrightarrow R(\pi(\cdot))$

$p(x)$ Distribution over all possible *policies*

- Maximizing $H(x)$ by searching all possible distributions (not feasible)
- $p(x)$ unknown (is the environment)
- Strategy: Iterative development of a distribution $q(x)$ that approximates $p(x)$

Sampling Theorem:

$$\mathbb{E}_{x\sim p(x)}\Big[H(x)\Big] = \int_x p(x)H(x)\,\mathrm{d}x = \int_x q(x)\frac{p(x)}{q(x)}H(x)\,\mathrm{d}x \quad (12)$$

$$= \mathbb{E}_{x\sim q(x)}\left[\frac{p(x)}{q(x)}H(x)\right] \quad (13)$$

- In eq. (13) Substituting $p(x) \Leftrightarrow q(x)$
- Goal: Optimization metric (approximation)
- Distance metric between two distributions **Kullback Leibler Divergence** (KL)

Kullback Leibler Divergence

Distance between $p(x)$ and $q(x)$

$$KL\big(p_1(x) \parallel p_2(x)\big) = \mathbb{E}_{x\sim p_1(x)} \log \frac{p_1(x)}{p_2(x)} \tag{14}$$

$$= \underbrace{\mathbb{E}_{x\sim p_1(x)}\Big[\log p_1(x)\Big]}_{\text{Entropy}} - \underbrace{\mathbb{E}_{x\sim p_1(x)}\Big[\log p_2(x)\Big]}_{\text{Cross Entropy}} \tag{15}$$

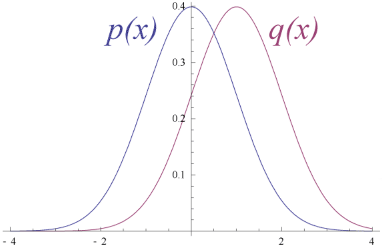
- Alternative Names: **Information Gain**, **relative Entropy**
- Not symmetric: $KL(p_1(x) \parallel p_2(x)) \neq KL(p_2(x) \parallel p_1(x))$, using sums instead: $KL_2(p_1(x) \parallel p_2(x)) = KL_2(p_2(x) \parallel p_1(x)) = KL(p_1(x) \parallel p_2(x)) + KL(p_2(x) \parallel p_1(x))$

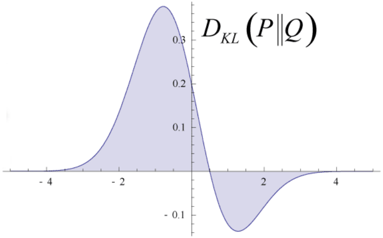
Kullback Leibler Divergence

Distance between $p(x)$ and $q(x)$

$$KL\big(p_1(x) \parallel p_2(x)\big) = \mathbb{E}_{x\sim p_1(x)}\Big[\log p_1(x)\Big] - \mathbb{E}_{x\sim p_1(x)}\Big[\log p_2(x)\Big] \tag{16}$$

$$= \int_{-\infty}^{\infty} p(x)\left(\log p(x) - \log q(x)\right) \mathrm{d}x \tag{17}$$





Combining Sampling and KL

Iterative Approximation

OFFIS

Iteratively improving the approximation $p(x)H(x)$:

$$q_{i+1}(x) = \arg\min_{q_{i+1}(x)} -\mathbb{E}_{x\sim q_i(x)} \frac{p(x)}{q(x)} H(x) \log q_{i+1}(x)$$

$$q_0(x) = p(x) \quad (18)$$

For *Reinforcement Learning*:

$$\pi_{i+1}(a|s) = \arg\min_{\pi_{i+1}} -\mathbb{E}_{z\sim\pi_i(a|s)} \Big[R(z) \geq \psi_i\Big] \log \pi_{i+1}(a|s) \quad (19)$$

- $H(x) \Leftrightarrow \Big[R(z) \geq \psi_i\Big]$
- Indicator Funktion $\Big[R(z) \geq \psi_i\Big] = 1$ if reward above threshhold, 0 else
- No normalization — works still

Cross Entropy Step-by-Step

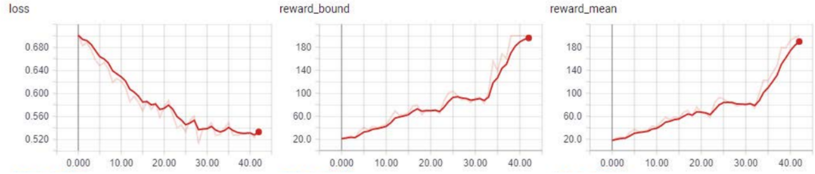
In a Nutshell

procedure CrossEntropy(env, batchSize = 16, percentile = 70)
    $ann \leftarrow$ GenerateRandomANN()
    for $batch \in$ PlayEpisodes($batchSize$) **do**
        $\boldsymbol{obs}_e, \boldsymbol{acts}_e, \boldsymbol{rews}_e \leftarrow$ FilterElite($batch, percentile$)
        $\boldsymbol{actScores}_e \leftarrow$ ann($\boldsymbol{obs}_e$)
        $loss \leftarrow$ CrossEntropy($\boldsymbol{actScores}_e, \boldsymbol{acts}_e$)
        $ann \leftarrow$ Optimize($ann, loss$)
    end for
end procedure

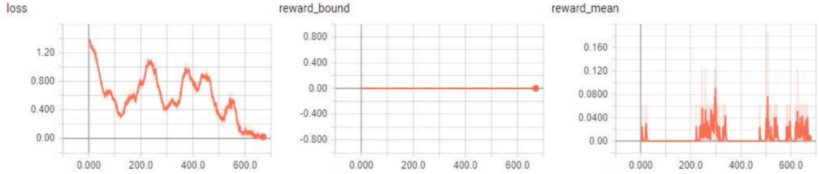
Influence of Episode Distribution

Pro and Con at the Same Time

Cartpole



Frozen Lake



Influence of Episode Distribution

Pro and Con at the Same Time

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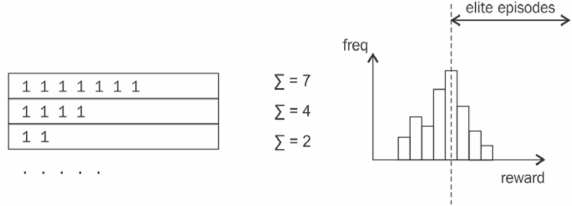
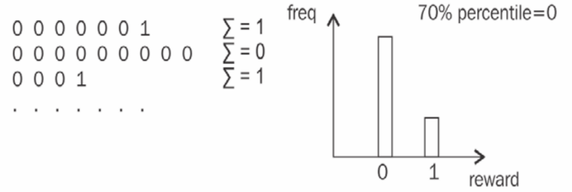


Frozen Lake



Overview CE

Strengths and Weaknesses of the Cross Entropy Method

Pros

- Simplicity: Easy to understand, implementations in 100 LoC possible
- Good convergence for short episodes with immediate rewards

Cons

- Episodes must be finite and short
- Episodes need high variance in rewards

Optimizations:

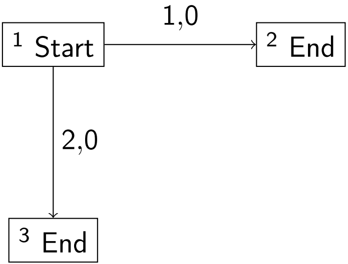
- Bigger *Batches* (prolonges training)
- Discount Factor $\gamma \in [0{,}9; 0{,}95]$ favors short episodes (easy to train)
- Hold *Elite Episodes* longer
- Reduce learning rate during ANN training (reduces speed of convergence)

The Bellman Principle of Optimality

Value of a State:

$$V(s) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R_t\right] \quad (20)$$

Example:

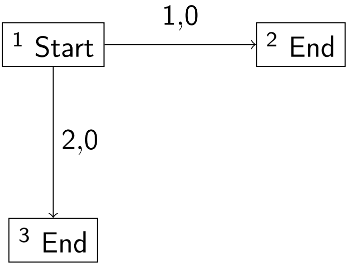


- $V(1)$? Unknown without π
- Even here infinite states
  - Always right:

Value of a State:

$$V(s) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R_t\right] \tag{20}$$

Example:



- $V(1)$? Unknown without π
- Even here infinite states
  - Always right: $V(1) = 1.0$

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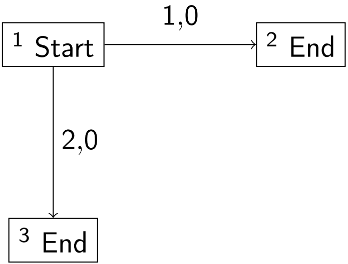
Example:



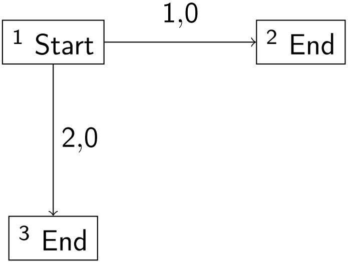
- $V(1)$? Unknown without π
- Even here infinite states
  - Always right: $V(1) = 1.0$
  - Always down:

Value Revisited
Value of a State

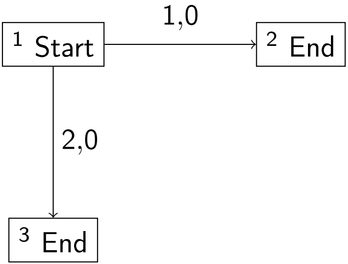
Value of a State:

$$V(s) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t R_t\right] \tag{20}$$

Example:



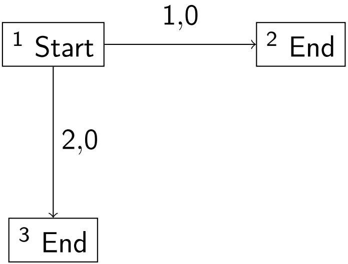
- $V(1)$? Unknown without π
- Even here infinite states
  - Always right: $V(1) = 1.0$
  - Always down: $V(1) = 2.0$



- $V(1)$? Unknown without π
- Even here infinite states
  - Always right: $V(1) = 1.0$
  - Always down: $V(1) = 2.0$
  - $p_{right} = 0.5$, $p_{down} = 0.5$:

Value Revisited

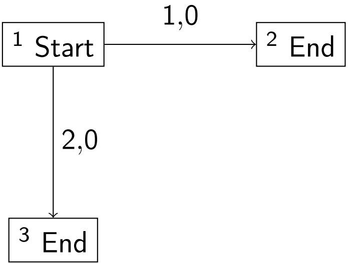
Value of a State



- $V(1)$? Unknown without π
- Even here infinite states
  - Always right: $V(1) = 1.0$
  - Always down: $V(1) = 2.0$
  - $p_{right} = 0.5$, $p_{down} = 0.5$: $V(1) = 1.0 \cdot 0.5 + 2.0 \cdot 0.5 = 1.5$

Value Revisited

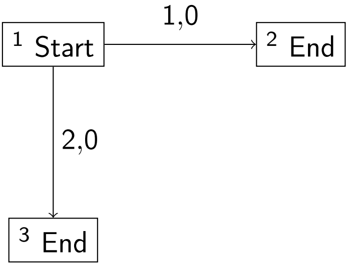
Value of a State



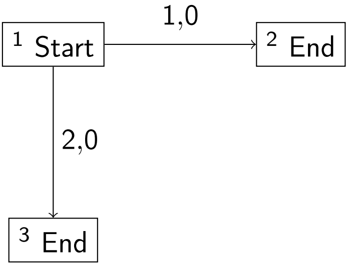
- $V(1)$? Unknown without π
- Even here infinite states
  - Always right: $V(1) = 1.0$
  - Always down: $V(1) = 2.0$
  - $p_{right} = 0.5$, $p_{down} = 0.5$: $V(1) = 1.0 \cdot 0.5 + 2.0 \cdot 0.5 = 1.5$
  - $p_{right} = 0.1$, $p_{down} = 0.9$:

Value Revisited

Value of a State



- $V(1)$? Unknown without π
- Even here infinite states
  - Always right: $V(1) = 1.0$
  - Always down: $V(1) = 2.0$
  - $p_{right} = 0.5$, $p_{down} = 0.5$: $V(1) = 1.0 \cdot 0.5 + 2.0 \cdot 0.5 = 1.5$
  - $p_{right} = 0.1$, $p_{down} = 0.9$: $V(1) = 1.0 \cdot 0.1 + 2.0 \cdot 0.9 = 1.9$



- $V(1)$? Unknown without π
- Even here infinite states
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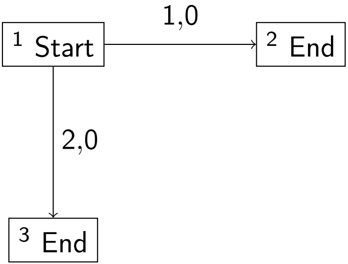
  - $p_{right} = 0.5$, $p_{down} = 0.5$: $V(1) = 1.0 \cdot 0.5 + 2.0 \cdot 0.5 = 1.5$
  - $p_{right} = 0.1$, $p_{down} = 0.9$: $V(1) = 1.0 \cdot 0.1 + 2.0 \cdot 0.9 = 1.9$

Value Revisited

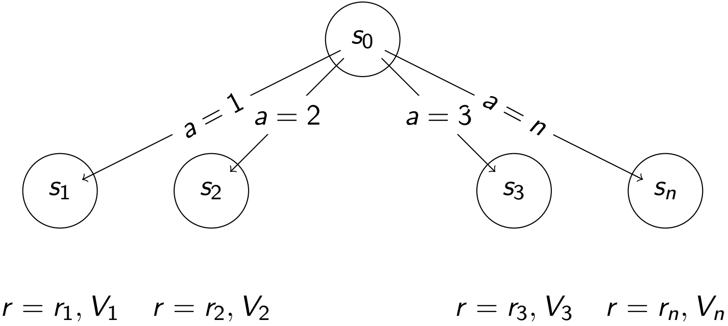
Value of a State



- $V(1)$? Unknown without π
- Even here infinite states
  - Always right: $V(1) = 1.0$
  - Always down: $V(1) = 2.0$
  - $p_{right} = 0.5,\ p_{down} = 0.5$: $V(1) = 1.0 \cdot 0.5 + 2.0 \cdot 0.5 = 1.5$
  - $p_{right} = 0.1,\ p_{down} = 0.9$: $V(1) = 1.0 \cdot 0.1 + 2.0 \cdot 0.9 = 1.9$
- And for more then 3 states...?

Value of a State

An abstract Look at $V(s)$



- An action k:

$$V_0(a = a_k) = r_k + \gamma V_k \tag{21}$$

- Best action:

$$V_0 = \max_{a \in 1 \ldots n} (r_a + \gamma V_a) \tag{22}$$

Value of a State

An abstract Look at $V(s)$

- Action 1:

$$V_0(a = a_1) = r_1 + \gamma V_1 \tag{23}$$

- Eine Handlung i, stochastisch:

$$V_0(a = a_1) = p_1(r_1 + \gamma V_1) + p_2(r_2 + \gamma V_2) + \cdots + p_n(r_n + \gamma V_n)$$
$$\sum_{i=1}^{n} p_i = 1{,}0 \tag{24}$$

- Formal für eine beliebige Handlung a:

$$V_0(a) = \mathbb{E}_{s \sim \mathbf{S}}\left[r_{s,a} + \gamma V_s\right] = \sum_{s \in \boldsymbol{S}} p_{a,0 \to s}(r_{s,a} + \gamma V_s) \tag{25}$$

Bellman Principle of Optimality

Finding the Maximum Value of a State

Bellman Equation for deterministic case:

$$V_0 = \max_{a \in 1 \dots n} (r_a + \gamma V_a) \tag{26}$$

Bellman Principle of Optimality:

$$V_0 = \max_{a \in \boldsymbol{A}} \mathbb{E}_{s \sim \boldsymbol{S}} \left[r_{s,a} + \gamma V_s \right] = \max_{a \in \boldsymbol{A}} \sum_{s \in \boldsymbol{S}} p_{a,0 \to s} (r_{s,a} + \gamma V_s) \tag{27}$$

Bellman Principle of Optimality

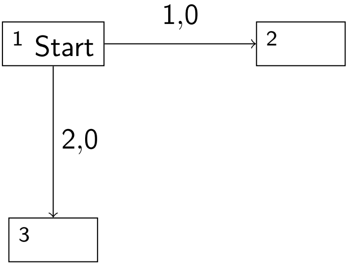
Finding the Maximum Value of a State

$$V_0 = \max_{a \in \boldsymbol{A}} \mathbb{E}_{s \sim \boldsymbol{S}} \left[r_{s,a} + \gamma V_s \right] = \max_{a \in \boldsymbol{A}} \sum_{s \in \boldsymbol{S}} p_{a,0 \rightarrow s} (r_{s,a} + \gamma V_s) \quad (28)$$

- Defining a state's value as the sum of...
  - *Rewards*, r
  - and *Values* $V(s)$ of following states $s \in \boldsymbol{S}$
  - multiplied by transition probability $p_{0 \mapsto s}$
  - given an action $a \in \boldsymbol{A}$
- Applies to *all* $V(s)$: **Recursion**
- In theory, best action obtainable by complete exploration of the state-action-value space

Recursion, Bellman, & Optimality

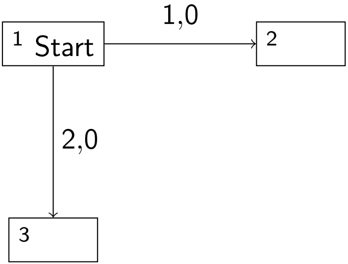
Solution to a very real Problem



- Ideal Strategy:

Recursion, Bellman, & Optimality

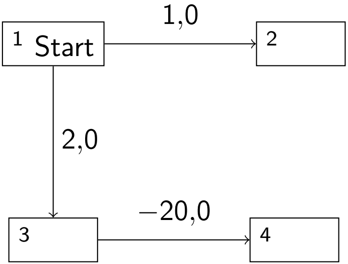
Solution to a very real Problem



> Ideal Strategy: $1 \rightarrow 3: r = 2$

Recursion, Bellman, & Optimality

Solution to a very real Problem



- Ideal Strategy: $1 \rightarrow 3$: $r = 2$
- Or not?! $1 \rightarrow 3 \rightarrow 4$: $r = -18$
- Value of a state depends on the following states!
- Recursive definition covers all following states (in theory).
- (Naive) Policy: For the current state, evaluate all reachable states and choose the action with the biggest value $r + V(s)$.

Value of an Action

Value of an action a in State s

$$Q_{s,a} = \mathbb{E}_{s' \sim \boldsymbol{S}} \left[r_{s,a} + \gamma V_{s'} \right] = \sum_{s' \in \boldsymbol{S}} p_{a,s \rightarrow s'} (r_{s,a} \gamma V_{s'}) \qquad (29)$$

> Expected immediate reward $r_{s,a}$ and discounted long-term reward of the target state

$$V_s = \max_{a \in \boldsymbol{A}} Q_{s,a} \qquad (30)$$

> Value of a state s, $V(s)$, is the value of the best possible action executable in s: expressing $V(s)$ via $Q_{s,a}$

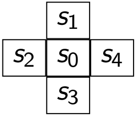
$$Q(s, a) = r_{s,a} + \gamma \max_{a' \in \boldsymbol{A}} Q(s', a') \qquad (31)$$

> Applying the Bellman Principle to actions

Applying the Bellman Principle of Optimality: from Value Iteration to Q Learning

$$Q(s, a) = r_{s,a} + \gamma \max_{a' \in \boldsymbol{A}} Q(s', a') \tag{32}$$

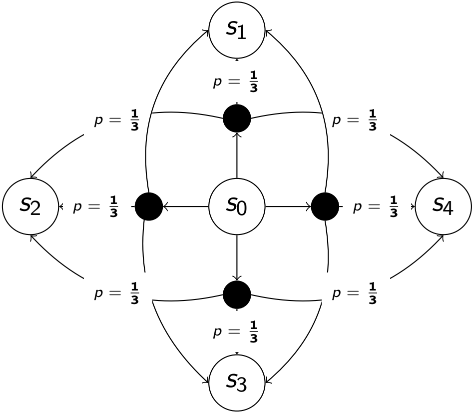
A simple Example:



s_0: Initial State
s_1, s_2, s_3, s_4: Final States
$p = \frac{1}{3}$ per action for slipping left/right

Q Learning

Basis of a Big Family of Algorithms

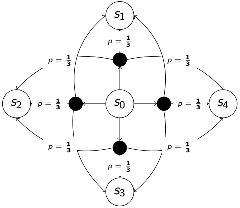


$$Q(s, a) = r_{s,a} + \gamma \max_{a' \in \boldsymbol{A}} Q(s', a') \tag{33}$$

- $Q(s,a) = 0 \; \forall s \in \{1, 2, 3, 4\}$
- $Q(s_0, up) = \frac{1}{3}V_1 + \frac{1}{3}V_2 + \frac{1}{3}V_4 = \frac{1}{3}1 + \frac{1}{3}2 + \frac{1}{3}4 = 2.31$
- $Q(s_0, left) = \ldots = 1.98$
- $Q(s_0, right) = \ldots = 2.64$
- $Q(s_0, down) = \ldots = 2.97$
- $V(s_0) = \max_{a \in \boldsymbol{A}} Q(s_0, a) = Q(s_0, down) = 2.97$

Q Learning

Q *Value* the Action Indicator



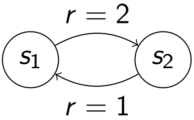
$Q(s_1, a)$	0
$Q(s_2, a)$	0
$Q(s_3, a)$	0
$Q(s_4, a)$	0
$Q(s_0, up)$	2.31
$Q(s_0, left)$	1.98
$Q(s_0, right)$	2.64
$Q(s_0, down)$	2.97

$$Q(s, a) = r_{s,a} + \gamma \max_{a' \in \boldsymbol{A}} Q(s', a') \qquad (34)$$

- Q better suited than V for selecting actions (value of an action, not value of a state)
- V computable from Q
- Missing: method for calculating Q/V (without knowing all transitions!)

Value Iteration

A naïve Approach to Q Learning



$$\boldsymbol{r} = [1, 2, 1, 2, 1, \ldots]$$

$$V(s_1) = 1 + \gamma(2 + \gamma(1 + \gamma(2 + \ldots)))$$
$$= \sum_{i=0}^{\infty} 1\gamma 2^{2i} + 2\gamma^{2i+1}$$

With $\gamma = 0{,}9$:

10	$0.9^{10} \approx 0.348$
50	$0.9^{50} \approx 0.00515$
100	$0.9^{100} \approx 0.0000265$

$$V(s_2) = 2 + \gamma(1 + \gamma(2 + \gamma(1 + \ldots)))$$
$$= \sum_{i=0}^{\infty} 2\gamma 2^{2i} + 1\gamma^{2i+1}$$

Value Iteration
Algorithm in a Nutshell

procedure ValueIteration(env)
    $\boldsymbol{Q} \leftarrow [0]$ ▷ $\forall s, a$
    for all $s \in \boldsymbol{S}, a \in s$ **do**
        $Q_{s,a} \leftarrow \sum_{s'} p_{a,s \rightarrow s'}(r_{s,a} + \gamma \max_{a'} Q_{s',a'})$ ▷ Bellman
Update
    end for
    return $\boldsymbol{Q}$
end procedure

Value Iteration

Algorithm in a Nutshell

procedure ValueIteration(env)
  $\boldsymbol{Q} \leftarrow [0]$ ▷ $\forall s, a$
  for all $s \in \boldsymbol{S}, a \in s$ **do**
    $Q_{s,a} \leftarrow \sum_{s'} p_{a,s \to s'}(r_{s,a} + \gamma \max_{a'} Q_{s',a'})$ ▷ Bellman
Update
  end for
  return $\boldsymbol{Q}$
end procedure

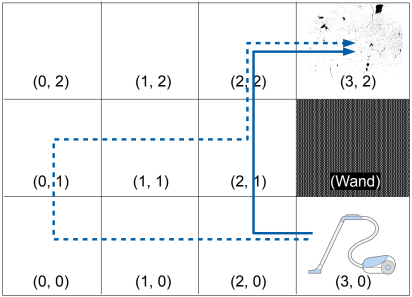
- State space must be discrete
- . . . and small enough!
- Transition probabilities from observations (s_0, s_1, a)

Deep Q Networks

Motivation

Capacity & Compute Power needed for *Value Iteration*

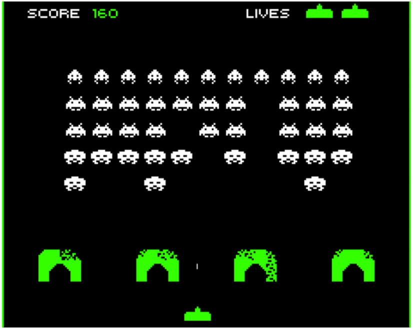
- Saving (s, a, r, s')
- Assumption: every value theoretically known and iterable
- Back-of-napkin calculation: 8.5 billion floating point numbers in in 32 GB RAM



Motivation

Capacity & Compute Power needed for *Value Iteration*

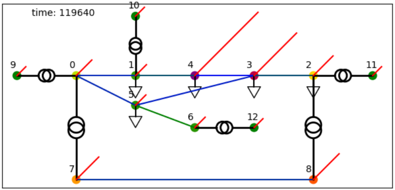
- Atari 2600 (Benchmark for DRL): $210 \times 160 = 33600$ pixels, 128 colors
- Each frame: $128^{33600} \approx 10^{70802}$ pictures (states!)
- 99(,9?) % of all iterations nonsensible
- *Space Invaders* & Co not discrete



Motivation

Capacity & Compute Power needed for *Value Iteration*

- Power grid mixed discrete/continuous (tap changer vs. generator scaling)
- State space in quasi-stationary calculations already complex (loat flow calculations, state estimation, . . .)



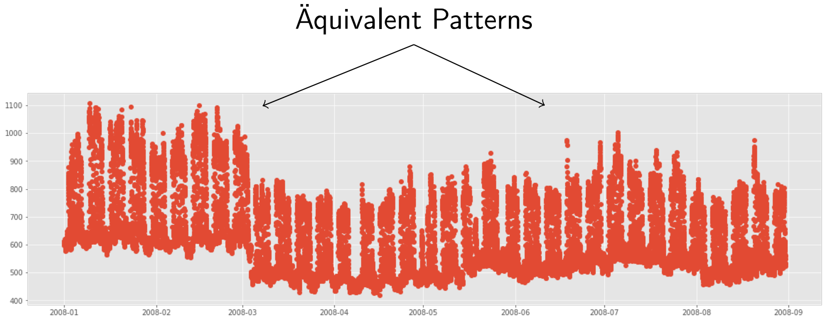
Tabular Q Learning

Optimizing Value Iteration

procedure TabularLearning(env, γ, α)
    $\boldsymbol{Q} \leftarrow [],\; R \leftarrow 0,\; \epsilon_e \leftarrow 1{,}0$
    repeat
        $s \leftarrow Read(env)$
        if $s \notin \boldsymbol{Q} \vee random() < \epsilon_e$ **then** ▷ Exploration vs. Exploitation
            $a \leftarrow RandomChoice(\boldsymbol{A})$
            $\epsilon_e \leftarrow \epsilon_e - 0{,}02$
        else
            $a \leftarrow \max_{a \in \boldsymbol{A}} Q_s$
        end if
        $s', r_{s,a} \leftarrow Act(env, a)$
        $Q_{s,a} \leftarrow (1-\alpha)Q_{s,a} + \alpha(r + \gamma max_{a' \in \boldsymbol{A}} Q_{s',a'})$ ▷ Bellman
        $R' \leftarrow R$
        $R \leftarrow R + \gamma r_{s,a}$
    until $|R - R'| < \epsilon_R$
    return Q
end procedure

Coping with Equivariance

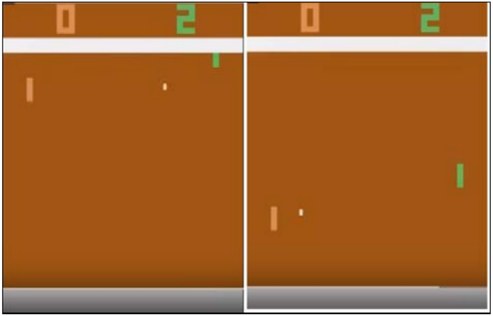
Representing Q as Matrix not Efficient Enough



- Difference—wrt actions—between both states?

Coping with Equivariance

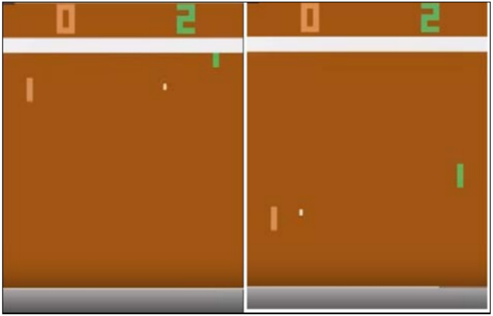
Representing Q as Matrix not Efficient Enough



> Difference—wrt actions—between both states?

Coping with Equivariance

Representing Q as Matrix not Efficient Enough



- Difference—wrt actions—between both states?
- None!
- But separate entry in $Q_{s,a}$: **Regression Problem**

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- Regression Problem: non-linear mapping $f : (s, a) \mapsto Q$
- f: **Artificial Neural Network**
- Adapting the algorithm:
  1. Init $Q(s, a)$ with potentially random approximaiton
  2. $(s, a, r, s') = \boldsymbol{Act}(env, a)$
  3. Calculate error:

$$\mathcal{L} = \begin{cases} (Q_{s,a} - r)^2 & \text{at the end of episode,} \\ (Q_{s,a} - (r + \gamma \max_{a' \in \boldsymbol{A}} Q_{s',a'}))^2 & \text{during the episode.} \end{cases} \quad (35)$$

  4. Change $Q(s,a)$ with gradient descent algorithm (**Stochastic Gradient Descent, SGD**)
  5. Repeat from (2) until convergence

Independent and Identically Distributed. . . ?

Base Assumption of SGD a Problem

- Base for Deep Q Learning borrowed from supervised Deep Learning:
- Assumption of **SGD**: i.i.d
- Neither nor at DRL
  1. Independent: (s, a, r, s') not independent, obviously
  2. Indentically: training data (exploration) differs from optimal policy (exploitation): (**exploration vs. exploitation**)
- Solution: **Replay Buffer**
  - Ring buffer
  - fixed size
  - more or less i.i.d., but still “fresh enough”

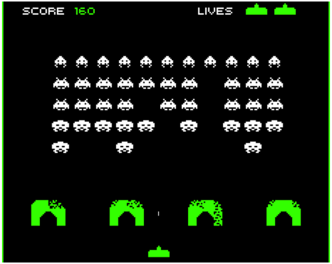
Correlation between Steps

Achilles' Heel of the Bellman Principle

$$Q_{s,a} = r + \gamma \max_{a' \in \boldsymbol{A}} Q_{s',a'} \quad (36)$$

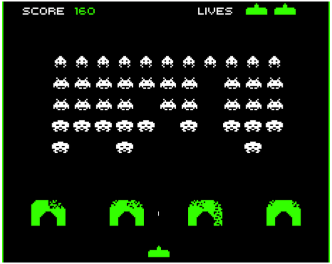
- Deriving $Q_{s,a}$ via $Q_{s',a'}$: **Bootstrapping**
- s and s' differ in just one step
- Update of $Q(s,a)$ influences $Q(s',a')$: **Training unstable** (After updating $Q(s,a)$, $Q(s',a')$ becomes worse if immediately explored; next update worsens, etc. ad infinitum)
- **Target Network**: copy of *Policy Network* for $Q_{s',a'}$; sync every N steps
- N a hyper parameter $N = [1{,}000; 10{,}000]$

Partially Observable Markov Decision Process



> How fast do the invaders move?

Partially Observable Markov Decision Process



- How fast do the invaders move?
- *Markov Decision Process* dictates that state is completely derivable from one observation
- In RL not always possbile: **Partially Observable Markov Decision Process, POMDP**
- Hack: Merge k observations (e.g., $k = 4$ frames in ATARI)

DQN Training I

Final Form

procedure DqnLearning(env, γ, α, N)

$\boldsymbol{Q} \leftarrow RandomWeights(), \hat{\mathbf{Q}} \leftarrow RandomWeights()$

$replayBuffer \leftarrow []$

$\epsilon \leftarrow 1{,}0,\ \ n \leftarrow 0$

repeat

$$a \leftarrow \begin{cases} RandomChoice(\boldsymbol{A}) & \textbf{if } Random() < \epsilon \\ \arg\max_a Q_{s,a} & \textbf{else} \end{cases}$$

$\epsilon \leftarrow \epsilon - 0{,}02$

$(s', r) \leftarrow Act(env, a)$

$replayBuffer \leftarrow replayBuffer \cup (s, a, r, s')$

$minibatch \leftarrow RandomSample(replayBuffer)$

for all $step = (s, a, r, s') \in minibatch$ **do**

$$y = \begin{cases} r & \textbf{if } EpisodeEnd(minibatch) \\ r + \gamma \max_{a' \in \boldsymbol{A}} \hat{Q}_{s',a'} & \textbf{else} \end{cases}$$

DQN Training II

Final Form

$\mathcal{L} = (Q_{s,a} - y)^2$
$\boldsymbol{Q} \leftarrow SGD(Q, y)$
$n \leftarrow n + 1$
if $n = N$ **then**
  $\hat{\boldsymbol{Q}} \leftarrow \boldsymbol{Q}$
  $n \leftarrow 0$
end if
end for
until *HasConverged*()
return $\boldsymbol{Q}$
end procedure

How to Proceed Further

Deep Q Learning is Only the Beginning

A Wrapup: What we Should Do Now

- DQN + Extensions (Rainbow Paper) very handy
- But suffers from the curse of dimensionality
- "Status Quo" for Power Systems: DQN, DDPG
- Still a long way in the power systems community until AlphaZero is applied
- Power Systems benchmark missing
- Framework for multi-agent in power systems missing
- Want to help? Drop a note: eric.veith@offis.de