Light Quanta on Beam Splitters

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Content:

1. Introduction

2. Quantum harmonic oscillator,
   photons and Wigner function

3. Beam splitter

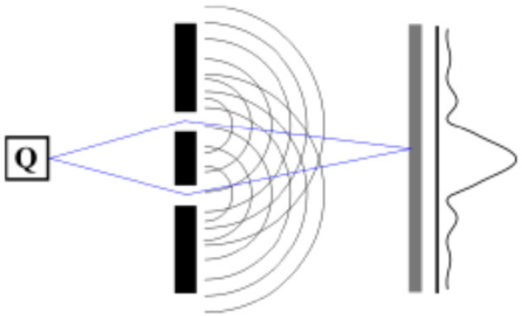
4. Coherent state

5. Quantum random numbers (QRN): Examples

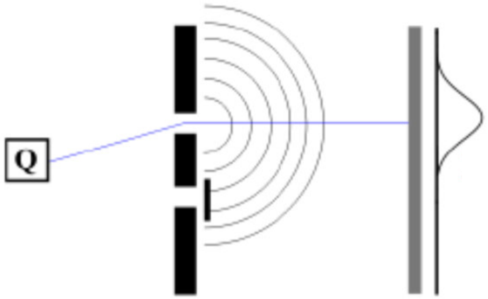
6. Summary

1. Introduction

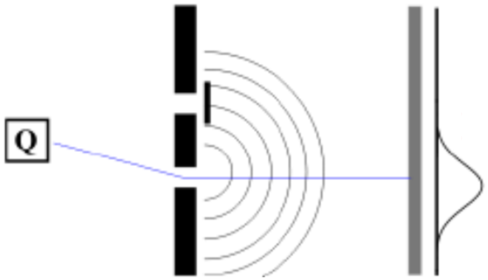
Double-slit experiment



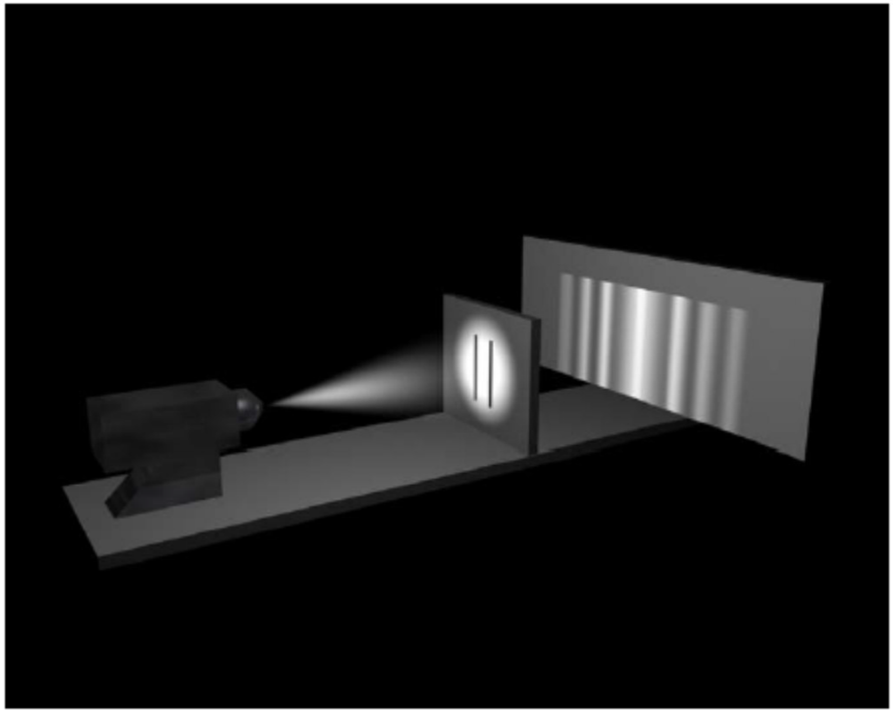
Interference pattern



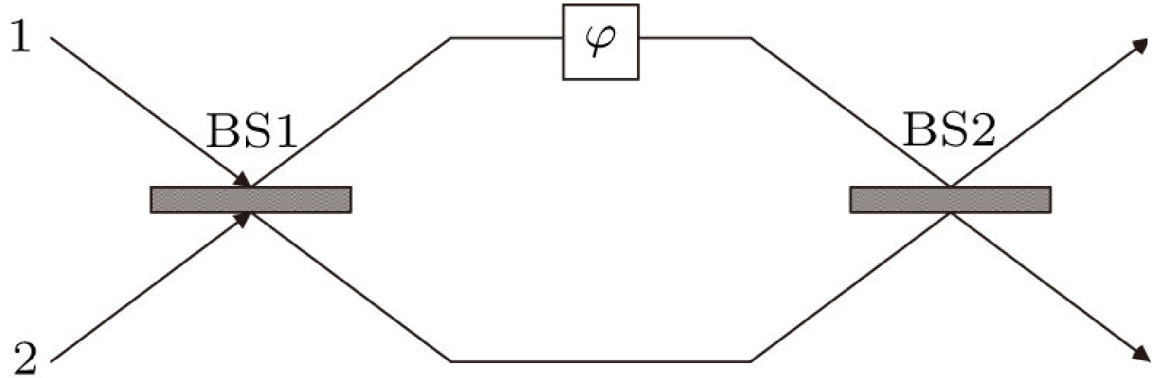
Lower slit covered

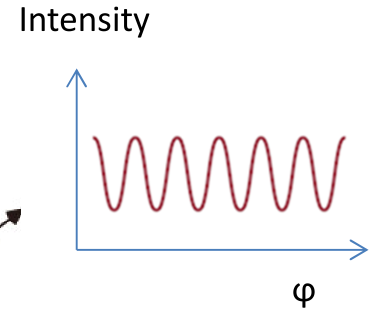


upper slit covered



Mach-Zehnder Interferometer





BS1 = 1. Beam Splitter
BS2 = 2. Beam Splitter
φ = Phase Shifter

2. Quantum harmonic oscillator, photons and Wigner function

QM harmonic oscillator

$$\hat{H} = \frac{1}{2}\left[\frac{1}{m}\hat{p}^2(t) + m\omega^2\hat{q}^2(t)\right] = \hbar\omega(\hat{N} + \tfrac{1}{2}\hat{1}),\ \hat{N} = \hat{a}^+\hat{a},\ \ \hat{N}|n\rangle = n|n\rangle$$

$$\hat{a} = \frac{1}{\sqrt{2\hbar}}(\lambda\,\hat{q} + \frac{i}{\lambda}\hat{p})$$

$$\hat{a}^+ = \frac{1}{\sqrt{2\hbar}}(\lambda\,\hat{q} - \frac{i}{\lambda}\hat{p})$$

$\lambda = \sqrt{m\omega}$

$$\hat{H}|n\rangle = E_n|n\rangle \quad \Rightarrow \quad \langle q|n\rangle = \psi_n(q) \propto H_n\left(\frac{q}{q_0}\right)\exp\left(-\frac{q^2}{2q_0^2}\right)$$

$|n\rangle$ = photon state
or light quantum
(n=0,1,2,…)

$$q_0 = \sqrt{\frac{\hbar}{m\omega}}$$

Hermite polynomials

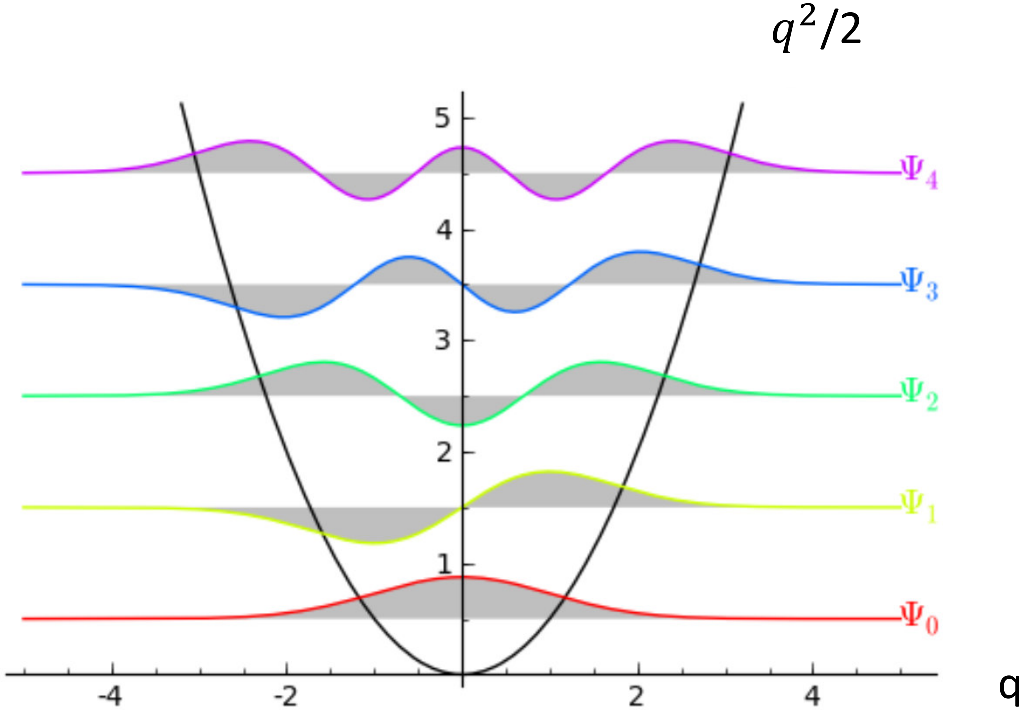
For light m:=$\hbar\omega/c^2$

$E_n = \hbar\omega(n + \frac{1}{2})$,

W(q,p)=Wigner function
q= position
p=momentum

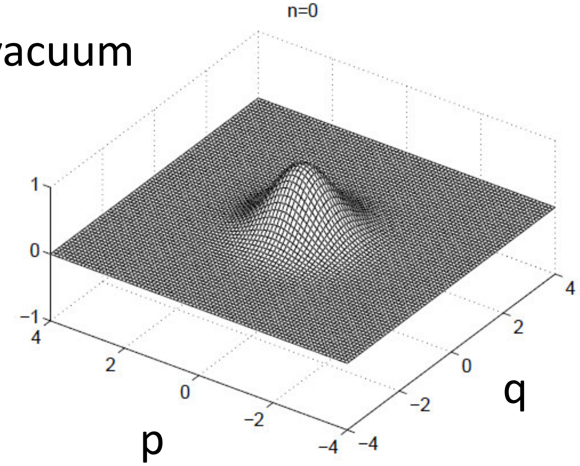
$$W(q,p) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}\psi_n\left(q - \frac{y}{2}\right)\psi_n^*\left(q + \frac{y}{2}\right)e^{iyp/\hbar}dy$$

QMHO-Potential

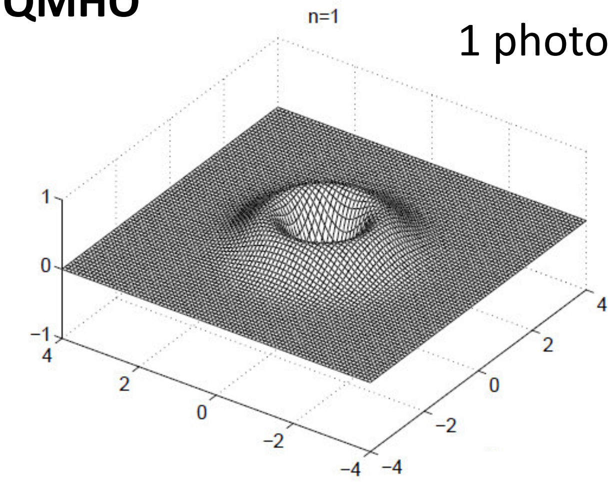


Wigner function of QMHO

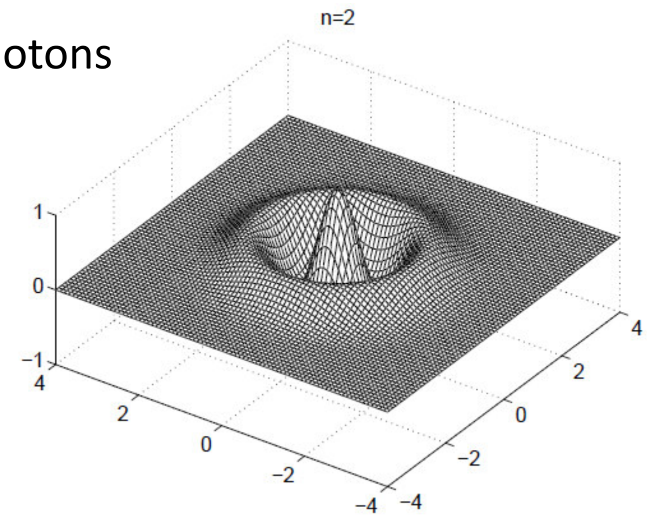
vacuum



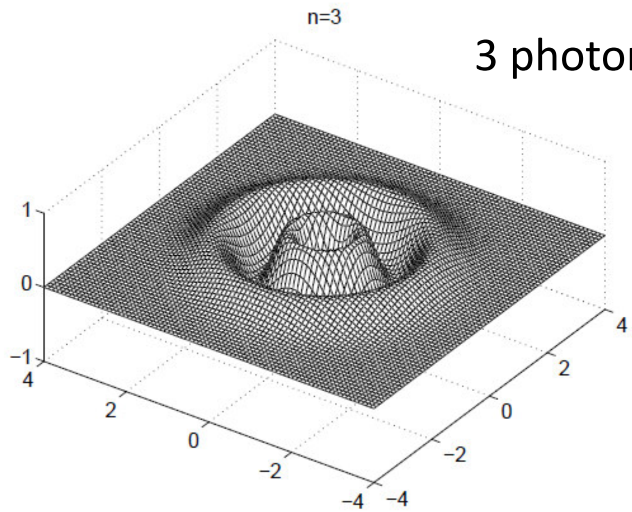
1 photon



2 photons

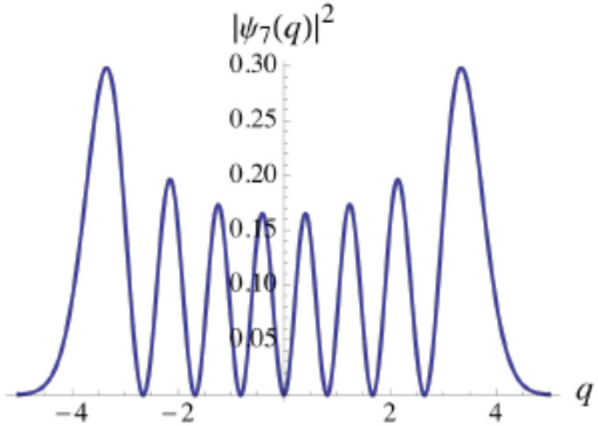


3 photons



QM harmonic oscillator

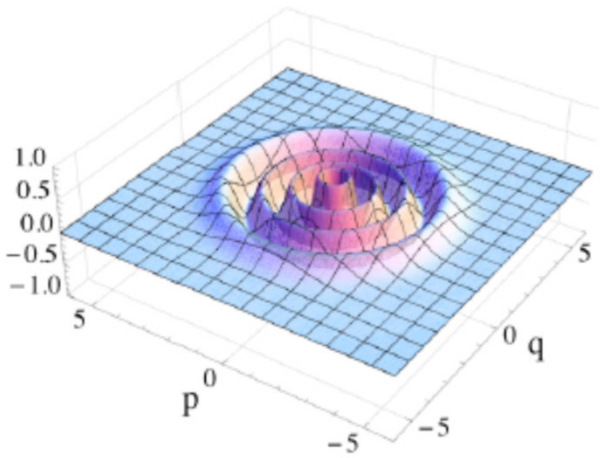
$|n=7\rangle$

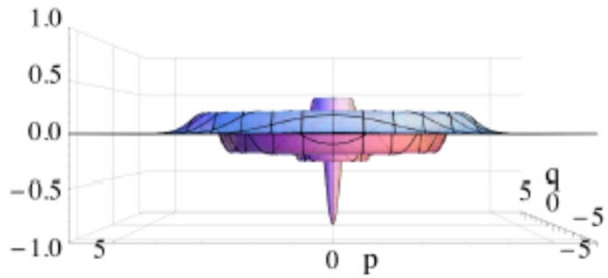


$\hbar = 1,$

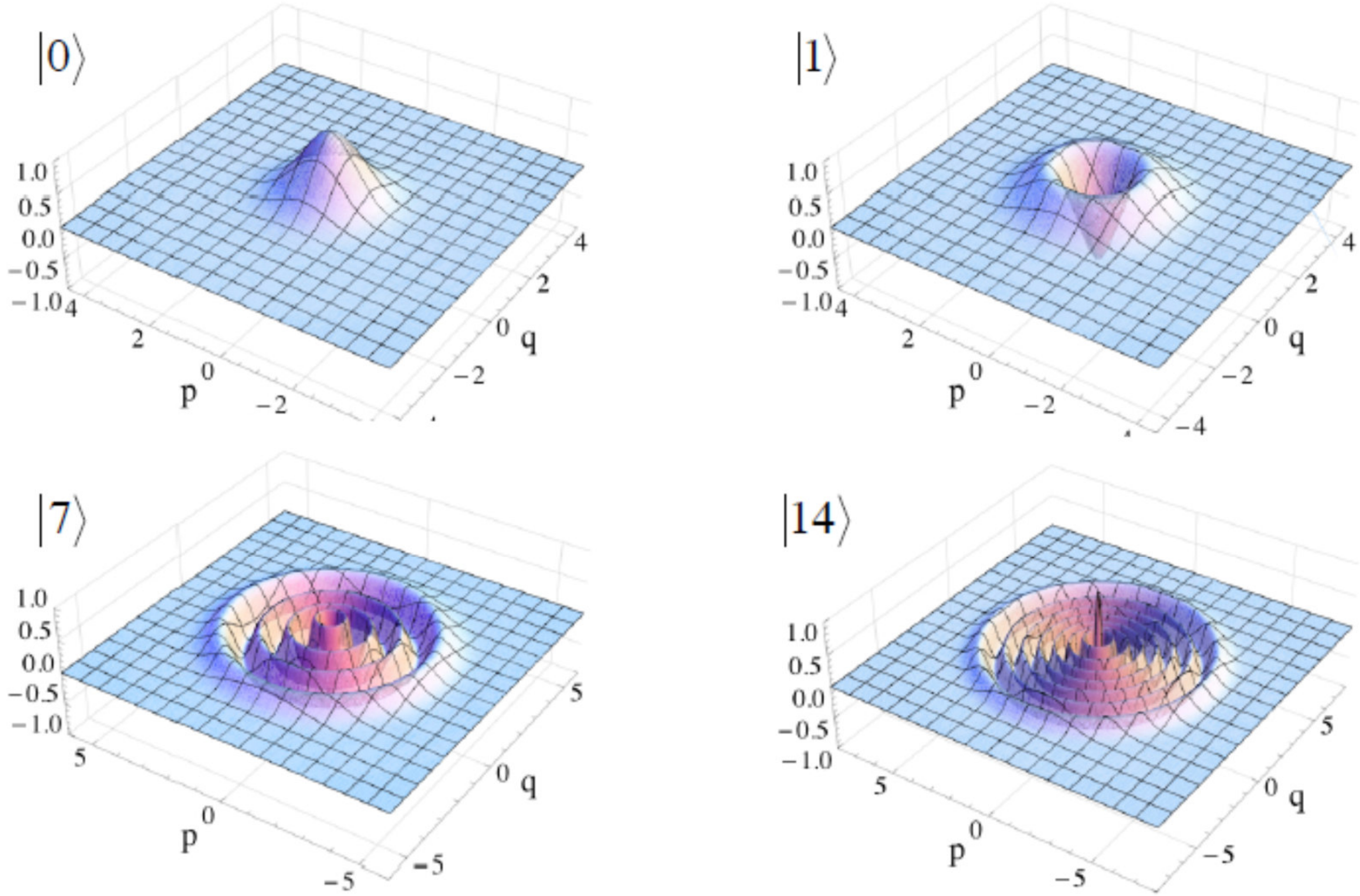
$q_0 = 1,$

$W \times \pi.$



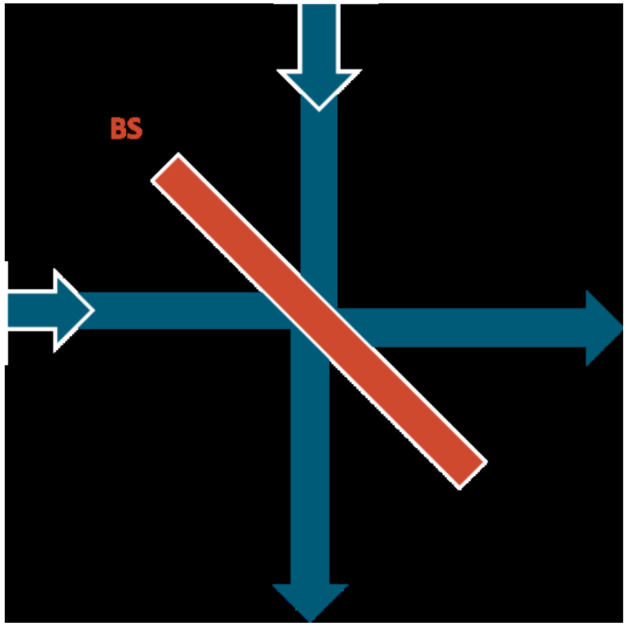


Fock states: Wigner function

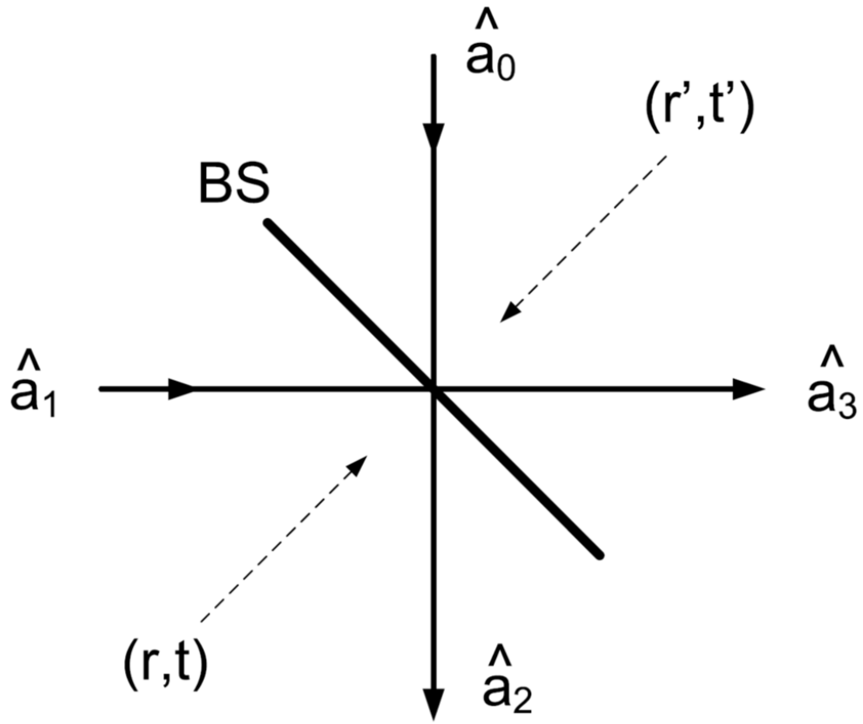


3. Beam splitter

Glass – Reflection and Transmission



Simplified & Balanced Beam Splitter

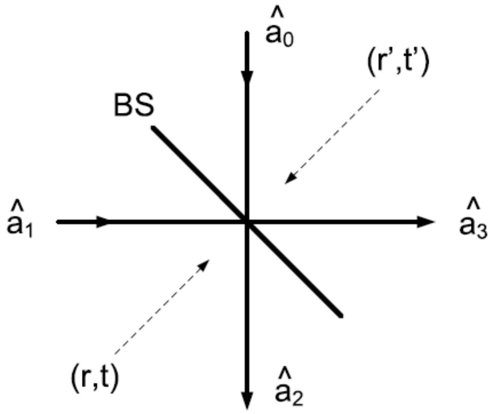


We choose for reflection coefficient: $r=i/\sqrt{2}$, $i=\exp(i\pi/2)$

Transmission coefficient : $t = 1/\sqrt{2}$

$$\hat{a}_2 = t\hat{a}_0 + r\hat{a}_1$$
$$\hat{a}_3 = r\hat{a}_0 + t\hat{a}_1$$

50:50 Beam splitter BS or $\hat{T}$



a) $$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix} = \hat{T} \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix}$$

with BS-operator $\hat{T}$

b) Alternative description of the BS:

$$\begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \hat{U}^+ \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix} \hat{U}$$

with unitary operator $\hat{U} = e^{-i\hat{H}\tau}$

Hamiltonian of BS:

$$\hat{H} = \kappa \left(\hat{a}_0^+ \hat{a}_1 + \hat{a}_0 \hat{a}_1^+ \right) \quad , \kappa = 1/2, \ \tau = \pi/2$$

$\hat{a}^+$ and $\hat{a}$ are creation and annihilation operators

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle, \ \hat{a}^+|n\rangle = \sqrt{n+1}|n+1\rangle \quad \Rightarrow \quad \hat{a}^+|0\rangle = |1\rangle, \ \hat{a}|1\rangle = |0\rangle$$

4. Coherent state

Coherent state (CS) |α>

BS not only for Fock-states |n> , but also with CS |α> :

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle = e^{-|\alpha|^2/2} e^{\alpha \hat{a}^+}|0\rangle =$$

$$= e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} (\hat{a}^+)^n |0\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

, $\widehat{D}(\alpha)$=displacement-operator

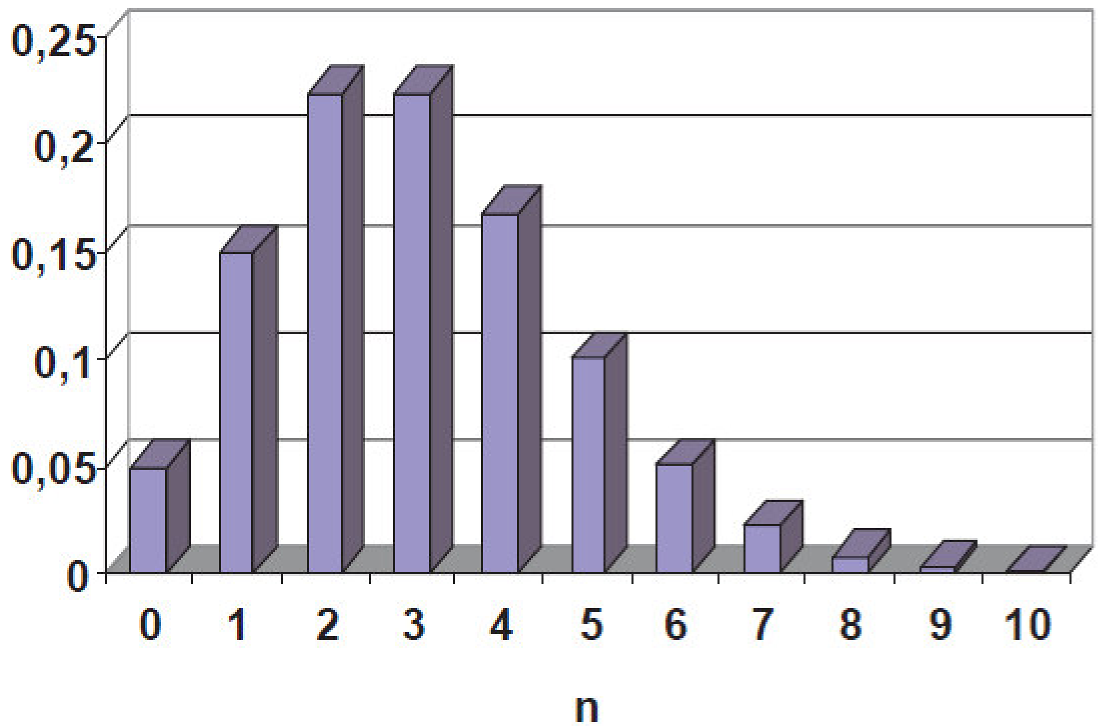
$\approx |0\rangle + \alpha\,|1\rangle$, $|\alpha|^2 \ll 1$

$|\alpha|^2$ = mean photon number

$$\hat{D}(\alpha) = e^{\alpha \hat{a}^+ - \alpha^* \hat{a}} ,$$

$\alpha \in \mathbb{C}$

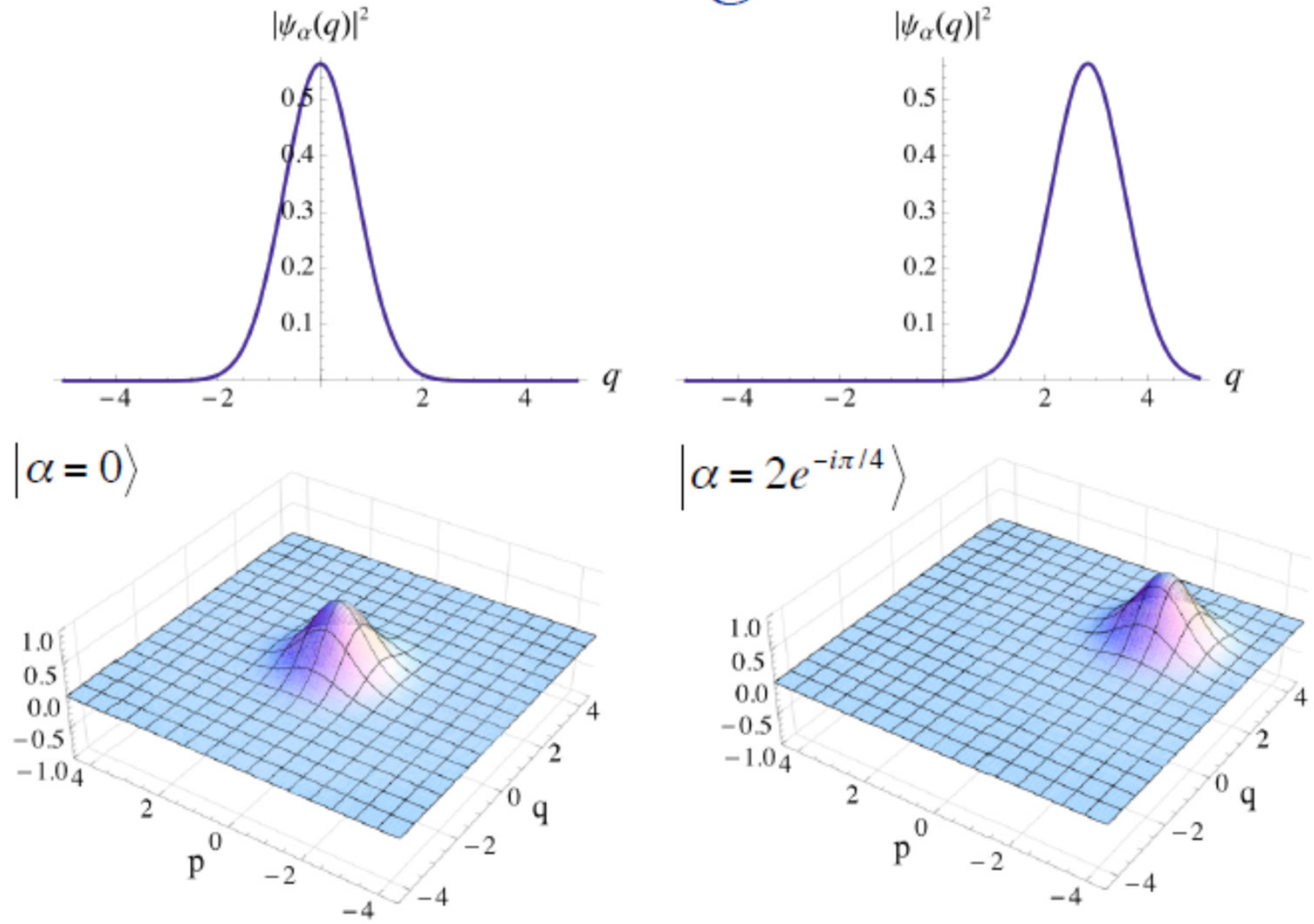
$$\langle\alpha|\alpha\rangle = e^{-|\alpha|^2} \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} = \sum_{n=0}^{\infty} P_n = 1$$



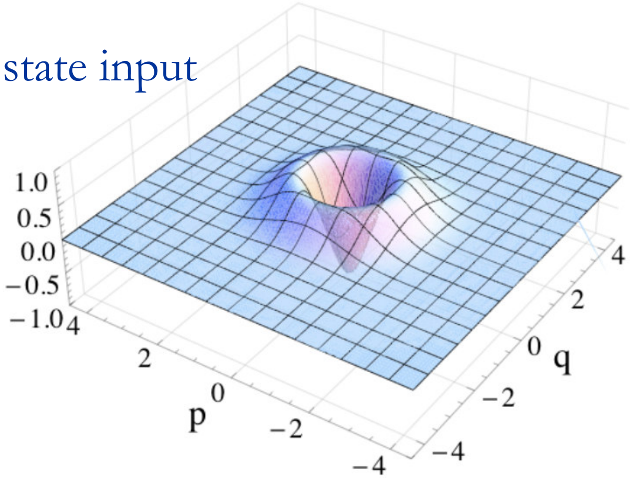
Poissonian distribution $P_n = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}$ for $\langle n \rangle = |\alpha|^2 = 3$

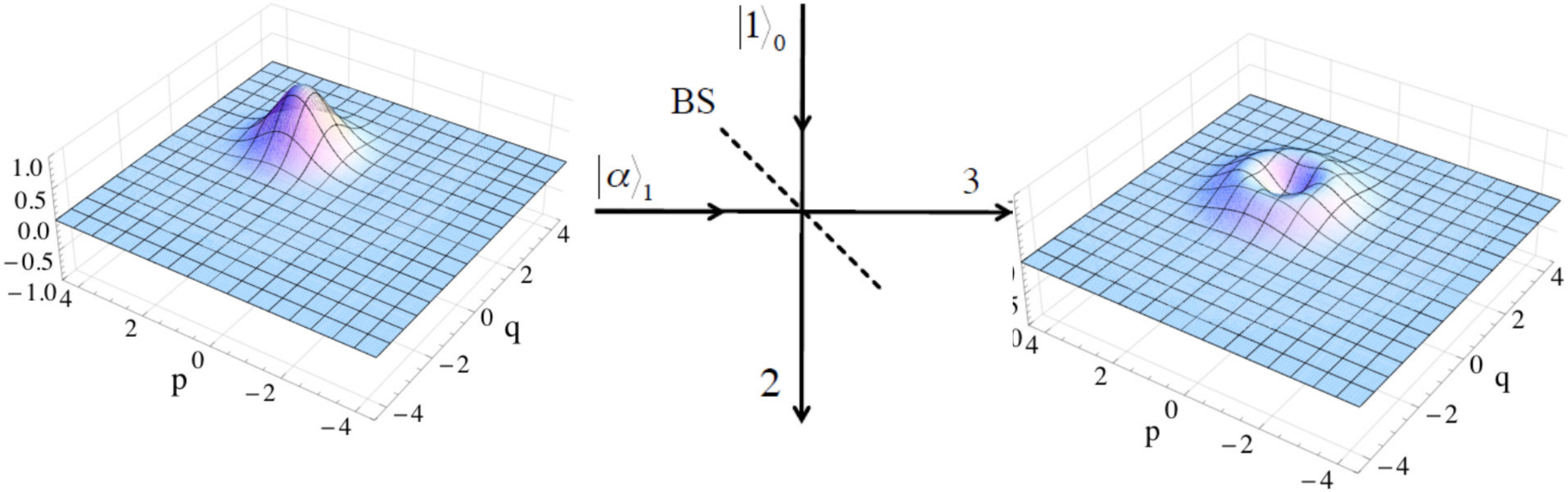
$|\alpha|^2 = mean\ photon\ number\ of\ CS$

Coherent states: Wigner function



BS: 1 photon & coherent state input





5. Quantum random numbers (QRN): Examples

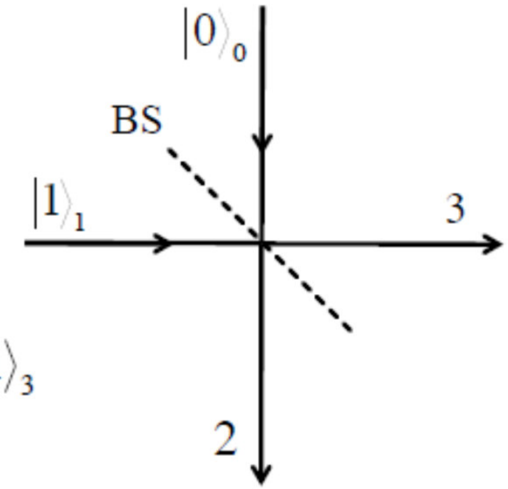
1. Example:

Beam splitter: 1 photon input

$$|0\rangle_0|0\rangle_1 \xrightarrow{BS} |0\rangle_2|0\rangle_3 \qquad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$|0\rangle_0|1\rangle_1 = \hat{a}_1^\dagger|0\rangle_0|0\rangle_1 \qquad \hat{a}_1^\dagger = r\hat{a}_2^\dagger + t\hat{a}_3^\dagger$$

$$\hat{a}_1^\dagger|0\rangle_0|0\rangle_1 \xrightarrow{BS} (r\hat{a}_2^\dagger + t\hat{a}_3^\dagger)|0\rangle_2|0\rangle_3 = r|1\rangle_2|0\rangle_3 + t|0\rangle_2|1\rangle_3$$



Balanced Beam Splitter equation:

A single input photon in mode 1 together with a vacuum input in mode 0 is equally transmitted and/or reflected with a probability of ½ :

$$|0\rangle_0|1\rangle_1 \quad \rightarrow^{BS}\rightarrow \quad \frac{1}{\sqrt{2}}(i\hat{a}_2^+ + \hat{a}_3^+)|0\rangle_2|0\rangle_3 =$$
$$= \quad \frac{1}{\sqrt{2}}(i|1\rangle_2|0\rangle_3 + |0\rangle_2|1\rangle_3) \ .$$

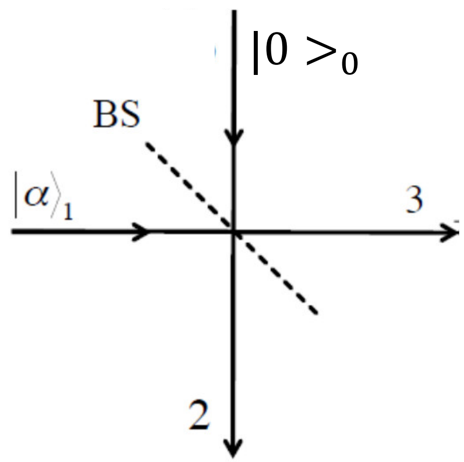
➡ QRN !

Remember:
We choose for reflection coefficient: $r=i/\sqrt{2}$, $i=\exp(i\pi/2)$
Transmission coefficient : $t = 1/\sqrt{2}$

2. Example:

A CS contains an infinite number of Fock states $|n>$.
For small mean photon number $|\alpha|^2$ $\approx$ *1/100)* we obtain quantum random numbers *(cf. paper)*

Input: $|0\rangle_0 \, |\alpha\rangle_1$



Output:

$$|\tfrac{i\alpha}{\sqrt{2}}\rangle_2 \, |\tfrac{\alpha}{\sqrt{2}}\rangle_3 \approx |0\rangle_2 \, |0\rangle_3 + \frac{\alpha}{\sqrt{2}} [\, i|1\rangle_2 \, |0\rangle_3 + |0\rangle_2 \, |1\rangle_3 \,]$$

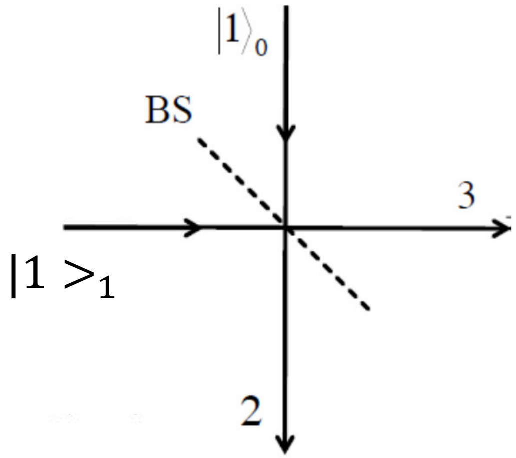
Here $|\alpha\rangle \approx |0\rangle + \alpha\,|1\rangle$, $|\alpha|^2 \ll 1$ has been used.

⇨ QRN !

3. Example:

Hong Ou Mandel *(HOM)* effect; Input of 2 single photons

Result: 2 photons with probability ½ on either output-mode 2 or 3



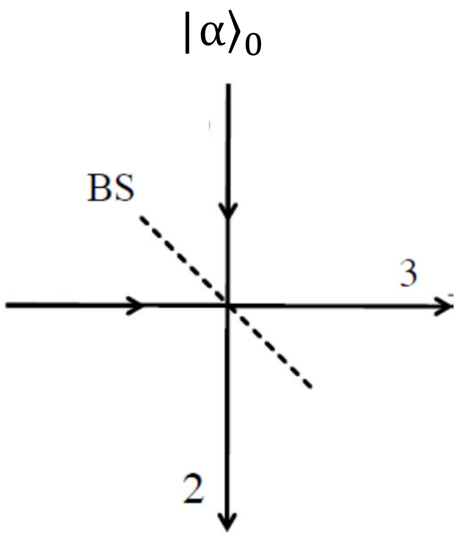
$$\begin{aligned} |1\rangle_0|1\rangle_1 &= \hat{a}_0^+\hat{a}_1^+|0\rangle_0|0\rangle_1 \rightarrow^{BS}\rightarrow \\ &= \frac{i}{\sqrt{2}}(|2\rangle_2|0\rangle_3 + |0\rangle_2|2\rangle_3) \end{aligned}$$

➡ QRN !

4. Example:

CS on input 0 and Schrödinger-cat-state on input 1

$(\,|\,\alpha\,\rangle_1 + |-\alpha\,\rangle_1\,)\,/\,N$



Output:

$$|\psi\rangle_{out} = \frac{1}{N}\left[\,|\sqrt{2}\alpha\rangle_2|0\rangle_3 + |0\rangle_2|\sqrt{2}\alpha\rangle_3\,\right] \quad ,$$

$$N = \sqrt{2(1+e^{-2|\alpha|^2})} \quad .$$

⇨ QRN !

6. Summary

- Double split experiment, Mach-Zehnder interferometer
- Quantum harmonic oscillator, photons and Wigner function
- Beam splitter
- Coherent state
- Quantum random numbers (QRN): 4 Examples

Thank you very much!