Tutorial: Chaotic System Control for Brain Stimulation & FPGA Hardware Implementation

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Outlines

- Chaotic Systems
- Hénon Map Analysis and Control
- Artificial Neural Network Design for Hénon Map
- Artificial Neural Network Design for Lorenz System
- Fixed-point Implementation
- Model and VHDL-based FPGA Design

One Idea and *Three* Methods

- One Idea:
  - Chaotic system simulation, analysis and control for pattern recognition of brain activities and brain stimulation.
- Three Methods:
  - Chaotic systems analysis and control
  - Artificial Neural Network (ANN) architecture design and optimization
  - FPGA fixed-point hardware implementation

The Idea:
Brain Research Program Overview

Brain Stimulation
- Parkinson's Disease – tremor
- Epilepsy – seizure

Chaotic Systems
- Dynamic Analysis and Control
- Artificial Neural Network based Model

Machine Learning
- Feature Extraction of EEG Signals
- Pattern Recognition and Classification

The Practical Goal: Brain Stimulation

- Electroencephalogram (EEG) uses electrodes attached to the scalp to capture brainwave signals;
- EEG signals captured from brain activities demonstrate chaotic behaviors (bifurcation etc.)
- Brain Stimulation
  - Deep brain stimulation
  - **Non-invasive brain stimulation**

    Eg. Direct current (tDCS), Electromagnetic, ultrasound

The Challenges and Remedies

- Challenges
  - EEG signals are individual dependent and the amount of available data is limited;
  - EEG signals are affected by noise
  - ANN training require big data
- Remedies
  - The outputs of chaotic systems are used to train ANN to simulate brain activities
  - FPGA hardware implementation for parallel processing and acceleration

Chaotic Systems

- A chaotic system is a bound system which obtains the existence of attractor.
- Outputs depends on initial values and system parameters;
- Predictability, probability and controllability;
- Examples:
  - 1D – Logistic map, Gaussian map
  - **2D – Hénon map**
  - **3D – Lorenz system**, Röseller system

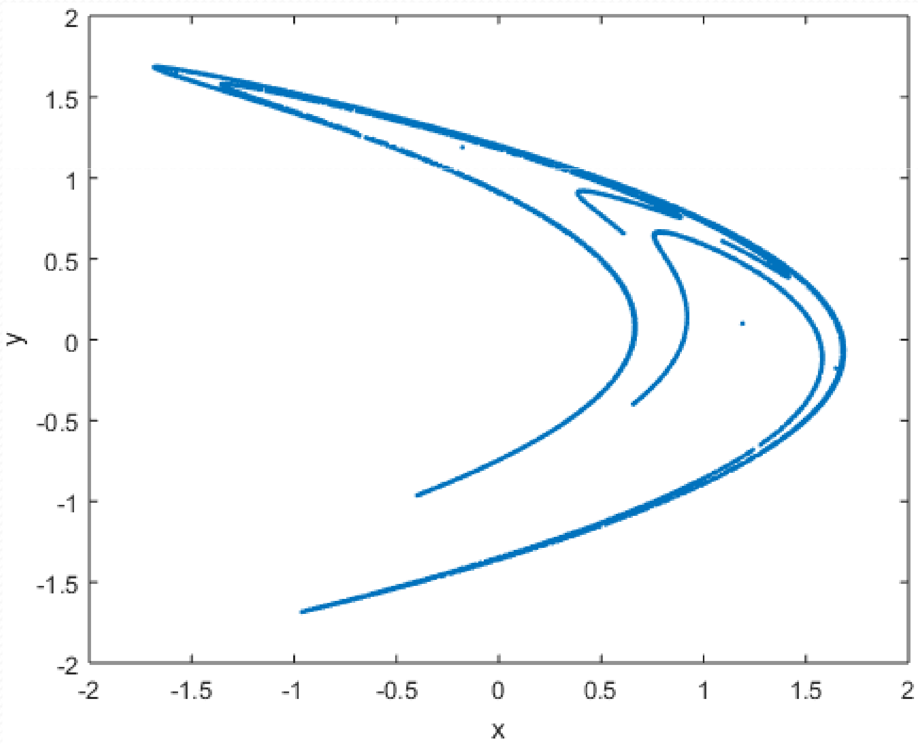
Hénon Map - Definition

Equations by definition:

$$x_{n+1} = 1 + y_n - ax_n^2$$
$$y_{n+1} = bx_n$$

Reformed equations :

$$x'_n = \frac{1}{\alpha}x_n, y'_n = \frac{\beta}{\alpha}y_n$$
$$x_{n+1} = \alpha + \beta y_n - x_n^2$$
$$y_{n+1} = x_n$$



Hénon Map Analysis

Jacobian Matrix:

$$J(x_1, y_1) = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix} \Bigg|_{(x_1, y_1)}$$

Hénon I:

$$J = \begin{pmatrix} -2\alpha x & 1 \\ \beta & 0 \end{pmatrix} = \begin{pmatrix} -2.4x & 1 \\ 0.4 & 0 \end{pmatrix}$$

$$Eig(J)_{(x_1=-1.1965)} = \begin{cases} \lambda_1 \approx 3.0047 \\ \lambda_2 \approx -0.1331 \end{cases}$$

$$Eig(J)_{(x_2=0.6965)} = \begin{cases} \lambda_1 \approx 0.2123 \\ \lambda_2 \approx -1.8839 \end{cases}$$

Hénon II:

$$J = \begin{pmatrix} -2x & \beta \\ 1 & 0 \end{pmatrix}$$

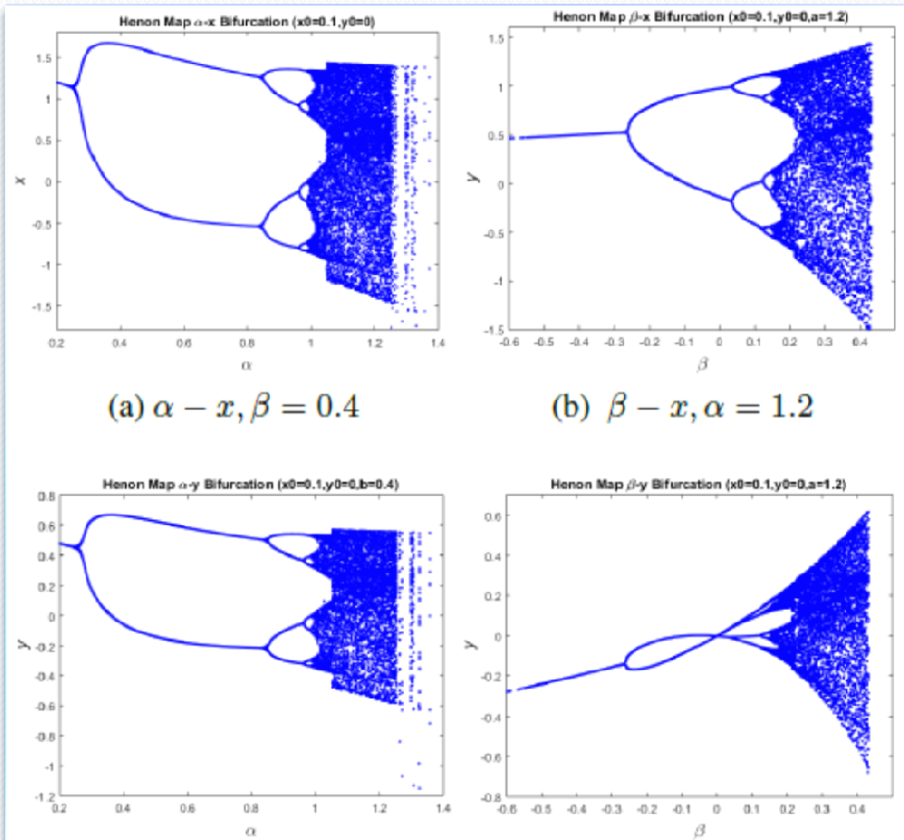
$$Eig(J)_{(x_1=0.8358)} = \begin{cases} \lambda_1 \approx 0.2123 \\ \lambda_2 \approx -1.8839 \end{cases}$$

$$Eig(J)_{(x_2=-1.4358)} = \begin{cases} \lambda_1 \approx 3.0047 \\ \lambda_2 \approx -0.1331 \end{cases}$$

Critical points of period N orbit is stable as long as:

$$|\lambda_1| < 1 \text{ and } |\lambda_2| < 1$$

Hénon Map - Bifurcation



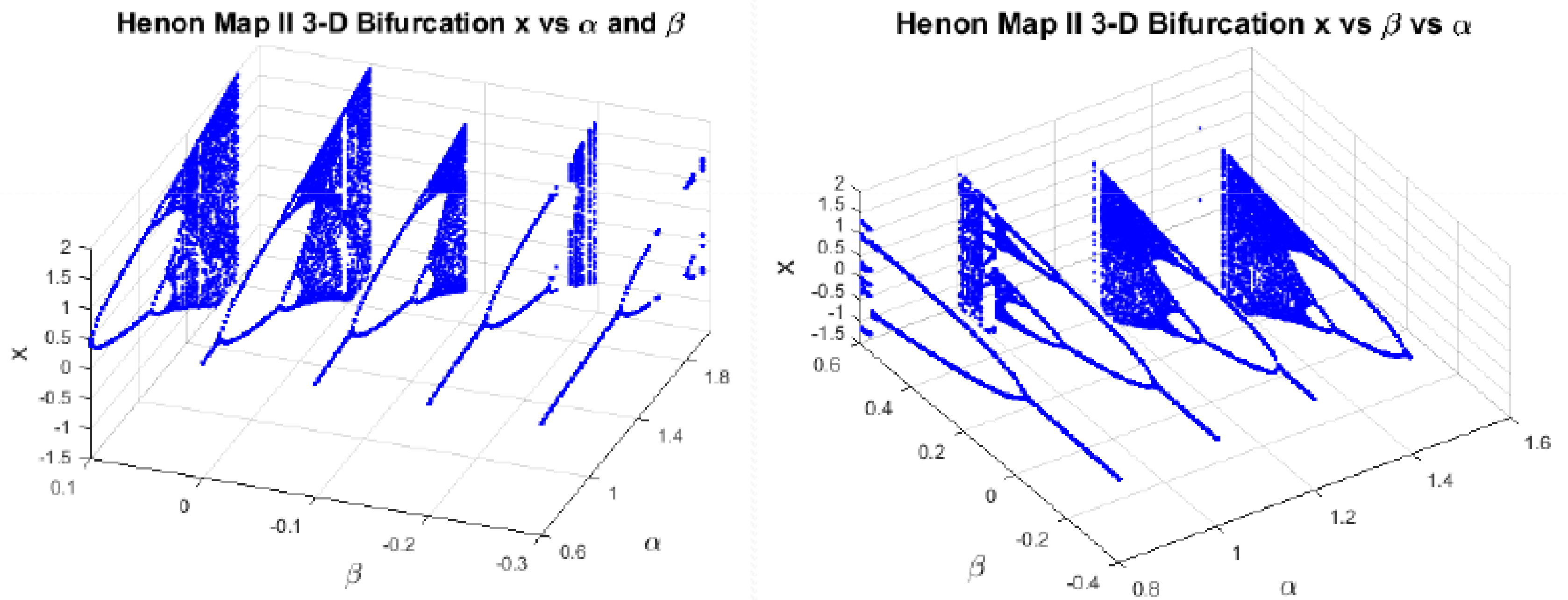
(a) $\alpha - x, \beta = 0.4$ (b) $\beta - x, \alpha = 1.2$

(c) $\alpha - y, \beta = 0.4$ (d) $\beta - y, \alpha = 1.2$

(a) & (c) The bifurcation points (h_1 =0) are found at :
α= 0.27 (period one doubling)
α= 0.85 (period two doubling)
α = 0.99 (period four doubling)

(b) & (d) The bifurcation points (h_1 =1) are found at :
β= 0.265 (period one doubling)
β = 0.035 (period two doubling)
β = 0.125 (period four doubling)

Hénon Map Bifurcation 3D

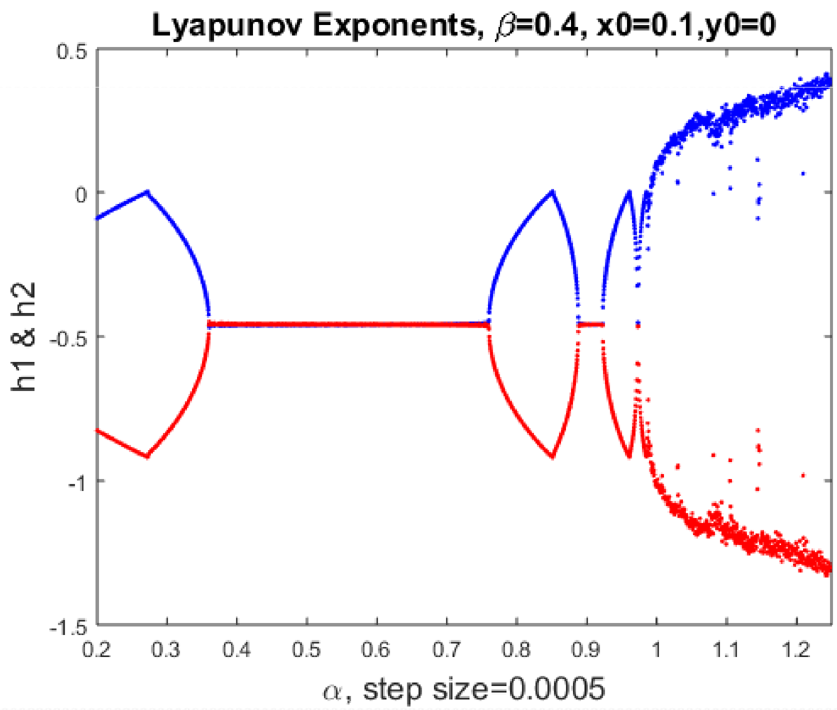


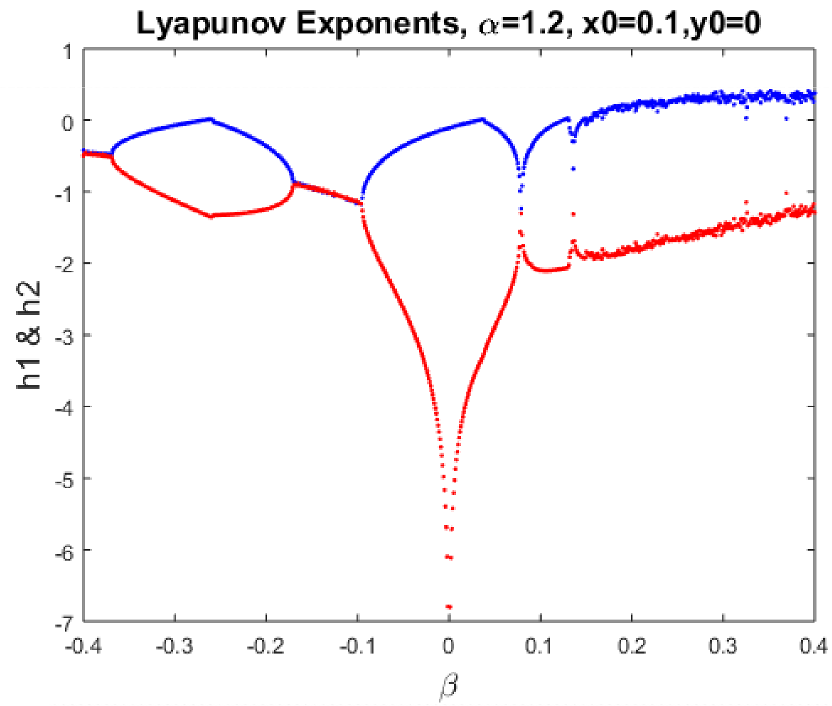
(a) x over α bifurcation with various β

(b) x over β bifurcation with various α

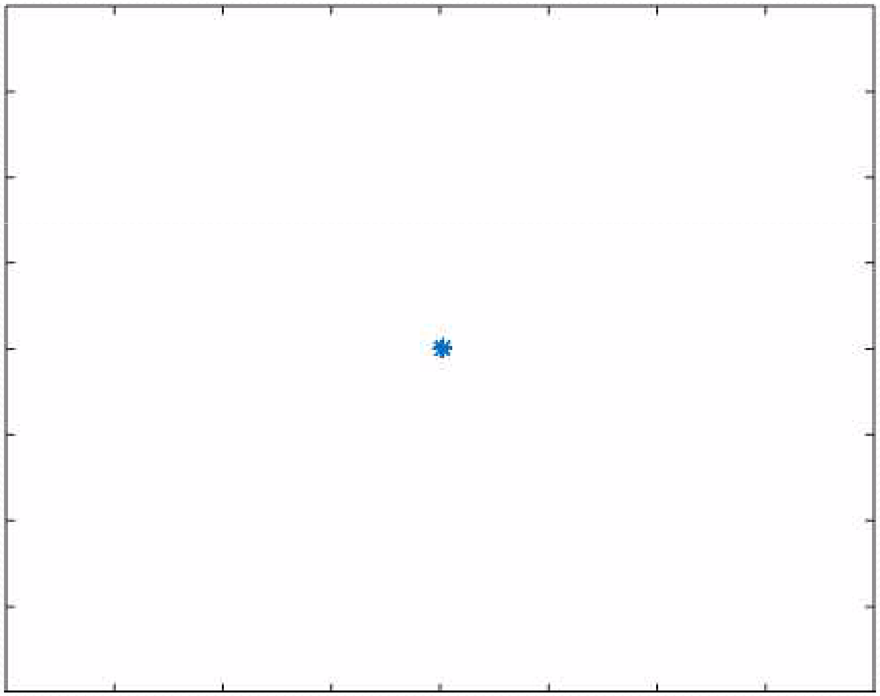
Hénon Map Lyapunov Exponents

$$L(x_0) = log\left(Eig \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} \left(J_i(x_0)\cdot J_i^T(x_0)\right)^{\frac{1}{2}}\right)$$

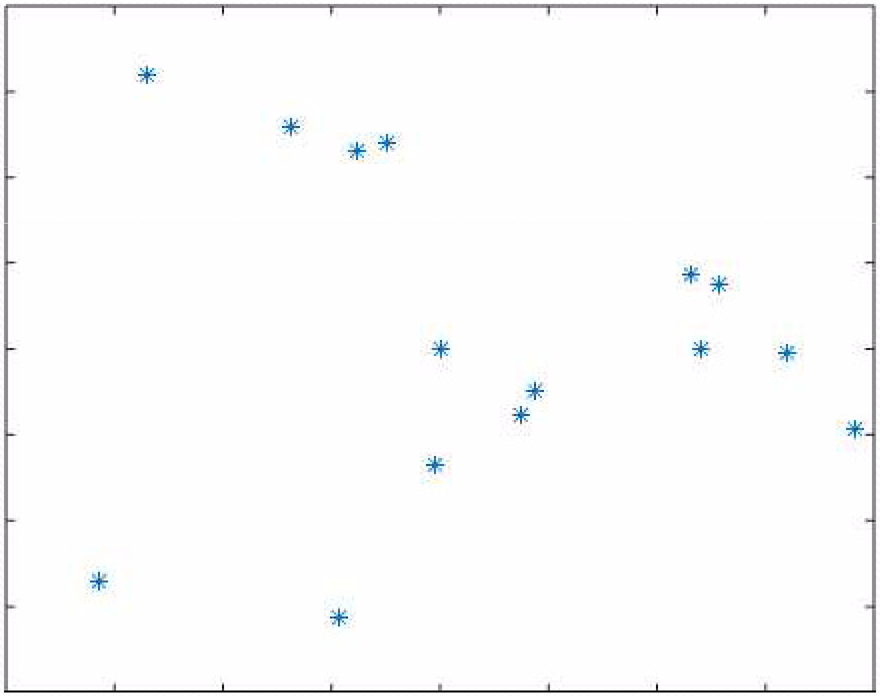




Hénon Map Bifurcation Animation



a=0.2~1.4, b=0.4



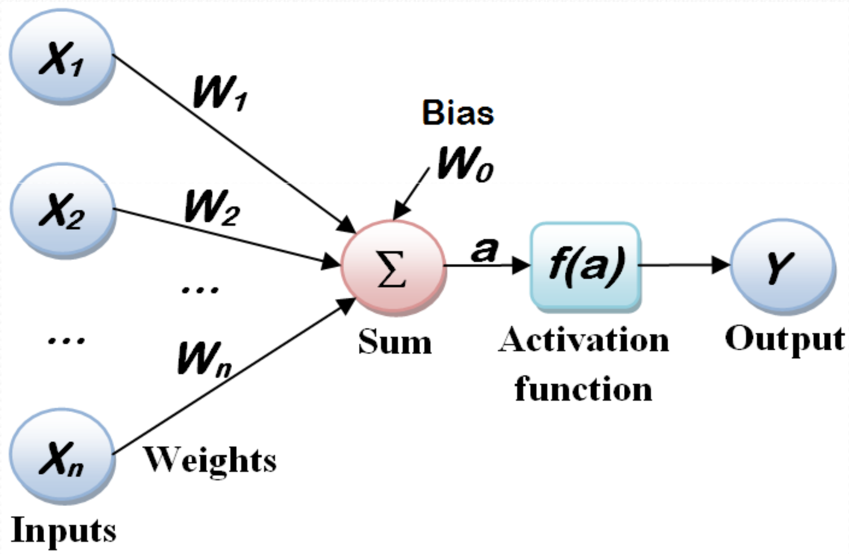
a=1.2, b=-0.6~0.4

ANN Model Design for Chaotic Systems

- An feed forward ANN can be trained using the output values of a chaotic system.
- The training process is carried out on a computer and the weights and bias are generated for all neurons in an ANN architecture.
- The complexity of the ANN architecture defines the implementation cost and speed. Therefore it is beneficial to use less number of hidden neurons to achieve the target training performance.

A Simple Neuron Model

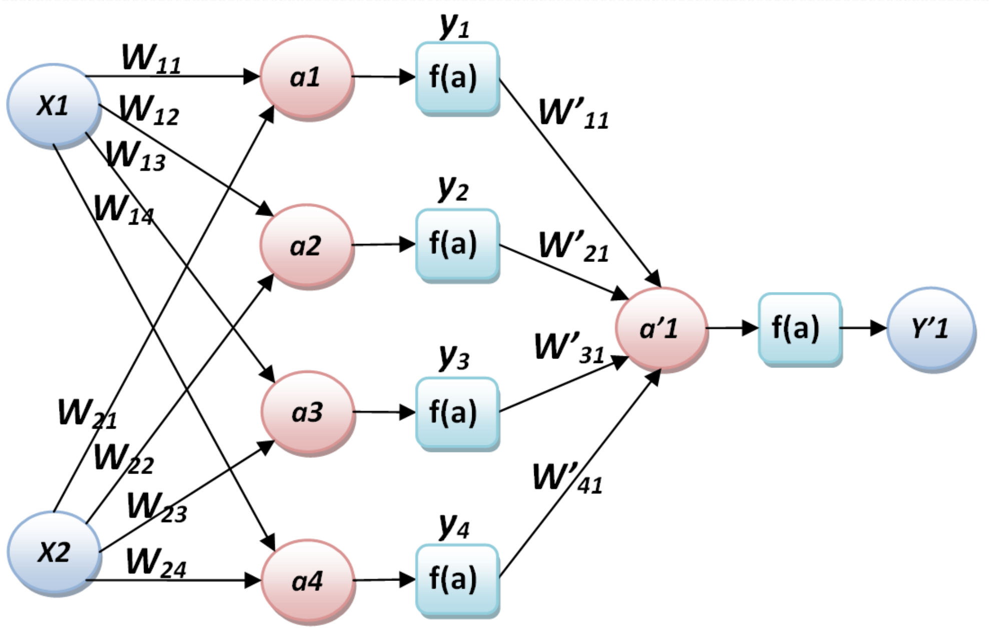
- Inputs
- Weights
- Biases
- Summed Weights
- Activation Function
- Outputs



Artificial Neural Network

$$a_j^l = \sum_{i=1}^{N_{l-1}} w_{j,i}^l x_i + b_{j,0}^l \qquad j = 1, 2, \ldots N_l$$

$$y_j^l = f_l(a_j^l)$$



ANN Training

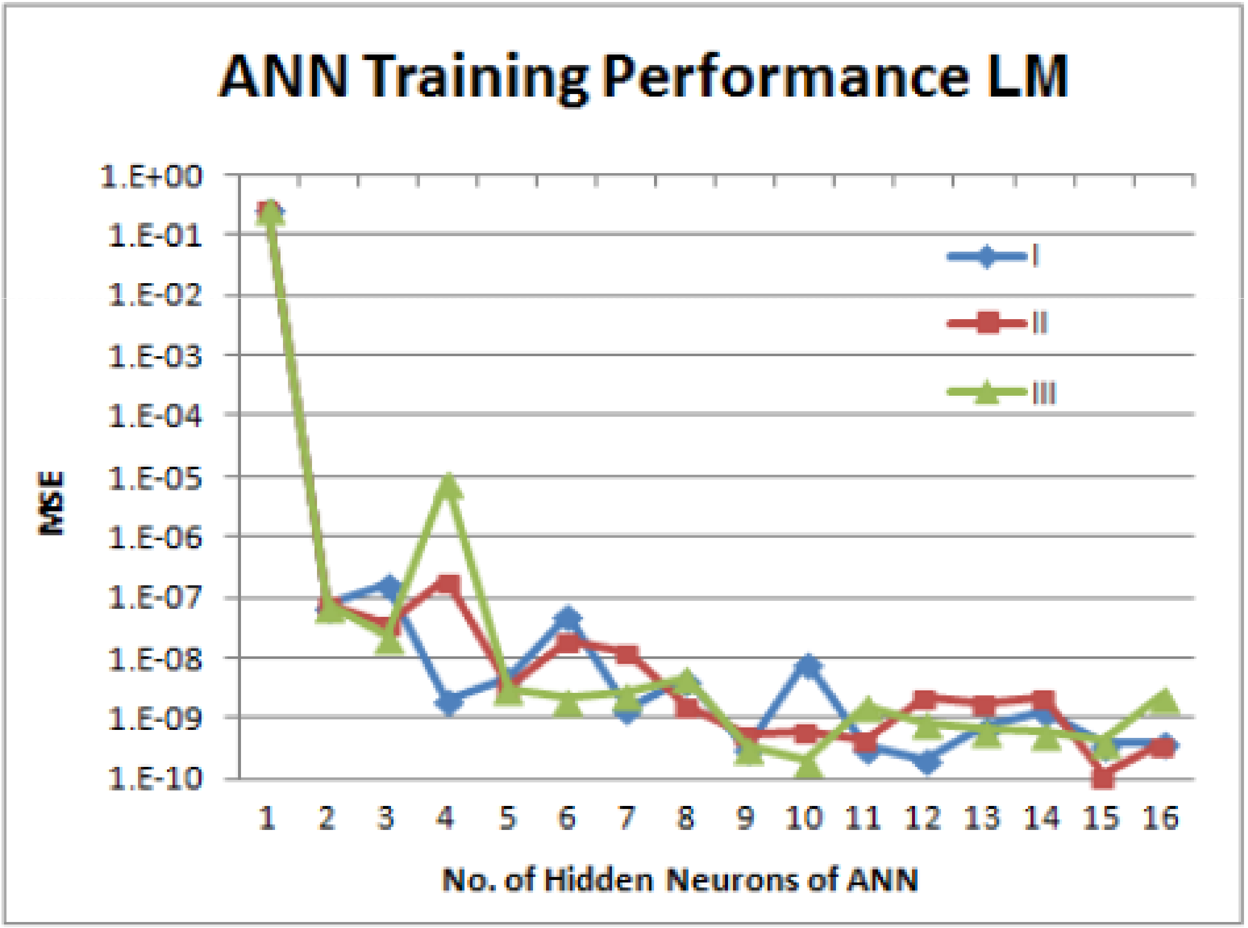
- 3 Training Algorithms:
  - Levenberg- Marquardt (LM)
  - Bayesian Regularization (BR)
  - Scaled Conjugate Gradient (SCG)
- 16 Architectures (1 to 16 hidden neurons) for each algorithm
- 3 Training iterations for per architecture per algorithm

ANN Training Performance

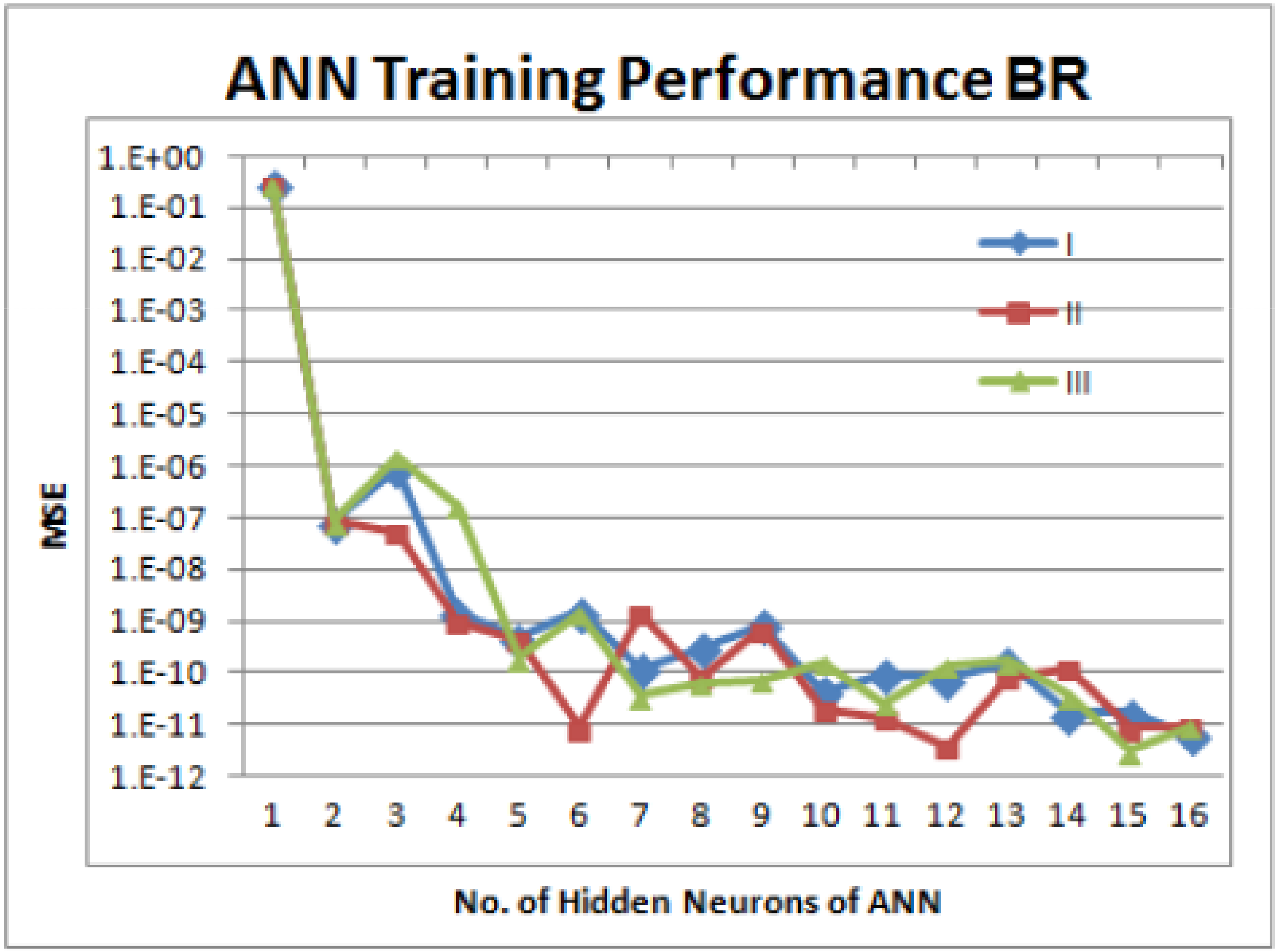
- The ANN training result is measured by the error between the calculated output y and the target training output ŷ.
- The performance of the ANN training process is evaluated by how fast and well the error converge to the target threshold.
- The most common method for measuring the output error is Mean Squared Error – MSE

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

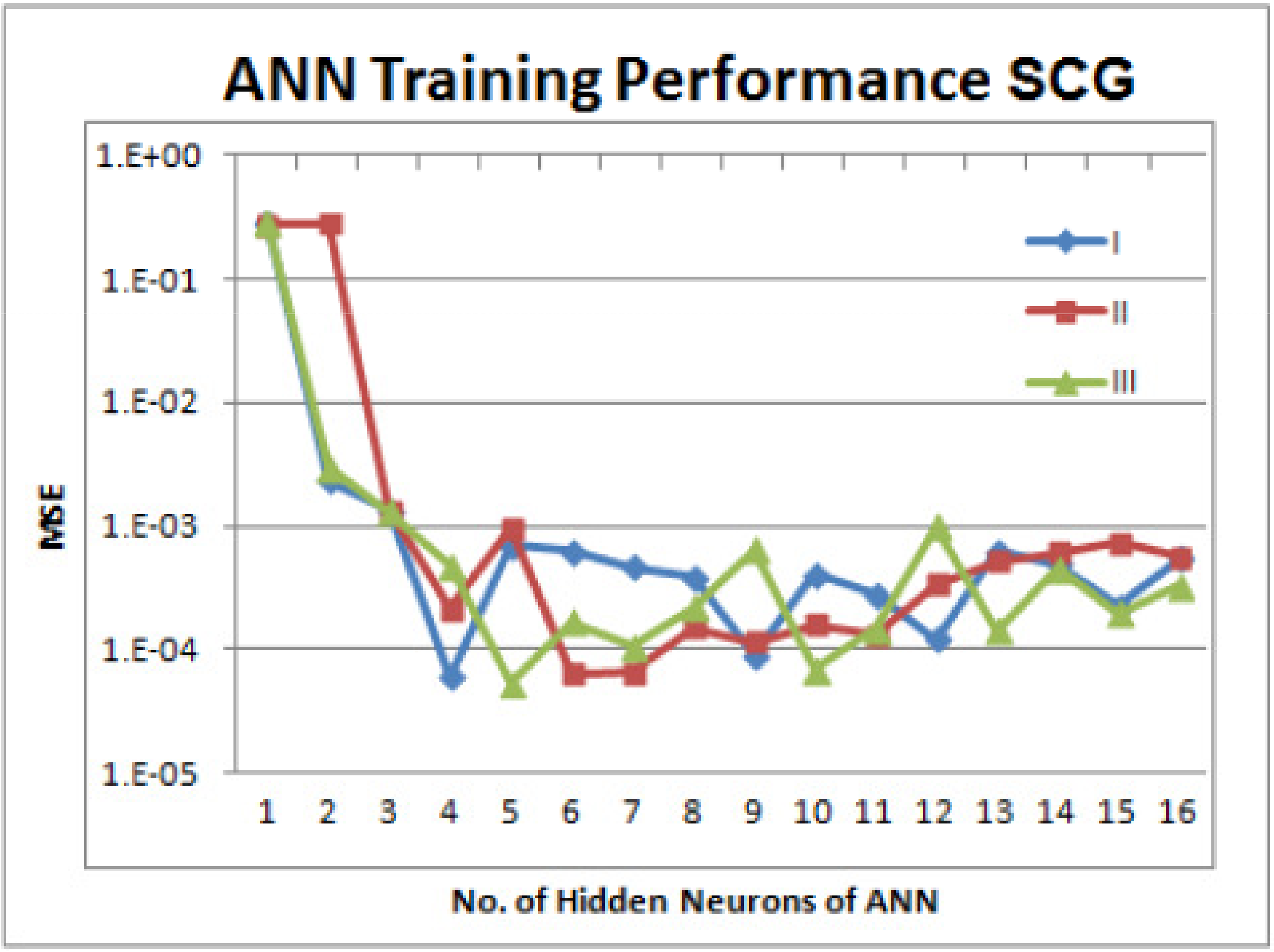
Hénon Map Training Results -LM



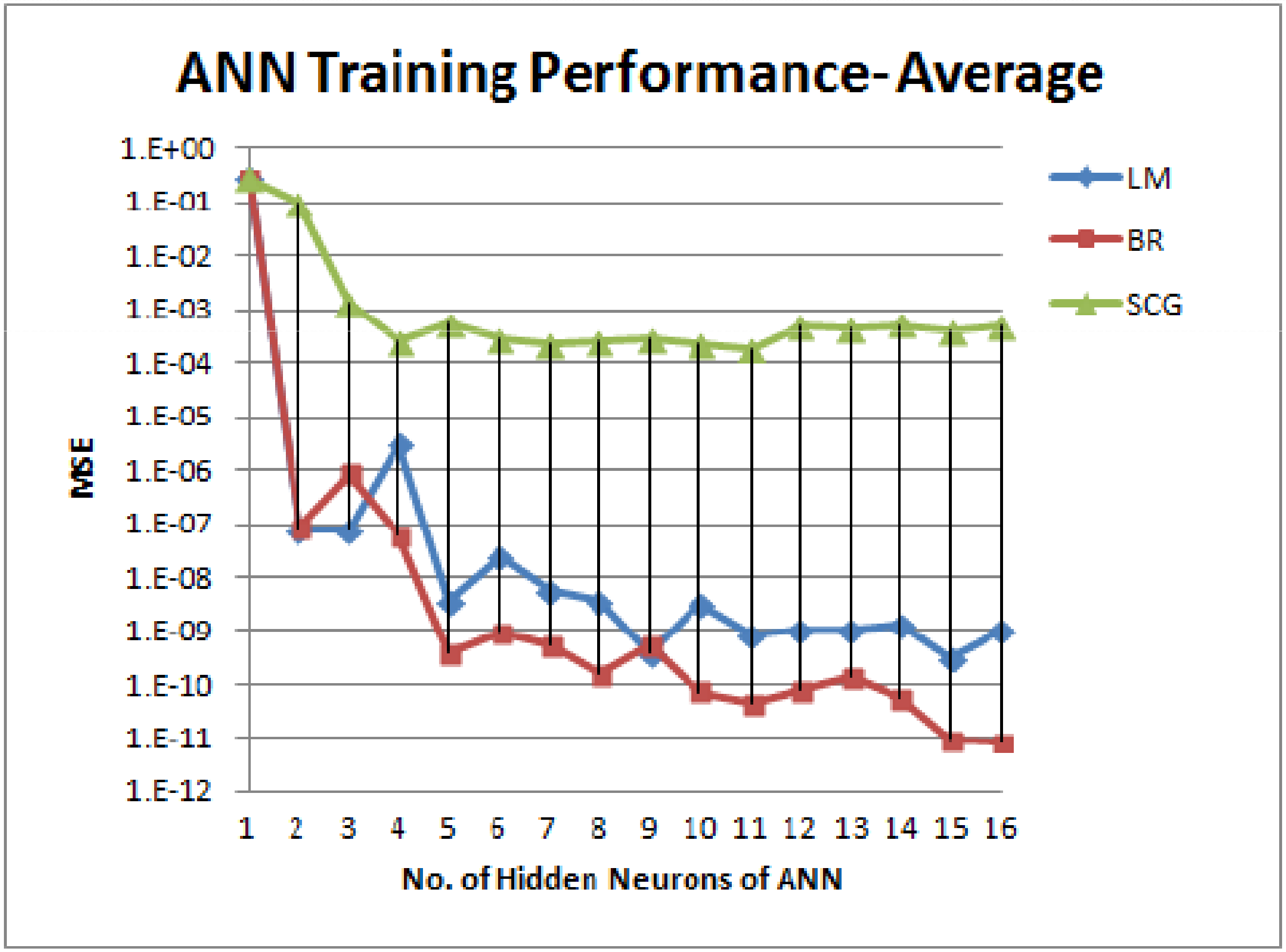
Hénon Map Training Results -BR



Hénon Map Training Results-SCG



Hénon Map Training Results



Hénon Map ANN Architecture

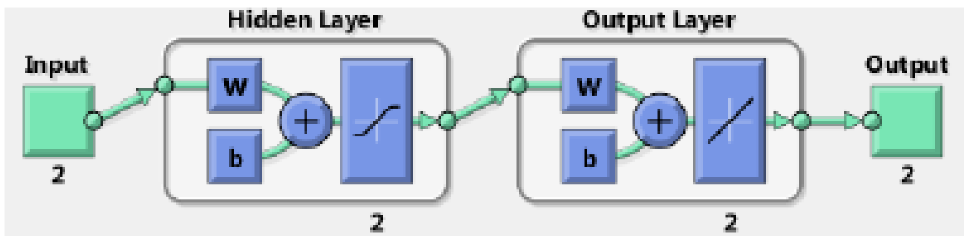


Figure 1. ANN Architecture for Hénon Map Chaotic System

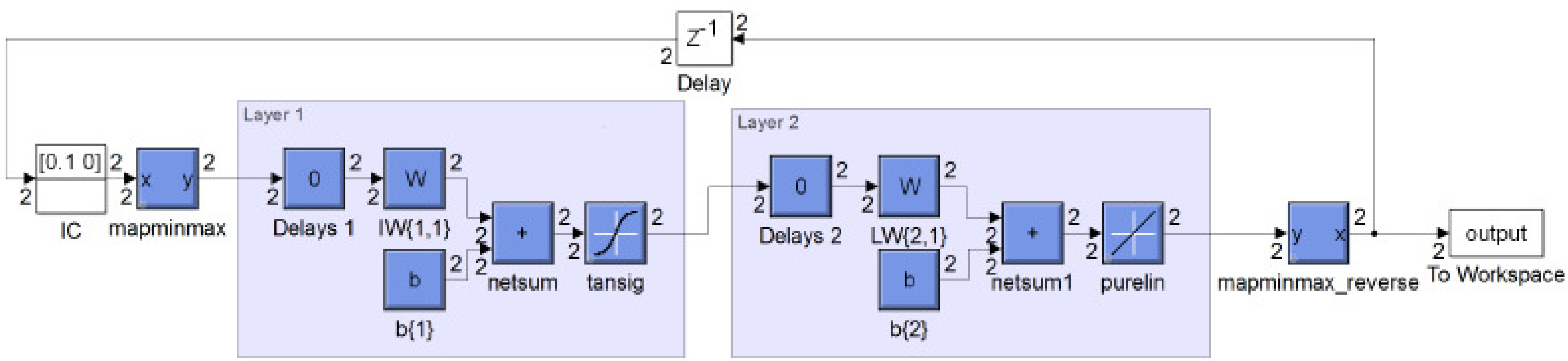
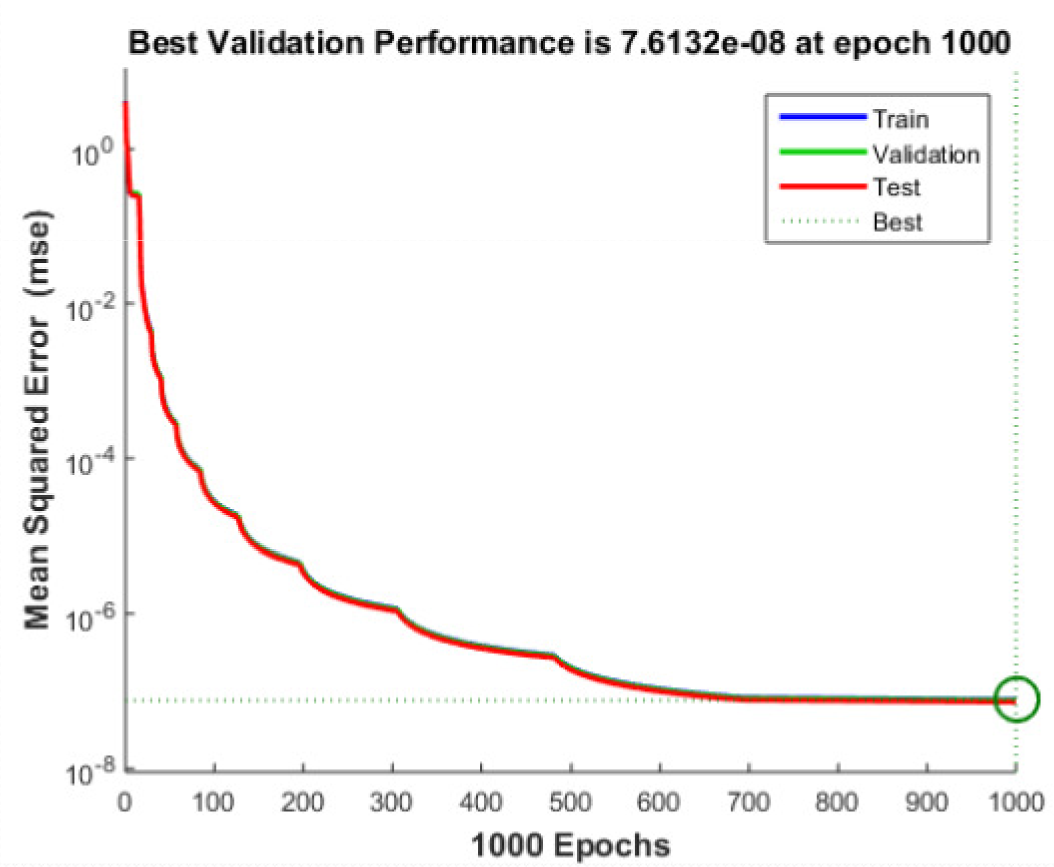
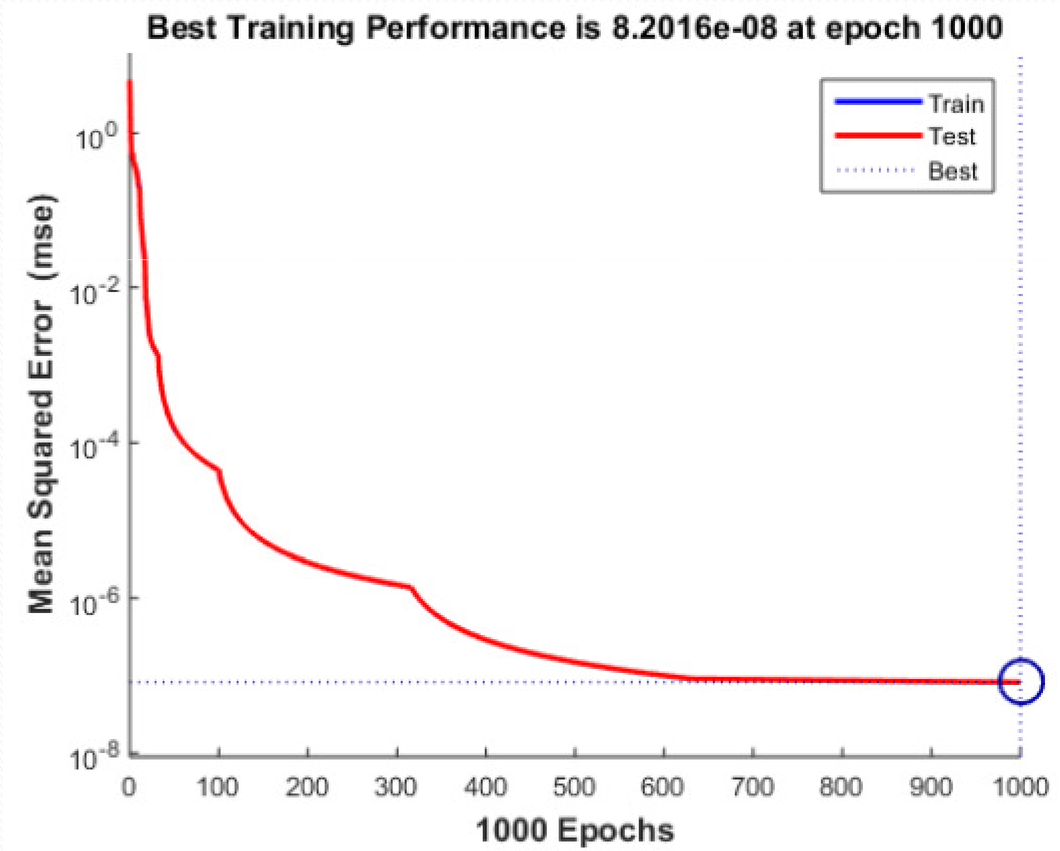


Figure 3. Simulink Model for ANN-based Hénon Map Chaotic System

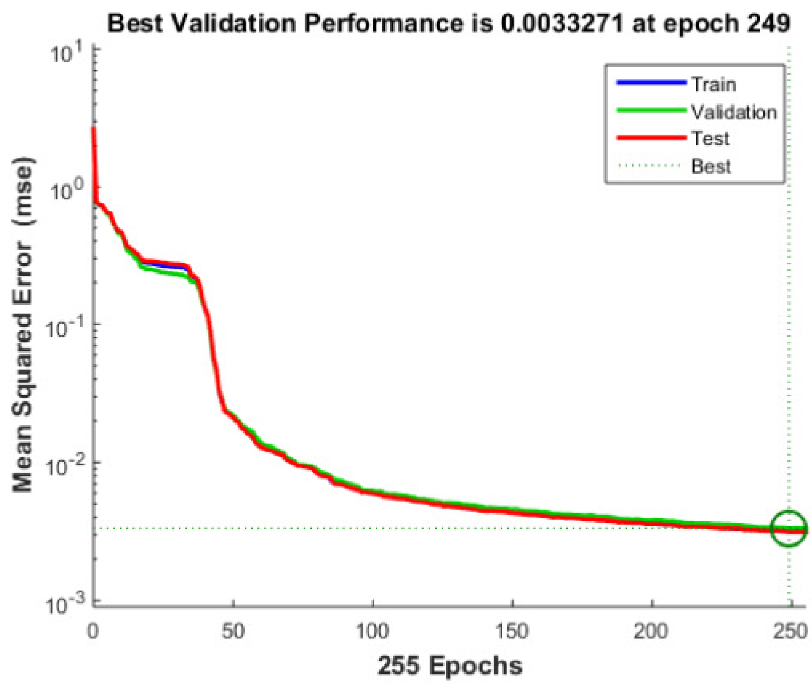
Hénon Map Training Performance 2-hidden neurons LM

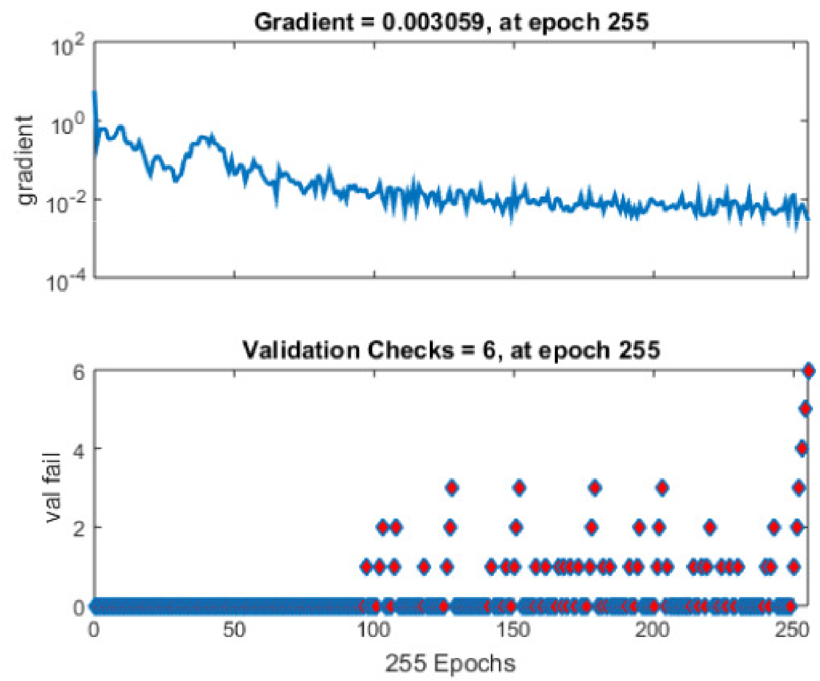


Hénon Map Training Performance 2-hidden neurons BR

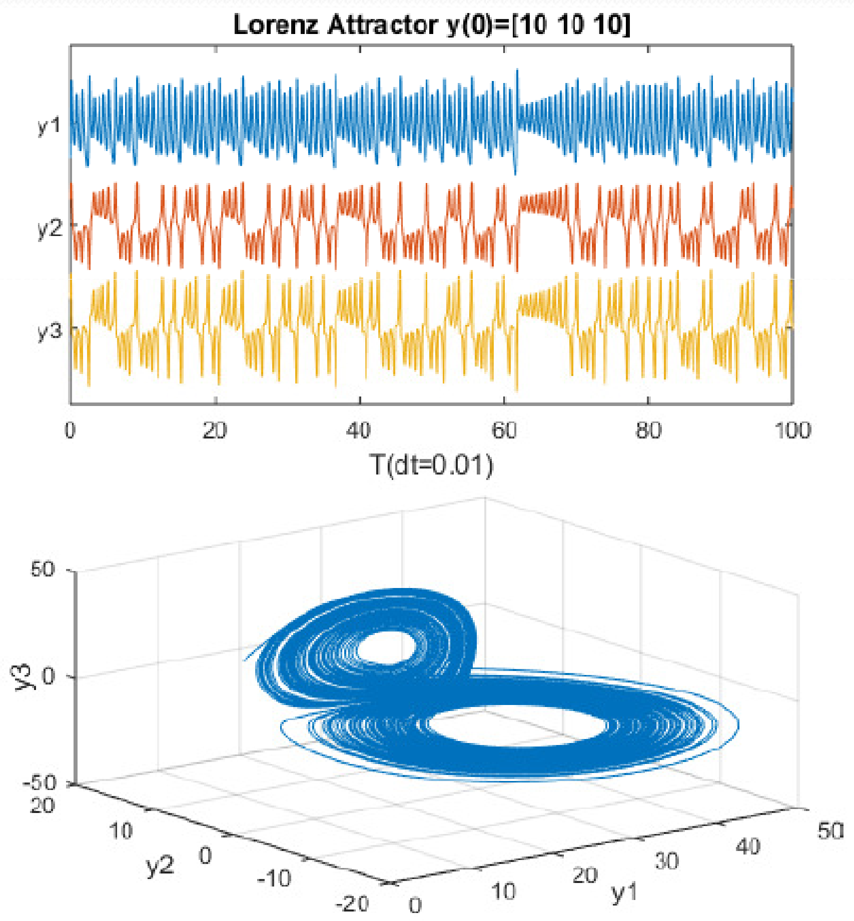


Hénon Map Training Performance 2-hidden neurons SCG





Lorenz Chaotic System

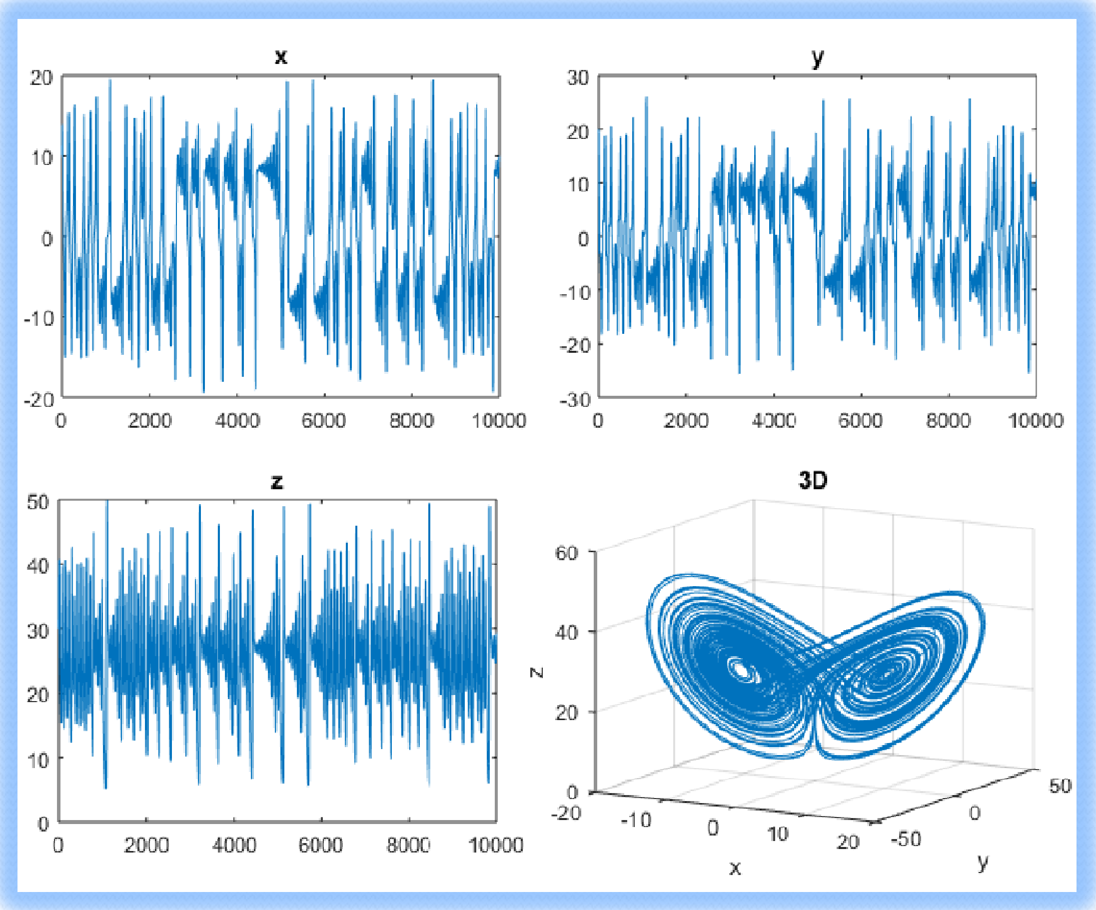


$$\frac{dx}{dt} = \sigma(y - x)$$

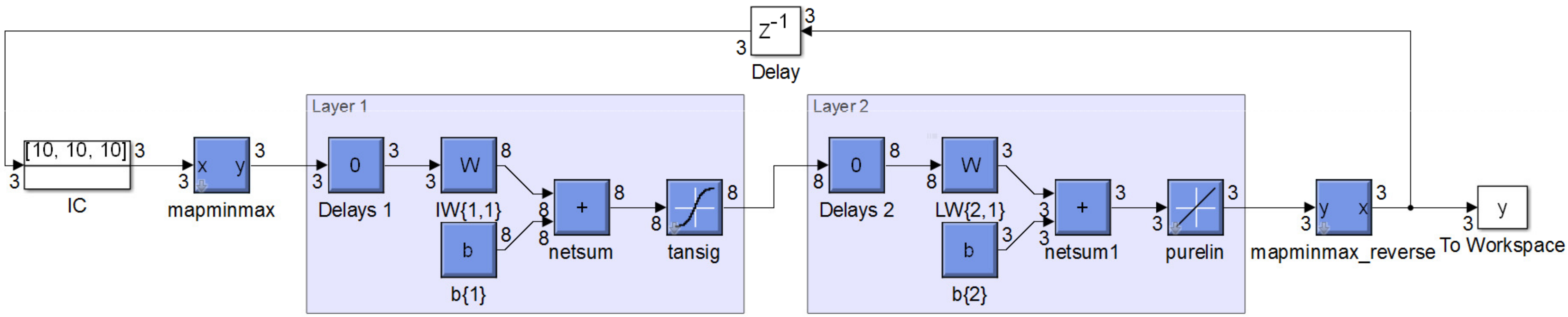
$$\frac{dy}{dt} = \rho x - y - xz$$

$$\frac{dz}{dt} = -\beta z + xy$$

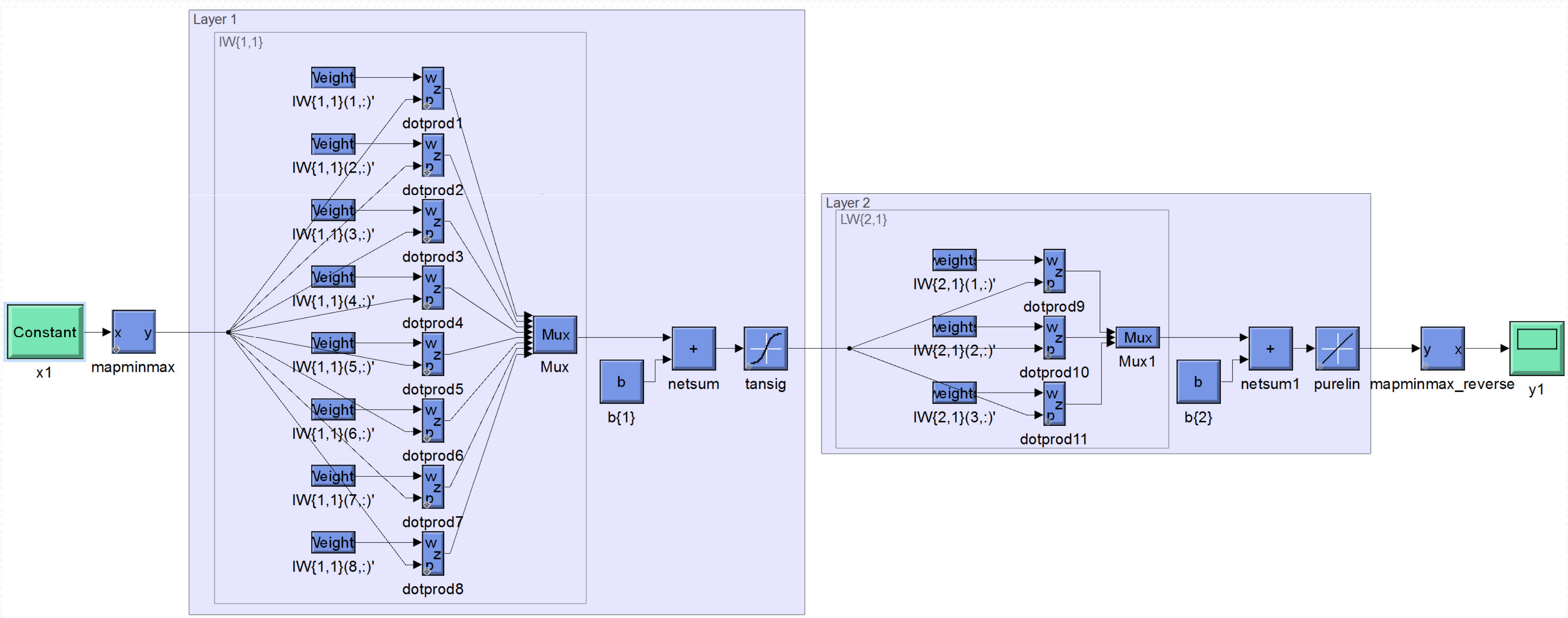
The Lorenz Butterfly (10,20,30)



Lorenz System ANN Model

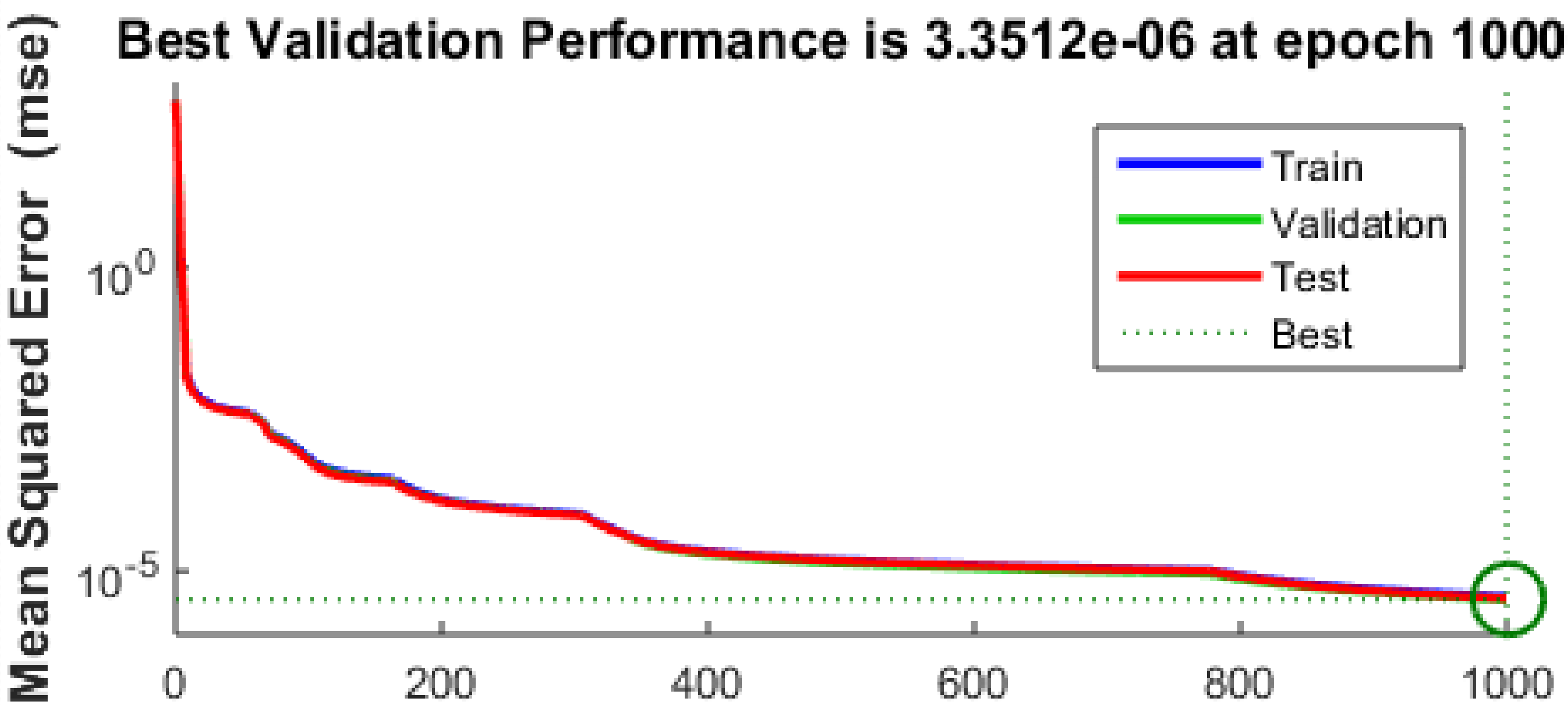


3x8x3 ANN Architecture



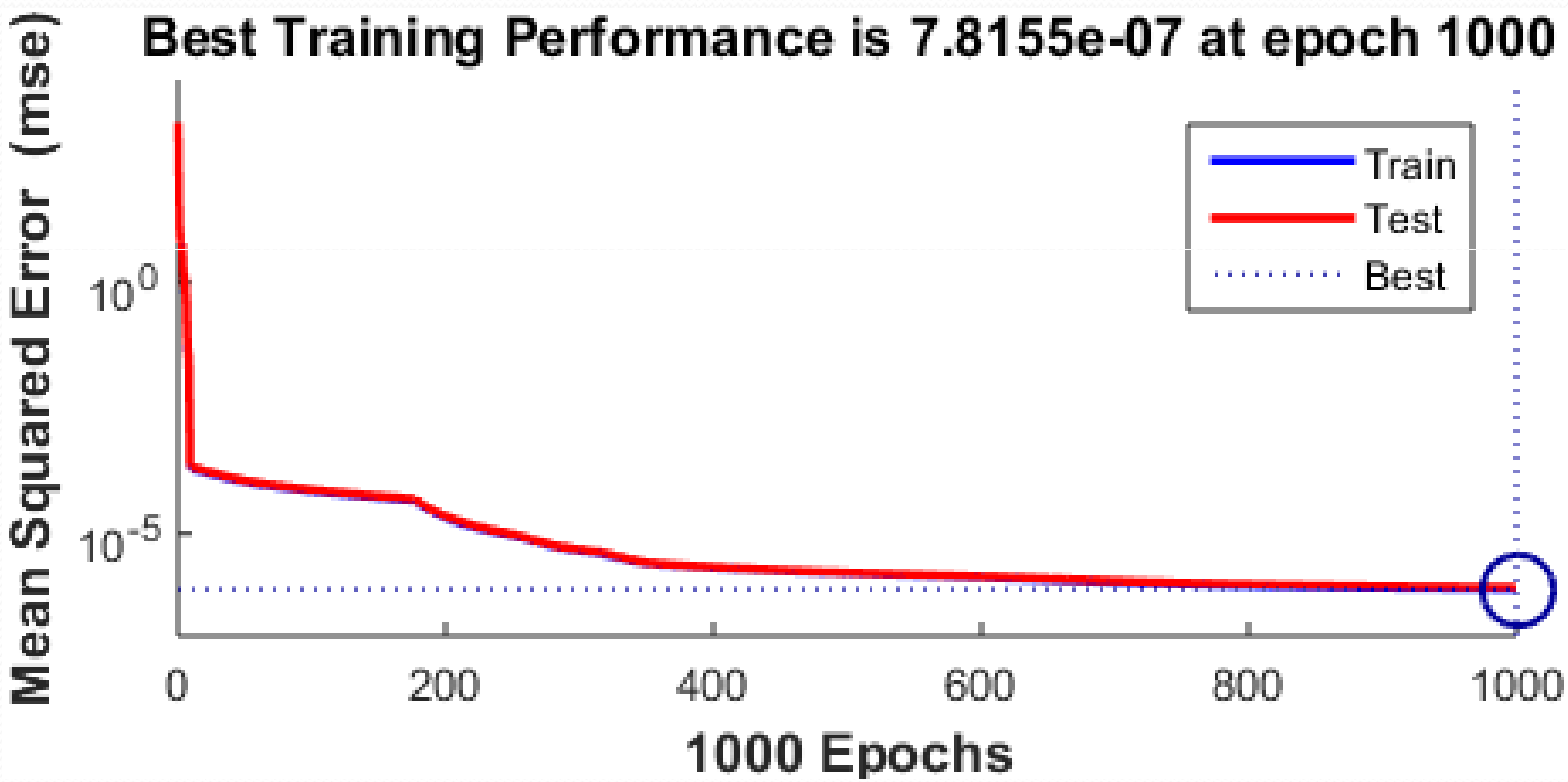
Training Performance – LM

– 8 hidden neurons



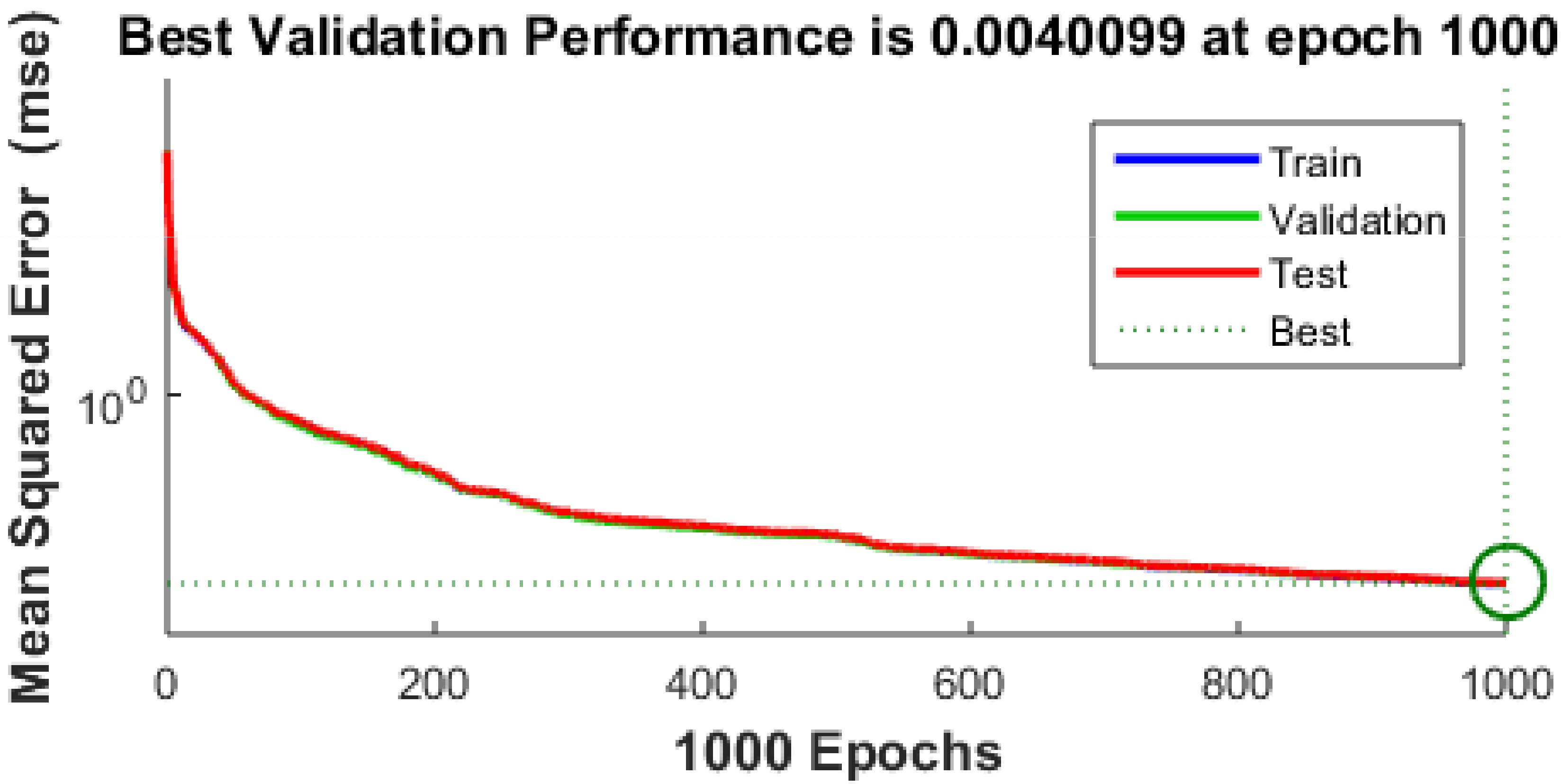
Training Performance – BR

– 8 hidden neurons

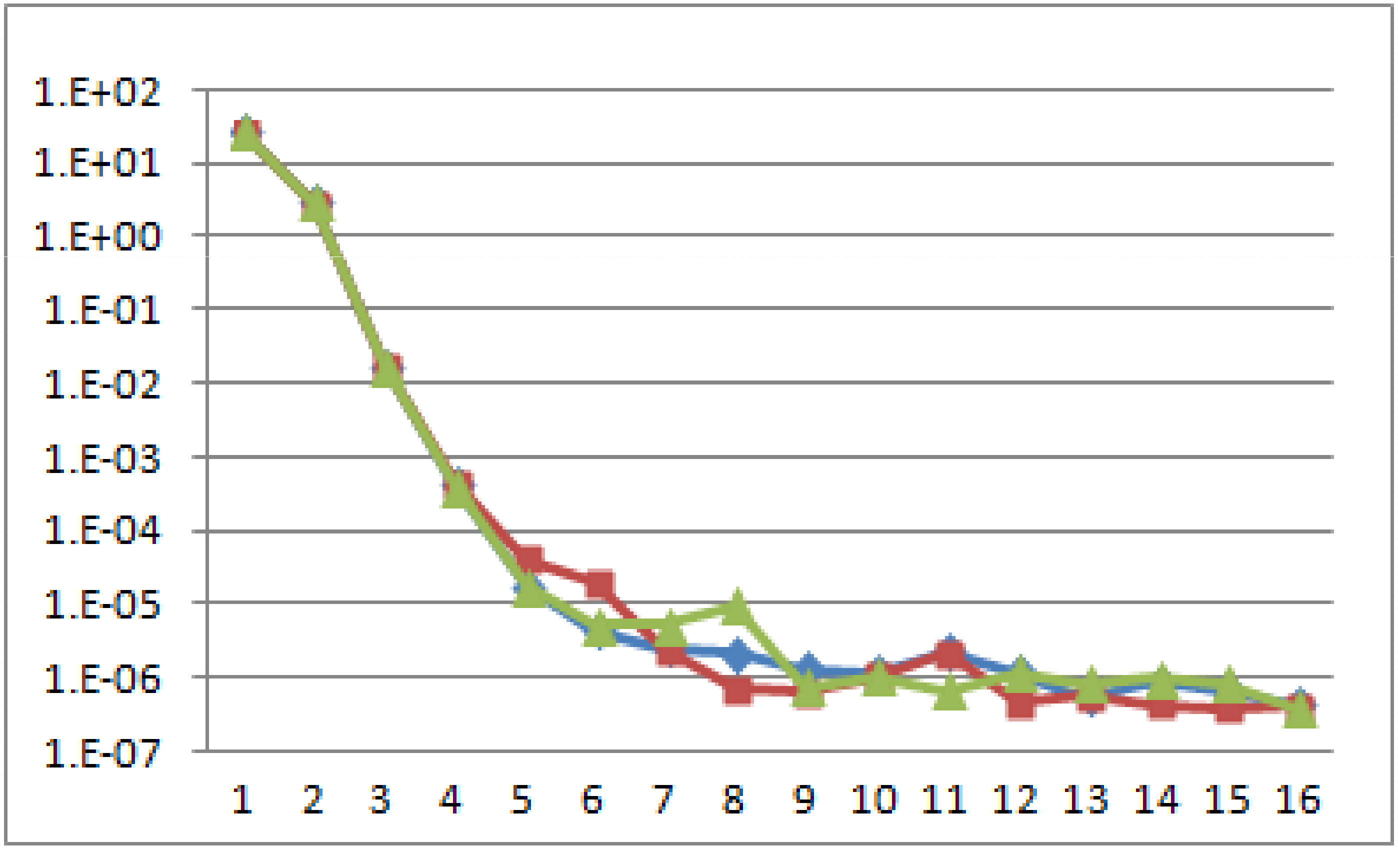


Training Performance – SCG

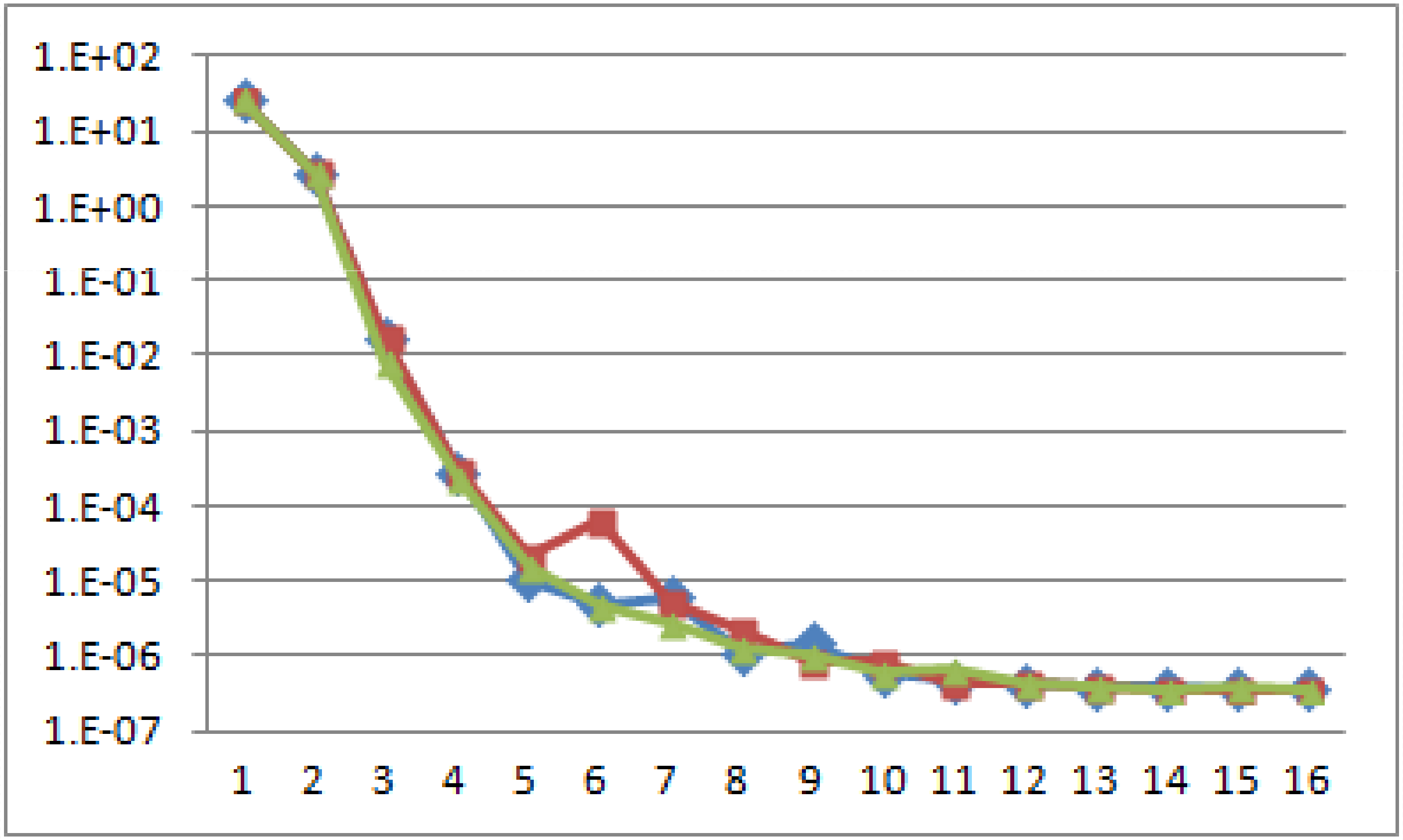
– 8 hidden neurons



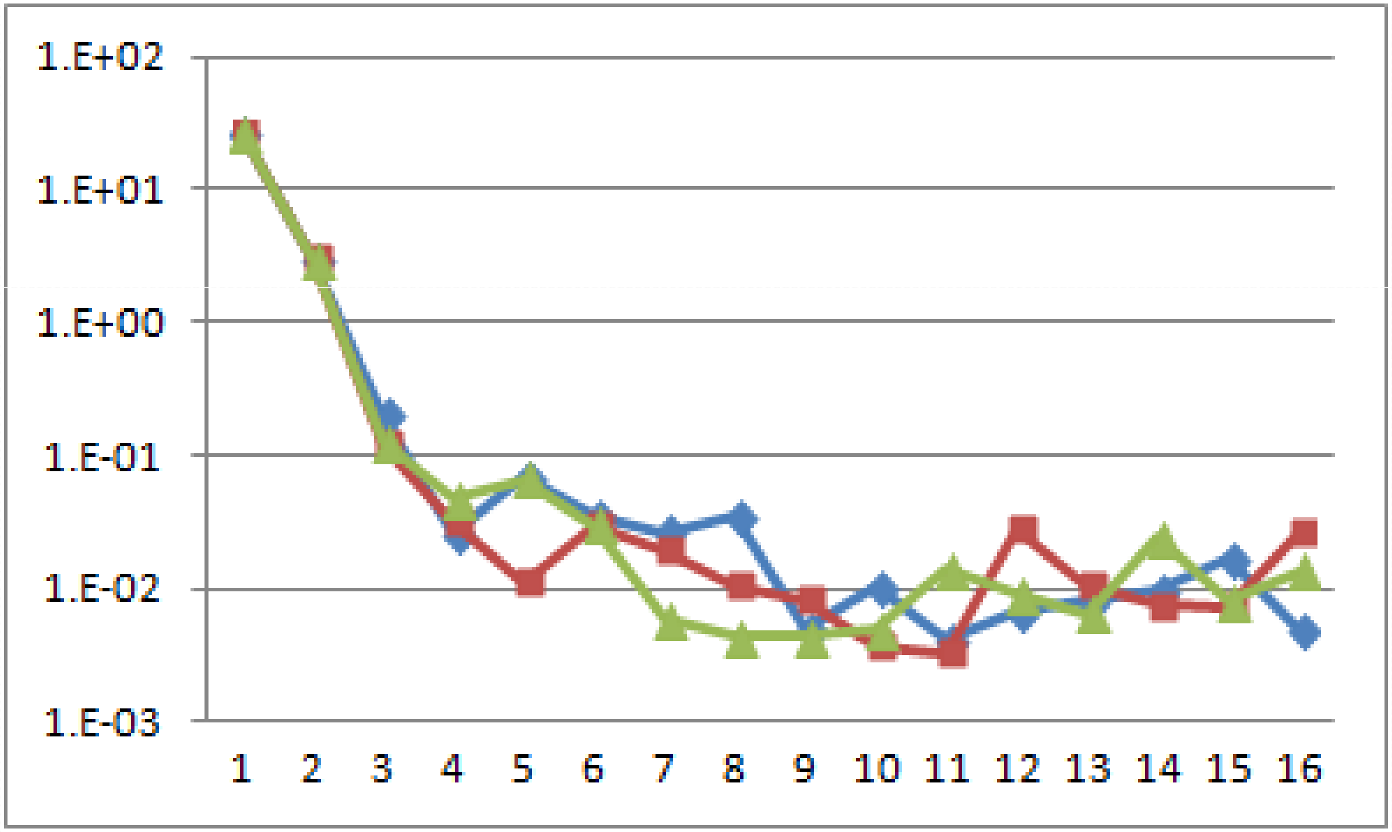
Best Training Performance- LM



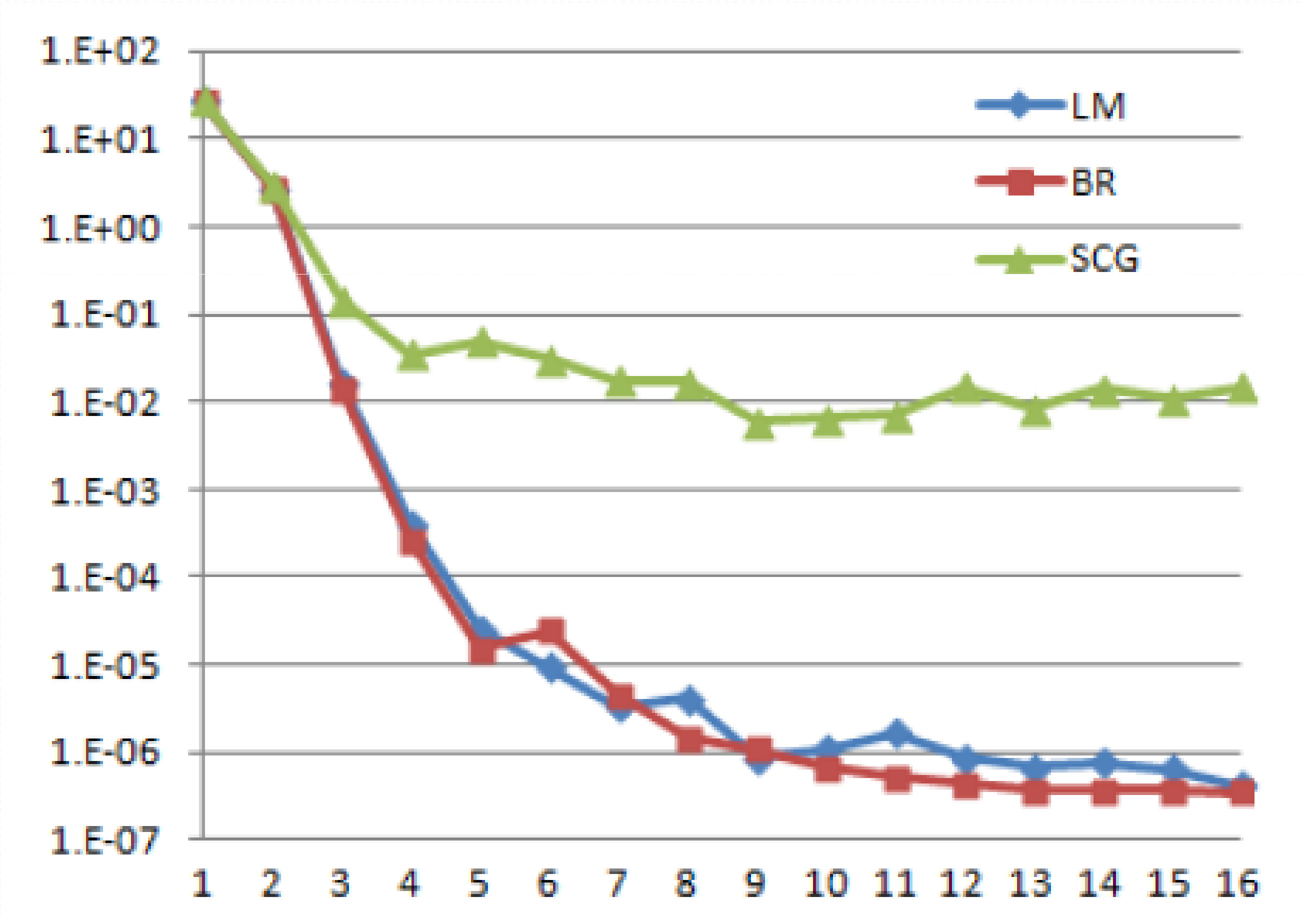
Best Training Performance- BR



Best Training Performance - SCG



Averaged Training Results



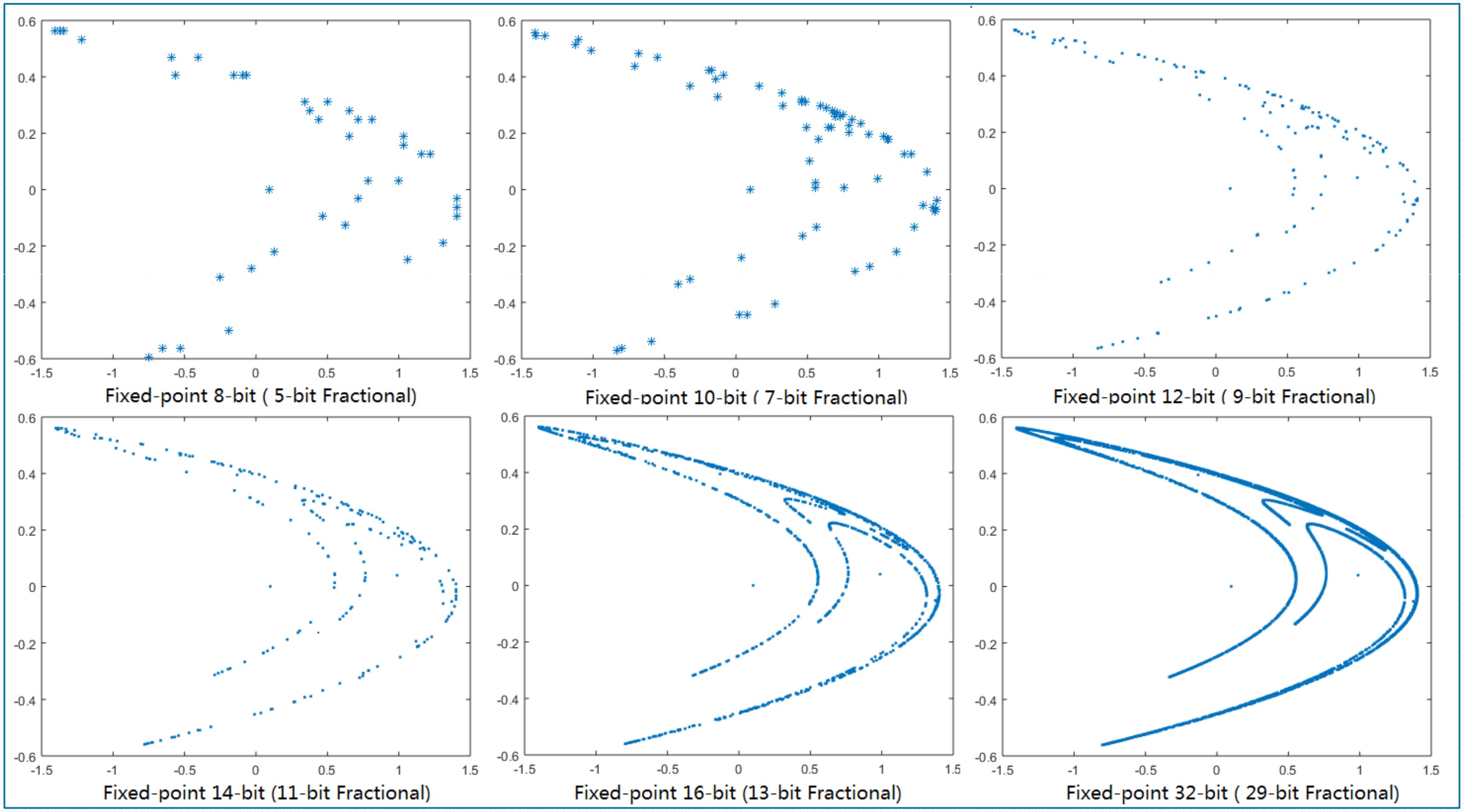
Fixed-point Representation

- The range of the singed fixed-point is represented by

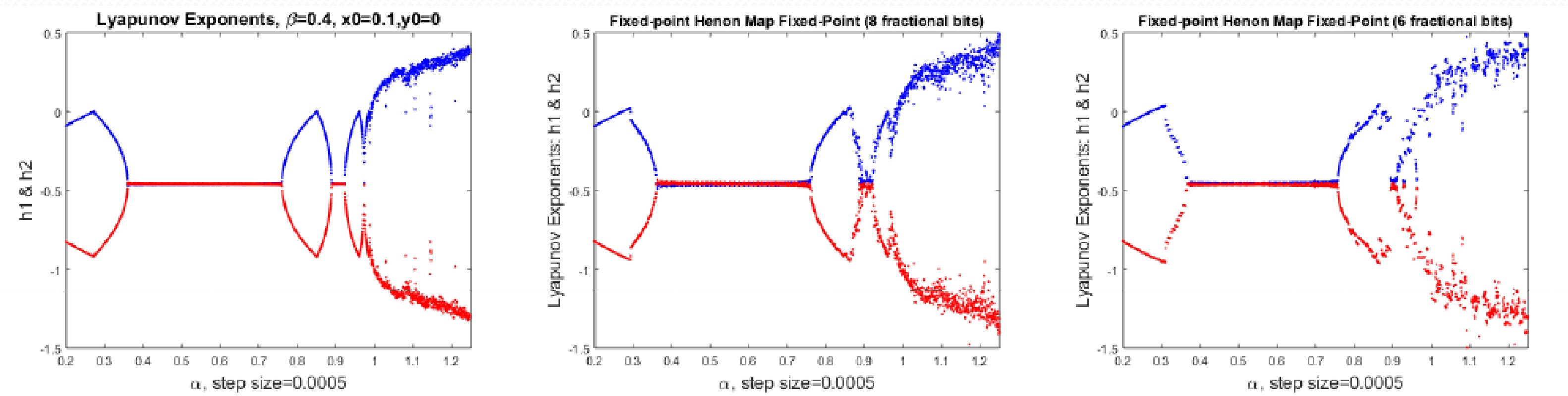
$$-(2^{N_i} - 2^{-N_f} + 1) \sim +(2^{N_i} - 2^{-N_f})$$

- where Ni be the number of integer bits, Nf be the number of fractional bits. The precision (step size) is 2^(-Nf).

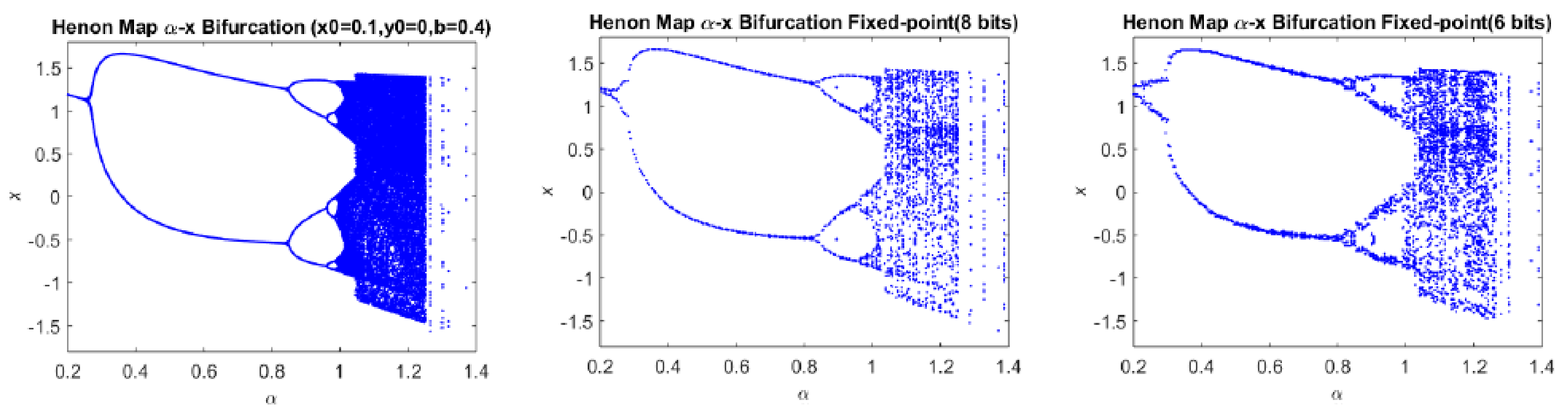
Hénon Map Fixed-point



Hénon Map Fixed-point Analysis

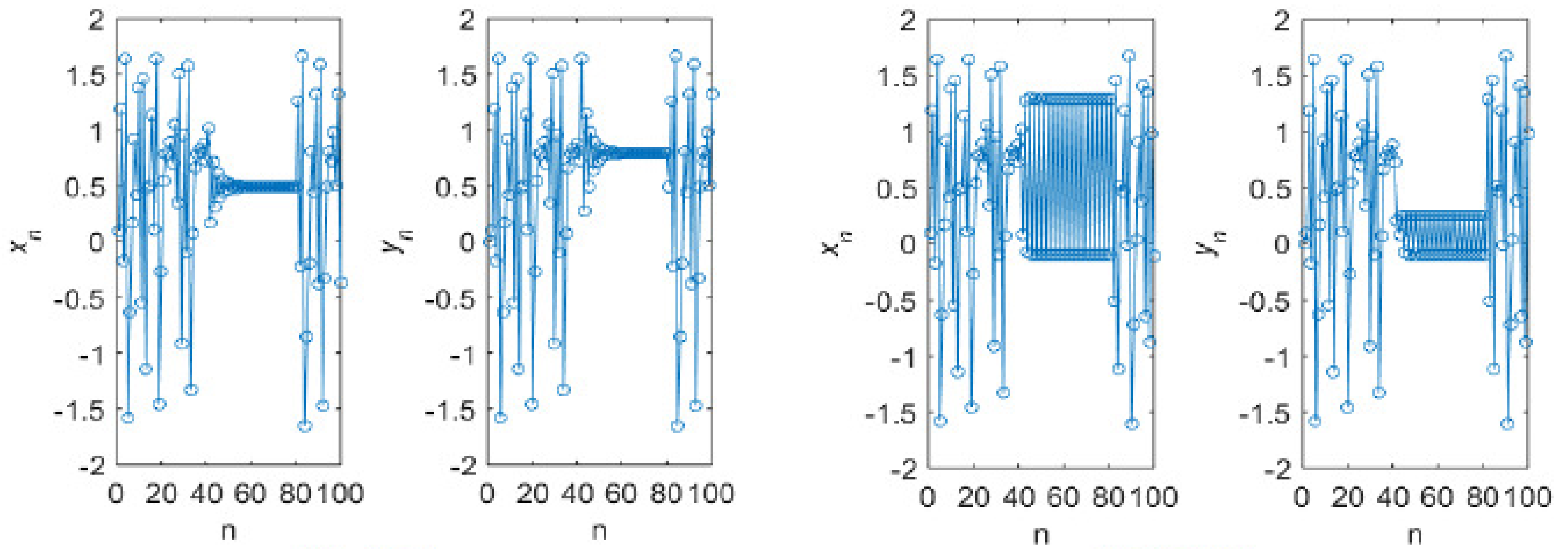


(a) Lyaponove Exponent - Floating Point (b) Lyaponove Exponent - Fixed-point 8b (c) Lyaponove Exponent - Fixed-point 6b



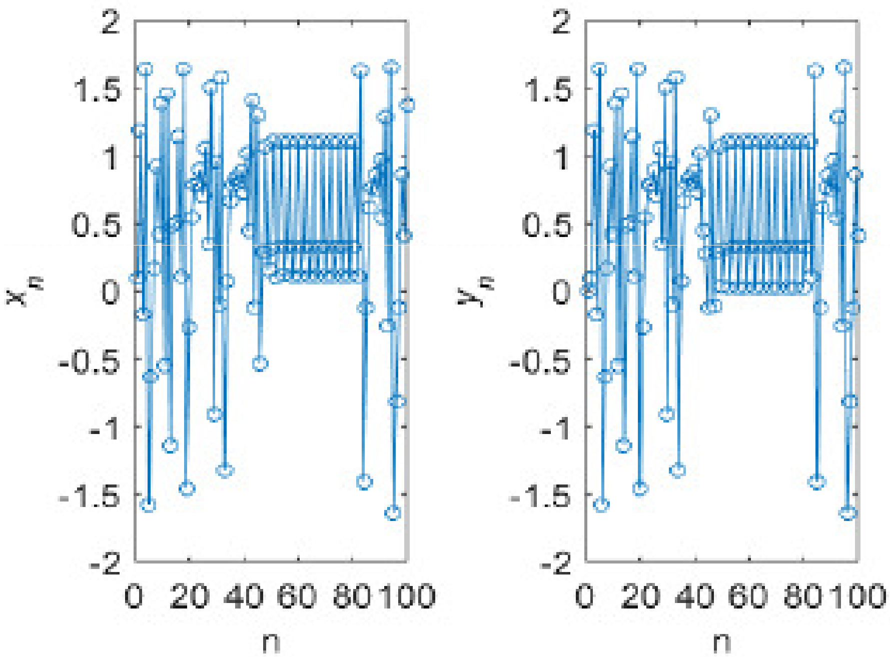
(d) Bifurcation - Floating Point (e) Bifurcation - Fixed-point 8 bit (f) Bifurcation - Fixed-point 6 bit

Hénon Map Chaotic Control: Periodic Proportional Pulses

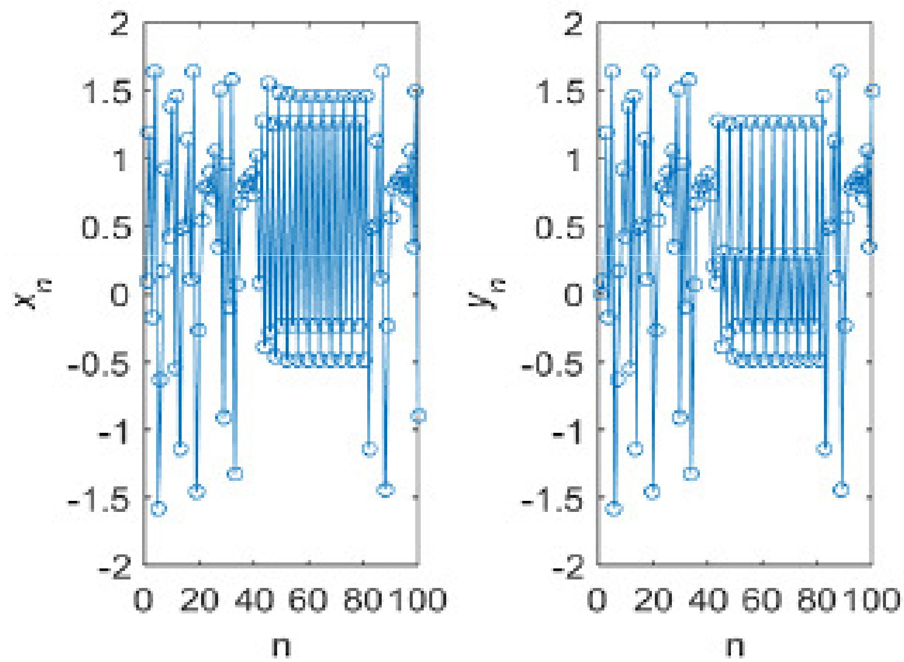


(a) Period One (b) Period Two

Periodic Proportional Pulses

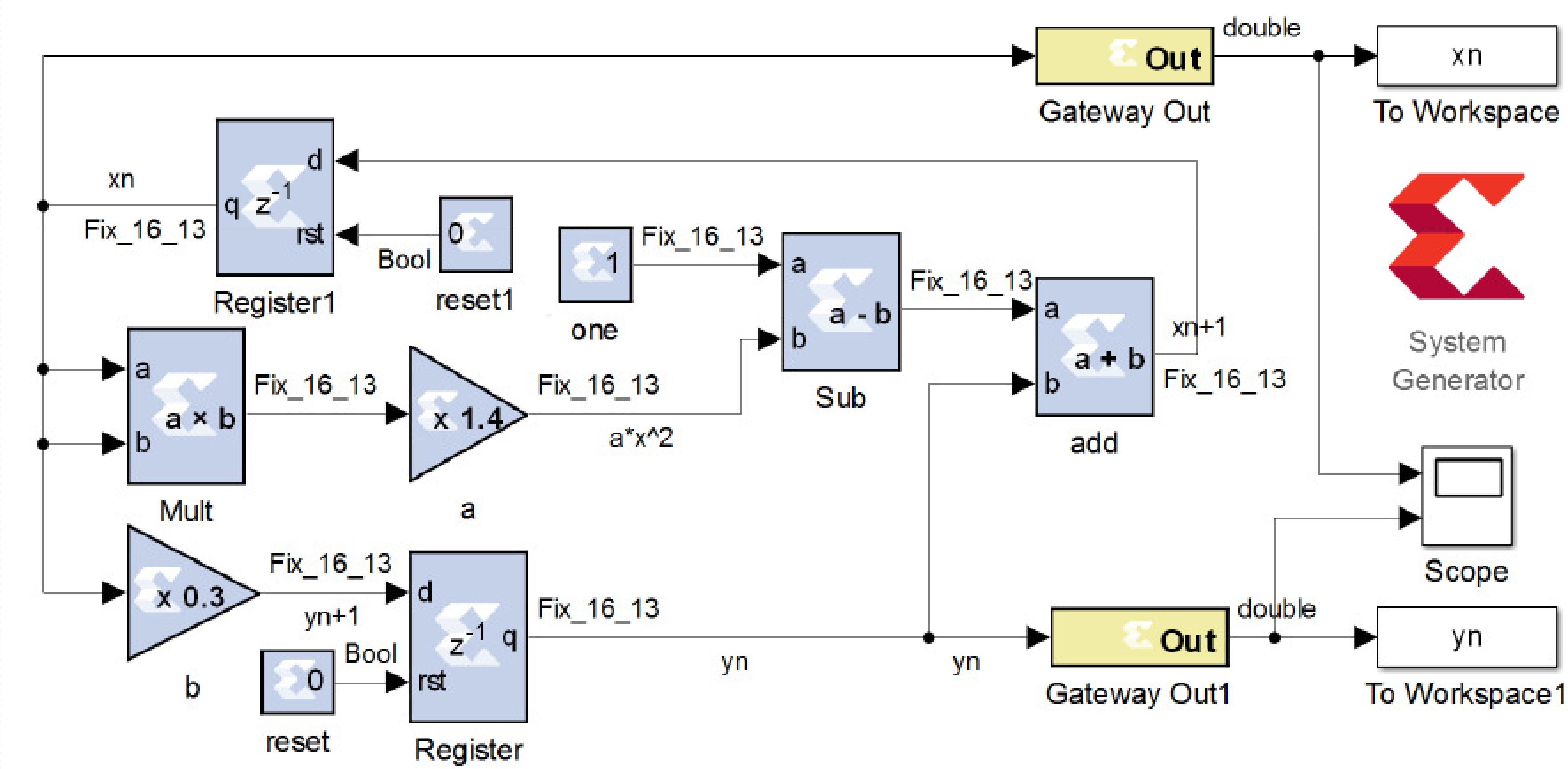


(c) Period Three



(d) Period Four

Model-based Hénon Map Design



VHDL Vs Model-Based Designs

Zynq 7020	VHDL Based Design I			VHDL Based Design II			Model Based Design[a]	
Data format	F32_29	F16_13	F16_13	F32_29	F16_13	F16_13	F32_18	F16_13
Sample period T_s	20 ns	20 ns	10 ns	20 ns	20 ns	10 ns	50 ns	20 ns
Worst Negative Slack	0.03 ns	7.593 ns	-2.034 ns	7.45 ns	11.857 ns	2.452 ns	24.37 ns	1.32 ns
Max Frequency(MHz)	50.08	80.60	–	79.68	122.80	132.49	39.01	53.53
No. of 4 input LUTs	172	16	16	123	16	16	366	150
No. of Registers	64	16	16	64	32	32	64	32
No. of Slices	44	4	5	40	10	10	126	56
No. of DSP	12	3	3	8	2	2	4	1
Total On-chip Power(W)	0.16	0.138	0.156	0.158	0.138	0.155	0.153	0.154

$$f_{max} = \frac{1}{T_s - WNS}$$

Design I : 3 multipliers; Design II: 2 multipliers; FPGA DSP: 18x18

Summary

One Idea

- Brain stimulation based on Chaotic systems simulation and Artificial Neural Network Design

Three Methods

- Chaotic systems analysis and control
- Artificial Neural Network (ANN) architecture design and optimization
- FPGA fixed-point hardware implementation

Q and A

Thank you!