# Super-resolving a Single Blurry Image Through Blind Deblurring Using ADMM

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# Single image super-resolution (SISR)

aims to recover a high-resolution (HR) image $\mathbf{x} \in \mathbb{R}^{N_h}$

from a low-resolution (LR) input image $\mathbf{y} \in \mathbb{R}^{N_l}$

$$\mathbf{y} = \mathbf{DBx} + \mathbf{n}$$

$\mathbf{D}: \mathbb{R}^{N_h} \rightarrow \mathbb{R}^{N_l} (N_l < N_h)$ is the downsampling matrix

$\mathbf{B}: \mathbb{R}^{N_h} \rightarrow \mathbb{R}^{N_h}$ is the blurring matrix

$\mathbf{n} \in \mathbb{R}^{N_l}$ is the additive noise

The SISR problem is typically severely ill-posed!

# Single image super-resolution (SISR)

$$\mathbf{y} = \mathbf{D}\mathbf{B}\mathbf{x} + \mathbf{n}$$

If $\mathbf{B}$ is the identity, then SISR reduces to the Image interpolation.

Most SISR cases assume $\mathbf{B}$ is known or predefined:

- Gaussian blur [Begin and Ferrie, 2004]
- Bicubic interpolation (BI) [Glasner, et al., 2009; Yang, et al., 2010]
- Gaussian blur followed by BI [Freeman and Liu, 2011]
- Simple pixel averaging [Fattal, 2007]
- ……

# Two important works

- An accurate blur model is critical to the success of SISR algorithms[Efrat et al., 2013]
- The PSF of camera is the wrong blur kernel from the LR image [Michaeli and Irani, 2013]

Both seek accurate blur kernels based on existing SISR algorithms, thus their complexisties are even more than those of the SISR ones.

# Single blind image super-resolution (SBISR)

$$\mathbf{y} = \mathbf{D}\mathbf{B}\mathbf{x} + \mathbf{n}$$

If $\mathbf{B}$ is unknown, then SISR becomes the single blind image super-resolution (**SBISR**).

Only a few works dedicated to the SBISR problem, have restrictive assumptions on the blur kernel:

- A parametric Gaussian model with unknown width [Begin and Ferrie, 2004; Qiao, et al., 2006; Wang, et al., 2005]
- Multiple parametric models [He, et al., 2009]
- A nonparametric model assuming the kernel has a single peak [He, et al., 2009]

# In this paper

We address the SBISR problem via a blind image deblurring (BID) method, bridge the gap between SBISR and BID, benefit from that some BID methods are arguably faster and easier to understand, than state-of-the-art SISR/SBISR methods, and reach competitive speed and restoration quality.

# SBISR and BID

SBISR:

recover a HR image $\mathbf{x} \in \mathbb{R}^{N_h}$ from a LR image $\mathbf{y} \in \mathbb{R}^{N_l}$

$$\mathbf{y} = \mathbf{DBx} + \mathbf{n}$$

$\mathbf{D}: \mathbb{R}^{N_h} \rightarrow \mathbb{R}^{N_l} (N_l < N_h)$ is the downsampling matrix

$\mathbf{B}: \mathbb{R}^{N_h} \rightarrow \mathbb{R}^{N_h}$ is the blurring matrix

$\mathbf{n} \in \mathbb{R}^{N_l}$ is the additive noise

BID:

recover a sharp image $\mathbf{x} \in \mathbb{R}^{N_h}$ from a blurry image $\mathbf{z} \in \mathbb{R}^{N_h}$

$$\mathbf{z} = \mathbf{Bx} + \mathbf{s}$$

$\mathbf{B}: \mathbb{R}^{N_h} \rightarrow \mathbb{R}^{N_h}$ is the blurring matrix

$\mathbf{s} \in \mathbb{R}^{N_h}$ is the additive noise

# SBISR and BID

With the same $\mathbf{B}$ and $\mathbf{x}$

BID: $\mathbf{z} = \mathbf{B}\mathbf{x} + \mathbf{s} \longmapsto \mathbf{B}\mathbf{x} = \mathbf{z} - \mathbf{s}$

$$\Downarrow$$

SBISR: $\mathbf{y} = \mathbf{D}\mathbf{B}\mathbf{x} + \mathbf{n}$

$$\Downarrow$$

$$\begin{aligned} \mathbf{y} &= \mathbf{D}(\mathbf{z} - \mathbf{s}) + \mathbf{n} \\ &= \mathbf{D}\mathbf{z} + (\mathbf{n} - \mathbf{D}\mathbf{s}) \end{aligned}$$

Due to the introduce of $\mathbf{D}$, the length of $\mathbf{y}$ is less than that of $\mathbf{z}$, namely, $\mathbf{y}$ has fewer known samples than $\mathbf{z}$.

we can solve the SBISR problem in an easier way via reformulating it into a BID problem.

# Reformulating SBISR into BID

SBISR: $\mathbf{y} = \mathbf{DBx} + \mathbf{n}$

The idea is to first interpolate the LR image $\mathbf{y} \in \mathbb{R}^{N_l}$ as $\mathbf{u} \in \mathbb{R}^{N_h}$

$$\mathbf{u} = \mathbf{Uy} = \mathbf{UDBx} + \mathbf{Un}$$

$\mathbf{U}: \mathbb{R}^{N_l} \rightarrow \mathbb{R}^{N_h}$ is the interpolation operator (e.g. *bicubic* or *bilinear*)

The resulting BID : $\mathbf{u} = \mathbf{Kx} + \mathbf{e}$

$\mathbf{K} = \mathbf{UDB}: \mathbb{R}^{N_h} \rightarrow \mathbb{R}^{N_h}$ is the new blurring matrix

$\mathbf{e} \in \mathbb{R}^{N_h}$ is the interpolation of $\mathbf{n}$

Instead of super-resolving $\mathbf{x}$ from $\mathbf{y}$, the HR image can be obtained via blind deblurring $\mathbf{x}$ from $\mathbf{u}$.

# The resulting BID problem

The regularization problem

$$(\hat{\mathbf{x}}, \hat{\mathbf{k}}) = \arg\min_{\mathbf{x},\mathbf{k}} \frac{\boldsymbol{\lambda}}{2} \|\mathbf{Kx} - \mathbf{u}\|_2^2 + \boldsymbol{\phi}_{\mathbf{GTV}}(\mathbf{x}) + \boldsymbol{\iota}_{\mathcal{S}}(\mathbf{k})$$

$$\boldsymbol{\phi}_{\mathbf{GTV}}(\mathbf{x}) = \sum_{\mathbf{i}} |[\mathbf{D}_h \mathbf{x}]_i|^p + |[\mathbf{D}_v \mathbf{x}]_i|^p \,, 0 \leq p \leq 1$$

$\mathbf{D}_h$ and $\mathbf{D}_h$ denote the horizontal and vertical derivative operator, respectively.

$\boldsymbol{\iota}_{\mathcal{S}}$ is the indicator of the set $\mathcal{S}$ which is defined as

$$\mathcal{S} = \{\mathbf{k}: \mathbf{k} \succcurlyeq \mathbf{0}, \|\mathbf{k}\|_1 = \mathbf{1}\}$$

# The algorithmic framework

**Algorithm** *Proposed algorithmic framework*

1. **Input**: Observed LR image $\mathbf{y}$, $\lambda$ and $\alpha > 1$.
2. **Step I**: Interpolate $\mathbf{y}$ via $\mathbf{u} = \mathbf{U}\mathbf{y}$.
3. **Step II**: Blind estimation of blur filter $\mathbf{k}$ from $\mathbf{u}$, by alternative loop over coarse-to-fine levels:
4. ▶ Update the image estimate

$$\hat{\mathbf{x}} \leftarrow \arg\min_{\mathbf{x}} \frac{\lambda}{2}\|\hat{\mathbf{K}}\mathbf{x} - \mathbf{u}\|_2^2 + \phi_{\mathrm{GTV}}(\mathbf{x}) \qquad (8)$$

where $\hat{\mathbf{K}}$ is the convolution matrix constructed by $\hat{\mathbf{k}}$ obtained from the blur filter estimation below.

5. ▶ Update the blur filter estimate

$$\hat{\mathbf{k}} \leftarrow \arg\min_{\mathbf{k}} \frac{\lambda}{2}\|\hat{\mathbf{X}}\mathbf{k} - \mathbf{u}\|_2^2 + \iota_S(\mathbf{k}) \qquad (9)$$

where $\hat{\mathbf{X}}$ is the convolution matrix constructed by $\hat{\mathbf{x}}$ obtained from the image estimation above.

6. ▶ Increase the parameter $\lambda$

$$\lambda \leftarrow \alpha\lambda. \qquad (10)$$

7. **Step III**: Non-blind estimation of HR image $\mathbf{x}^*$ from $\mathbf{u}$ through solving (8) with final $\hat{\mathbf{k}}$ (obtained by Step II).
8. **Output**: the HR image $\mathbf{x}^*$ and the blur estimate $\hat{\mathbf{k}}$.

Can be efficiently solved by alternating direction method of multipliers (ADMM)

# The alternating direction method of multipliers (ADMM)

[Gabay and Mercier, 1976; Boyd et al., 2011; Almeida and Figueiredo, 2013]

ADMM has been as a popular tool to solving imaging inverse problems

$$\min_{\mathbf{x}} \sum_{j}^{J} g_j(\mathbf{B}^{(j)}\mathbf{x}) \qquad (11)$$

**Algorithm** *ADMM for solving (11)*

1. Set $k = 0$, $\beta > 0$, $\mathbf{v}_0^{(1)}, \cdots, \mathbf{v}_0^{(J)}$, $\mathbf{d}_0^{(1)}, \cdots, \mathbf{d}_0^{(J)}$.
2. **repeat**
3. $\quad \mathbf{r}_k = \sum_{j=1}^{J} (\mathbf{B}^{(j)})^T (\mathbf{v}_k^{(j)} + \mathbf{d}_k^{(j)})$
4. $\quad \mathbf{x}_{k+1} = \left[\sum_{j=1}^{J} (\mathbf{B}^{(j)})^T \mathbf{B}^{(j)}\right]^{-1} \mathbf{r}_k$
5. $\quad$ **for** $j = 1, \cdots, J$
6. $\quad\quad \mathbf{v}_{k+1}^{(j)} = \text{Prox}_{g_j/\tau}\left(\mathbf{B}^{(j)}\mathbf{x}_{k+1} - \mathbf{d}_k^{(j)}\right)$
7. $\quad\quad \mathbf{d}_{k+1}^{(j)} = \mathbf{d}_k^{(j)} - (\mathbf{B}^{(j)}\mathbf{x}_{k+1} - \mathbf{v}_{k+1}^{(j)})$
8. $\quad$ **end for**
9. $\quad k \leftarrow k + 1$
10. **until** some stopping criterion is satisfied.

In line 6 of above algorithm, the proximity operator of $g_j/\tau$: $\text{Prox}_{g_j/\tau}$ is defined as

$$\text{Prox}_{g_j/\tau}(\mathbf{v}) = \arg\min_{\mathbf{x}} \left(g_j(\mathbf{x}) + \frac{\tau}{2}\|\mathbf{x} - \mathbf{v}\|^2\right). \qquad (12)$$

# x update using the ADMM

$$\hat{\mathbf{x}} \leftarrow \arg\min_{\mathbf{x}} \frac{\lambda}{2}\|\hat{\mathbf{K}}\mathbf{x} - \mathbf{u}\|_2^2 + \phi_{\text{GTV}}(\mathbf{x}) \qquad (8)$$

$$g_1(\cdot) = \frac{\lambda}{2}\|\cdot - \mathbf{u}\|_2^2, \; g_2(\cdot) = g_3(\cdot) = \|\cdot\|_p^p,$$

$$\mathbf{B}^{(1)} = \hat{\mathbf{K}}, \; \mathbf{B}^{(2)} = \mathbf{D}_h, \; \mathbf{B}^{(3)} = \mathbf{D}_v$$

**Algorithm** *ADMM for solving (8)*

**Initialize** $k = 0$, $\tau_1 > 0$, $\mathbf{v}_0^{(1)}, \mathbf{v}_0^{(2)}, \mathbf{v}_0^{(3)}$, $\mathbf{d}_0^{(1)}, \mathbf{d}_0^{(2)}, \mathbf{d}_0^{(3)}$.

**repeat**

$\mathbf{z}_k^{(1)} = \mathbf{v}_k^{(1)} + \mathbf{d}_k^{(1)}$

$\mathbf{z}_k^{(2)} = \mathbf{v}_k^{(2)} + \mathbf{d}_k^{(2)}$

$\mathbf{z}_k^{(3)} = \mathbf{v}_k^{(3)} + \mathbf{d}_k^{(3)}$

$\mathbf{r}_k = \hat{\mathbf{K}}^T\mathbf{z}_k^{(1)} + \mathbf{D}_h^T\mathbf{z}_k^{(2)} + \mathbf{D}_h^T\mathbf{z}_k^{(3)}$

$\mathbf{x}_{k+1} = \left[\hat{\mathbf{K}}^T\hat{\mathbf{K}} + \mathbf{D}_h^T\mathbf{D}_h + \mathbf{D}_v^T\mathbf{D}_v\right]^{-1}\mathbf{r}_k$

$\mathbf{v}_{k+1}^{(1)} = \text{Prox}_{g_1/\tau_1}\left(\hat{\mathbf{K}}\mathbf{x}_{k+1} - \mathbf{d}_k^{(1)}\right)$

$\mathbf{d}_{k+1}^{(1)} = \mathbf{d}_k^{(1)} - (\hat{\mathbf{K}}\mathbf{x}_{k+1} - \mathbf{v}_{k+1}^{(1)})$

$\mathbf{v}_{k+1}^{(2)} = \text{Prox}_{g_2/\tau_1}\left(\mathbf{D}_h\mathbf{x}_{k+1} - \mathbf{d}_k^{(2)}\right)$

$\mathbf{d}_{k+1}^{(2)} = \mathbf{d}_k^{(2)} - (\mathbf{D}_h\mathbf{x}_{k+1} - \mathbf{v}_{k+1}^{(2)})$

$\mathbf{v}_{k+1}^{(3)} = \text{Prox}_{g_3/\tau_1}\left(\mathbf{D}_v\mathbf{x}_{k+1} - \mathbf{d}_k^{(3)}\right)$

$\mathbf{d}_{k+1}^{(3)} = \mathbf{d}_k^{(3)} - (\mathbf{D}_v\mathbf{x}_{k+1} - \mathbf{v}_{k+1}^{(3)})$

$k \leftarrow k + 1$

**until** some stopping criterion is satisfied.

# k update using the ADMM

$$\hat{\mathbf{k}} \leftarrow \arg\min_{\mathbf{k}} \frac{\lambda}{2}\|\hat{\mathbf{X}}\mathbf{k} - \mathbf{u}\|_2^2 + \iota_{\mathcal{S}}(\mathbf{k}) \qquad (9)$$

$$g_1(\cdot) = \frac{\lambda}{2}\|\cdot - \mathbf{u}\|_2^2, \; g_2(\cdot) = \iota_{\mathcal{S}}(\cdot),$$
$$\mathbf{B}^{(1)} = \hat{\mathbf{X}}, \; \mathbf{B}^{(2)} = \mathbf{I},$$

**Algorithm** *ADMM for solving (9)*

1. **Initialize** $k = 0$, $\tau_2 > 0$, $\mathbf{v}_0^{(1)}, \mathbf{v}_0^{(2)}, \mathbf{d}_0^{(1)}, \mathbf{d}_0^{(2)}$.
2. **repeat**
3. $\mathbf{z}_k^{(1)} = \mathbf{v}_k^{(1)} + \mathbf{d}_k^{(1)}$
4. $\mathbf{z}_k^{(2)} = \mathbf{v}_k^{(2)} + \mathbf{d}_k^{(2)}$
5. $\mathbf{r}_k = \hat{\mathbf{X}}^T \mathbf{z}_k^{(1)} + \mathbf{z}_k^{(2)}$
6. $\mathbf{k}_{k+1} = \left[\hat{\mathbf{X}}^T \hat{\mathbf{X}} + \mathbf{I}\right]^{-1} \mathbf{r}_k$
7. $\mathbf{v}_{k+1}^{(1)} = \text{Prox}_{g_1/\tau_2}\left(\hat{\mathbf{X}}\mathbf{k}_{k+1} - \mathbf{d}_k^{(1)}\right)$
8. $\mathbf{d}_{k+1}^{(1)} = \mathbf{d}_k^{(1)} - (\hat{\mathbf{X}}\mathbf{k}_{k+1} - \mathbf{v}_{k+1}^{(1)})$
9. $\mathbf{v}_{k+1}^{(2)} = \text{Prox}_{g_2/\tau_2}\left(\mathbf{k}_{k+1} - \mathbf{d}_k^{(2)}\right)$
10. $\mathbf{d}_{k+1}^{(2)} = \mathbf{d}_k^{(2)} - (\mathbf{k}_{k+1} - \mathbf{v}_{k+1}^{(2)})$
11. $k \leftarrow k + 1$
12. **until** some stopping criterion is satisfied.

# On synthetic blurry images

Test the baby image (size: 512×512) blurred by eight PSFs provided by [Levin et al., 2009]. In the algorithm, the operator **U** here has two options:

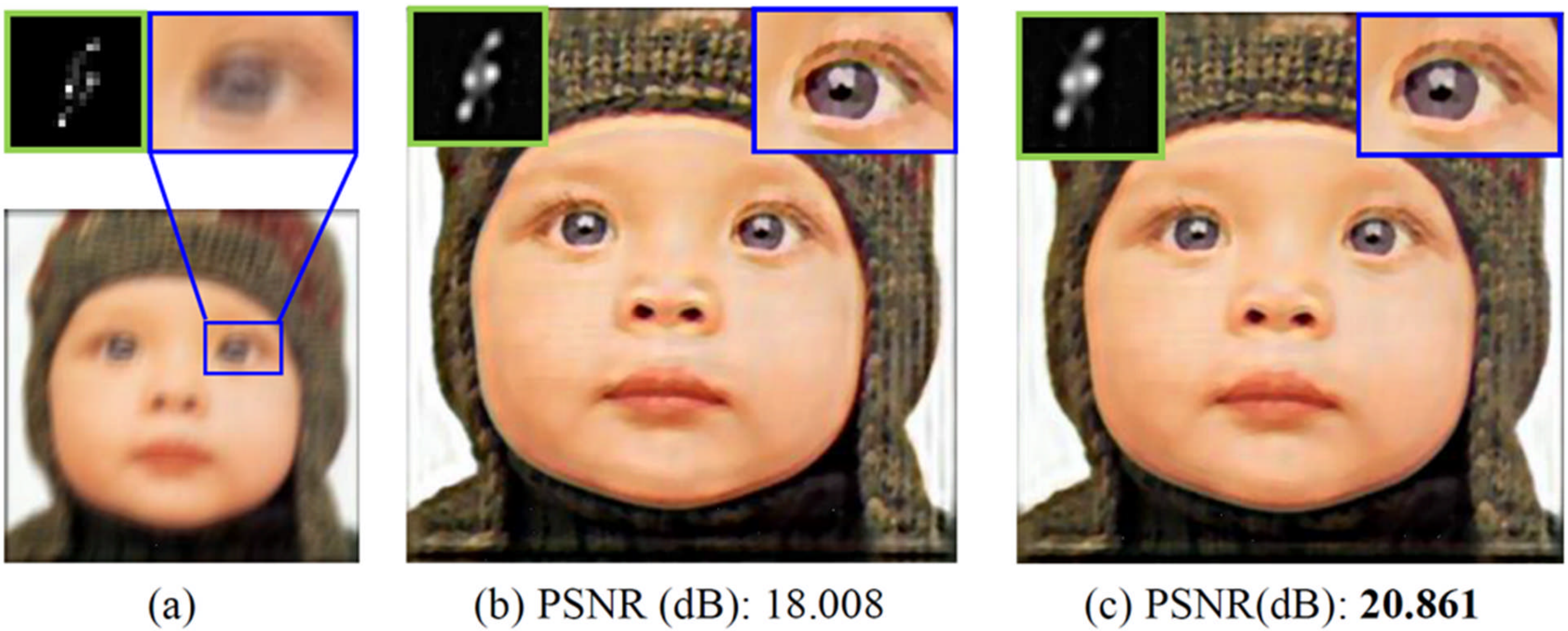



Figure 1. Estimated HR images, PSFs and PSNRs. (a) are input LR blurry image (size: $256 \times 256$, obtained by (1)) and one of the eight PSFs (corresponding to $\mathbf{B}$ in (1)); (b) and (c) are estimated HR images, PSFs (corresponding to $\mathbf{K}$ in (5)) and PSNRs by the proposed method with the *bicubic* and *bilinear* interpolation operators, respectively.

# On synthetic blurry images

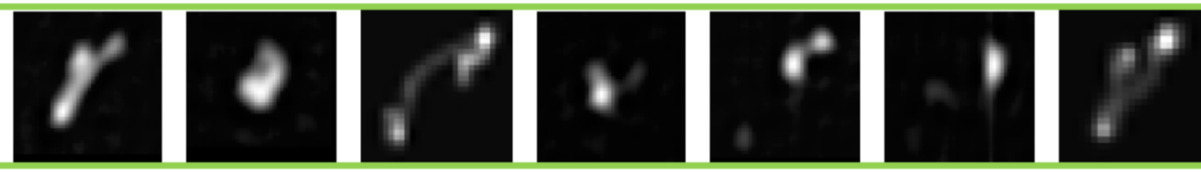
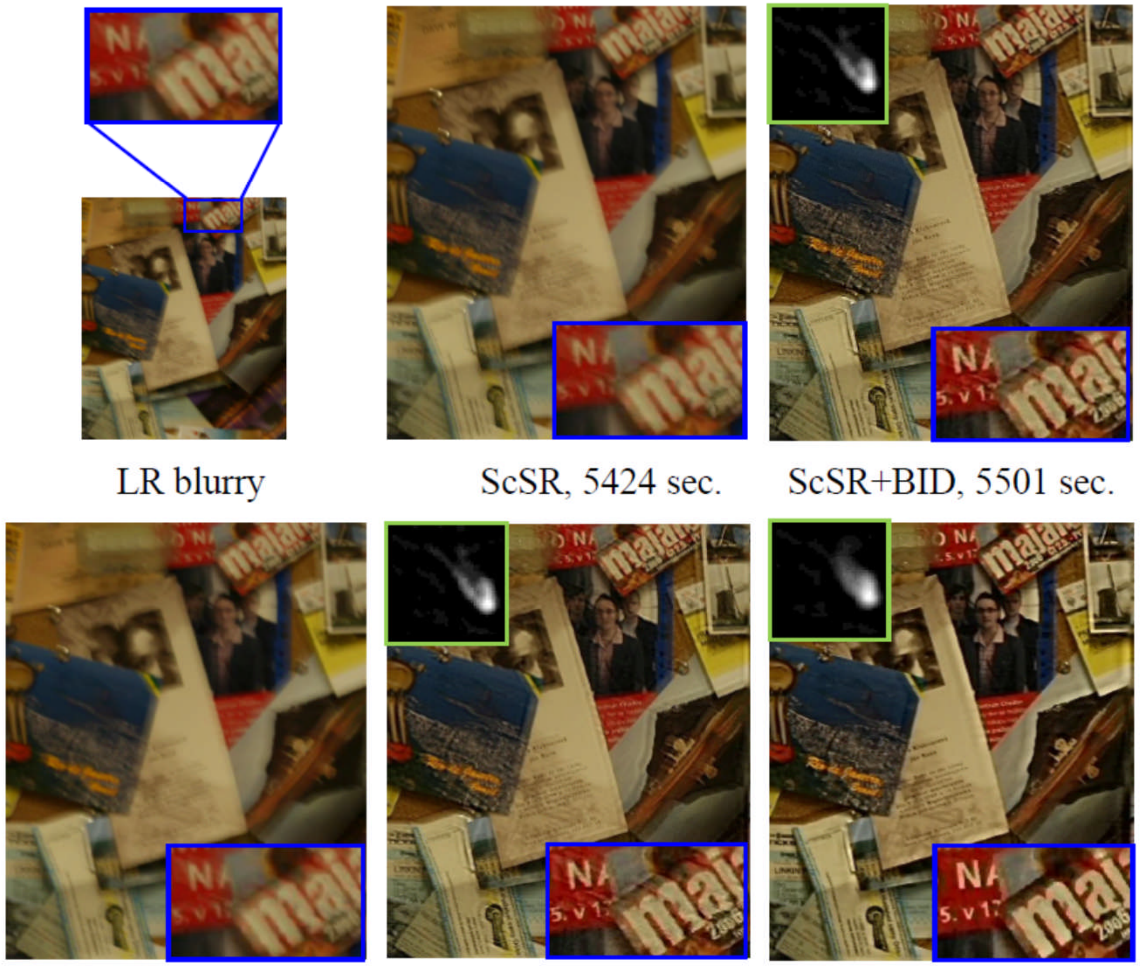
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Other seven PSFs | |  | | | | | | |
| *bicubic* | Estimated PSFs |  | | | | | | |
| | PSNR (dB) | **20.809** | 17.228 | **16.856** | 21.171 | 16.512 | 17.040 | **16.364** |
| *bilinear* | Estimated PSFs |  | | | | | | |
| | PSNR (dB) | 19.933 | **19.213** | 16.569 | **22.184** | **16.776** | **17.454** | 16.286 |

Figure 2. Other seven PSFs and their corresponding estimated PSFs and PSNRs by the proposed method with the *bicubic* and *bilinear* operators, respectively.

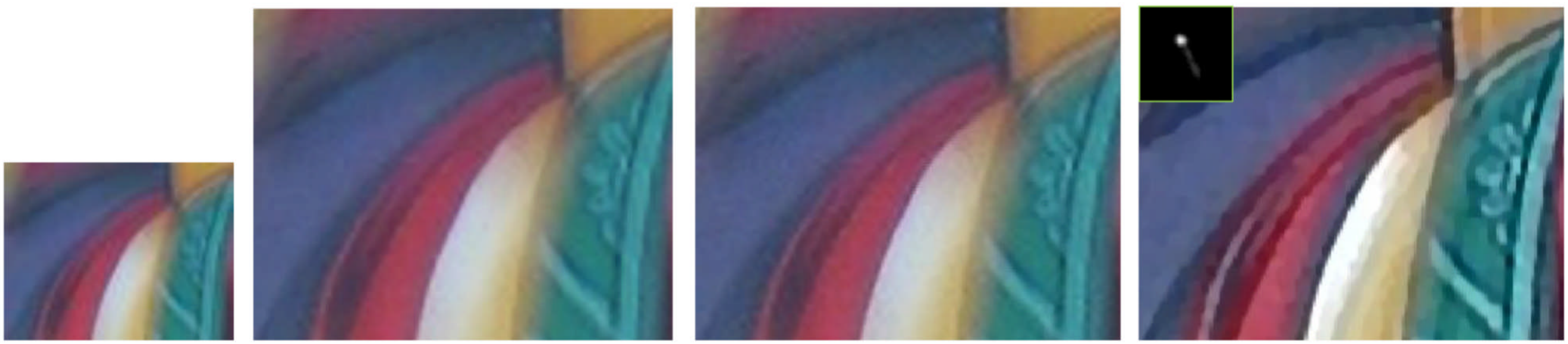
# On real images


LR blurry | ScSR, 5424 sec. | ScSR+BID, 5501 sec.

SRCNN, 230 sec. | SRCNN+BID, 306 sec. | **Proposed**, **78** sec.

Figure 3. Results on a real LR blurry image (size: $900 \times 540$).

# On real images



LR | ScSR, 2052 sec. | SRCNN, 187 sec. | **Proposed**, **57** sec.

Figure 4. Results on a real LR image (size: $324 \times 464$).

# Conclusions

- Have proposed a new approach for single blind image super-resolution (SBISR) via a blind image deblurring (BID) method, bridging the gap between SBISR and BID, benefitting from that some BID methods are arguably faster and easier to understand, than state-of-the-art SISR/SBISR methods, and reaching competitive speed and restoration quality.

- Experiments on synthetic and real images show that the effectiveness and competitiveness of the proposed method.

**Thanks for your attention!**