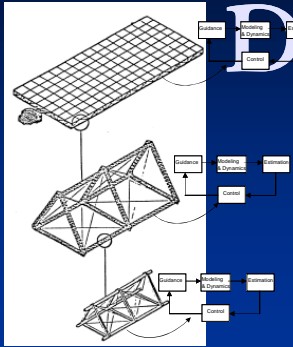
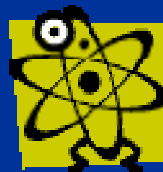
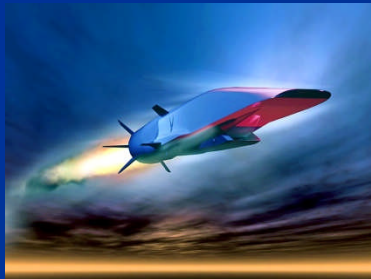
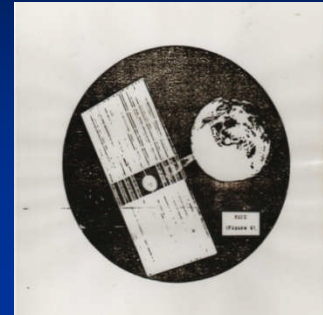


# Emerging New Directions in Infinite-Dimensional Adaptive Control



Mark J. Balas  
Distinguished Professor  
Aerospace Engineering Department  
Embry-Riddle Aeronautical University  
Daytona Beach, FL, USA



Thank you Petre

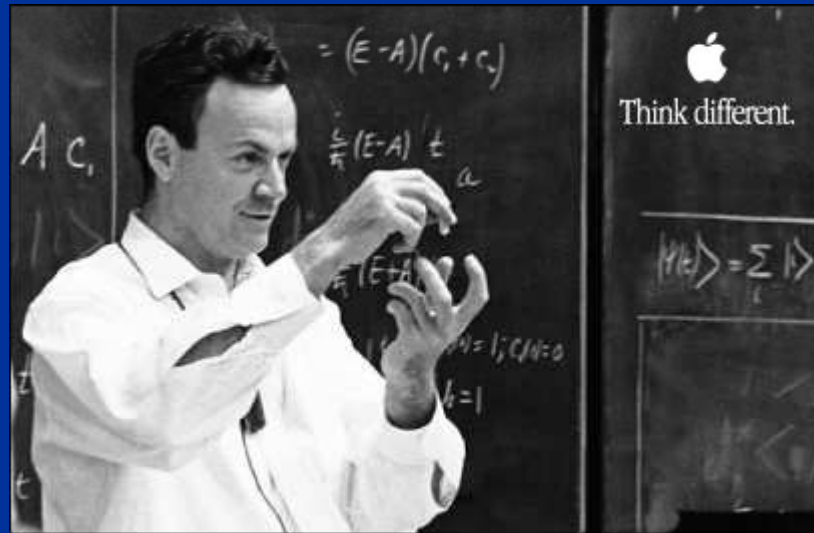


ADAPTIVE 2015, The Seventh International Conference  
on Adaptive and Self-Adaptive Systems and Applications  
March 22 - 27, 2015 - Nice, France

Infinite-Dimensional Adaptive  
Control Theory

“~~Physics~~ is like sex: sure, it may give some practical results, but that's not why we do it.”

— Richard P. Feynman



In a tile motif on the back of the Ross Dress For Less building on Lake Ave, Pasadena, CA

# F-16 Flexible Structure Model: Fluid-Structure Interaction



Flutter

USAF-Edwards AFB  
Flight Test Center



# Many Emerging Solutions



Aerodynamically  
Shaped  
Graduate  
Student

# Hypersonic Aircraft X51A Wave Rider



Reality



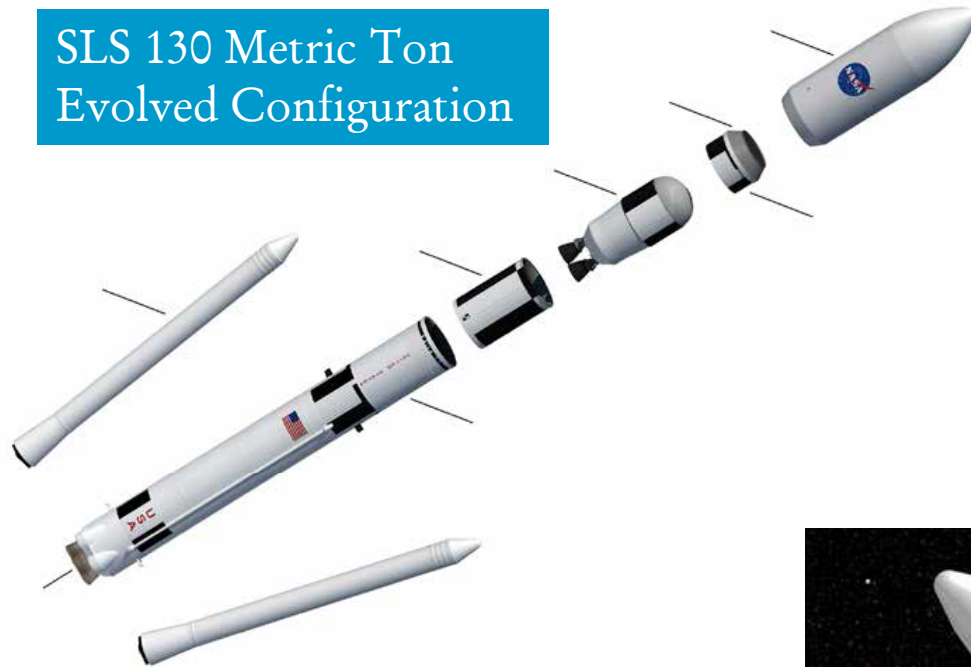
6 Minutes at Mach 5.1

The X-51A WaveRider is an unmanned, autonomous supersonic combustion, ramjet-powered hypersonic flight-test demonstrator for the U.S. Air Force. The X-51A demonstrates a scalable, robust endothermic hydrocarbon-fueled scramjet propulsion system in flight, as well as high temperature materials, airframe/engine integration and other key technologies within the hypersonic range of Mach 4.5 to 6.5.

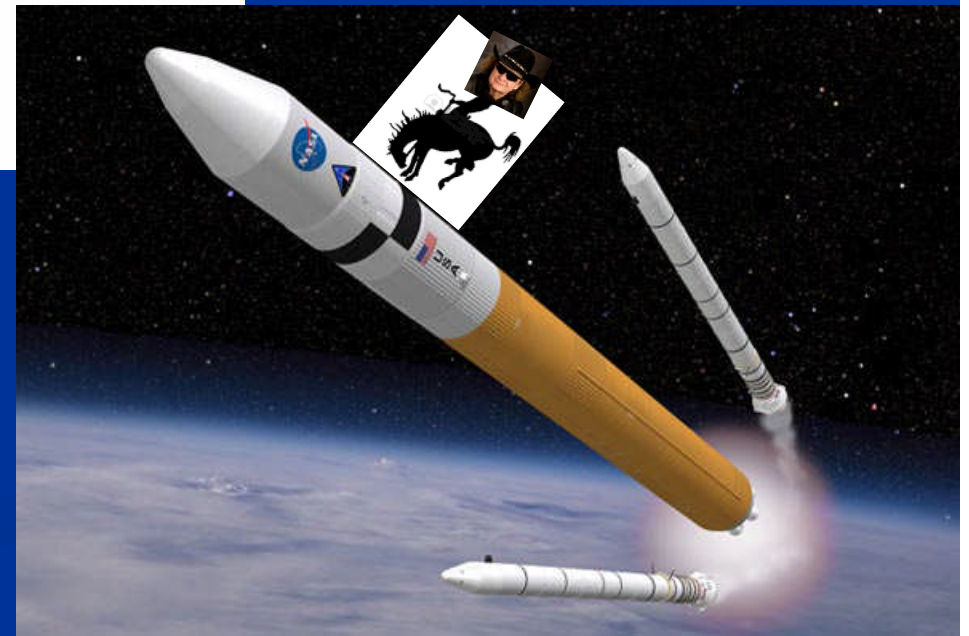
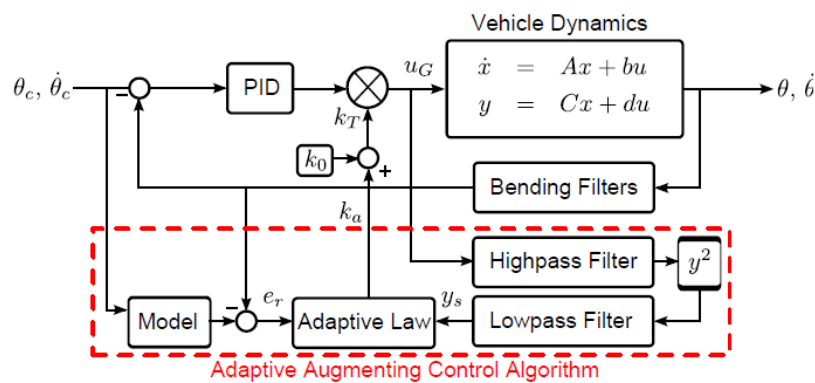
# NASA Space Launch System

## SLS

SLS 130 Metric Ton Evolved Configuration



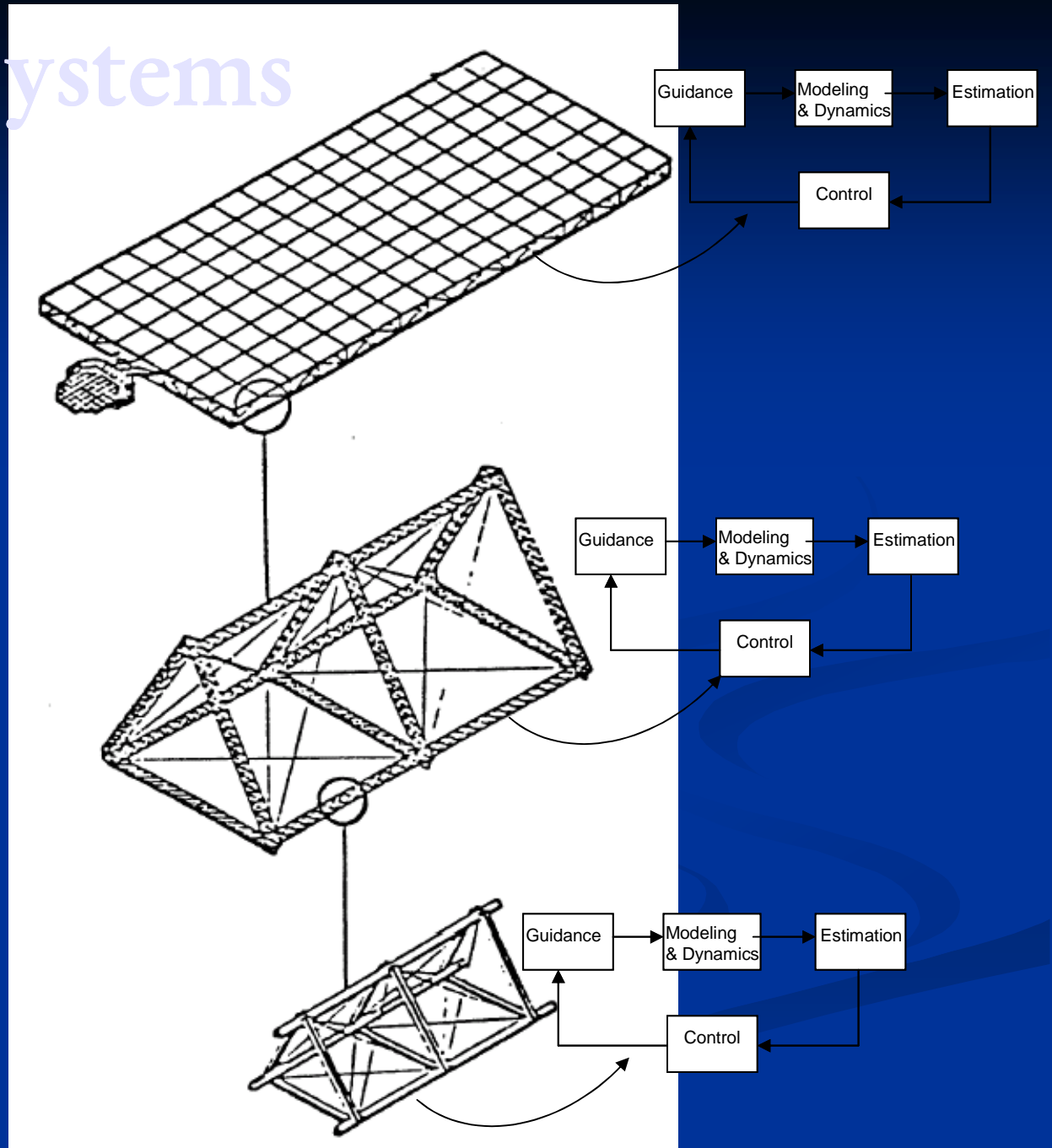
NASA MSFC



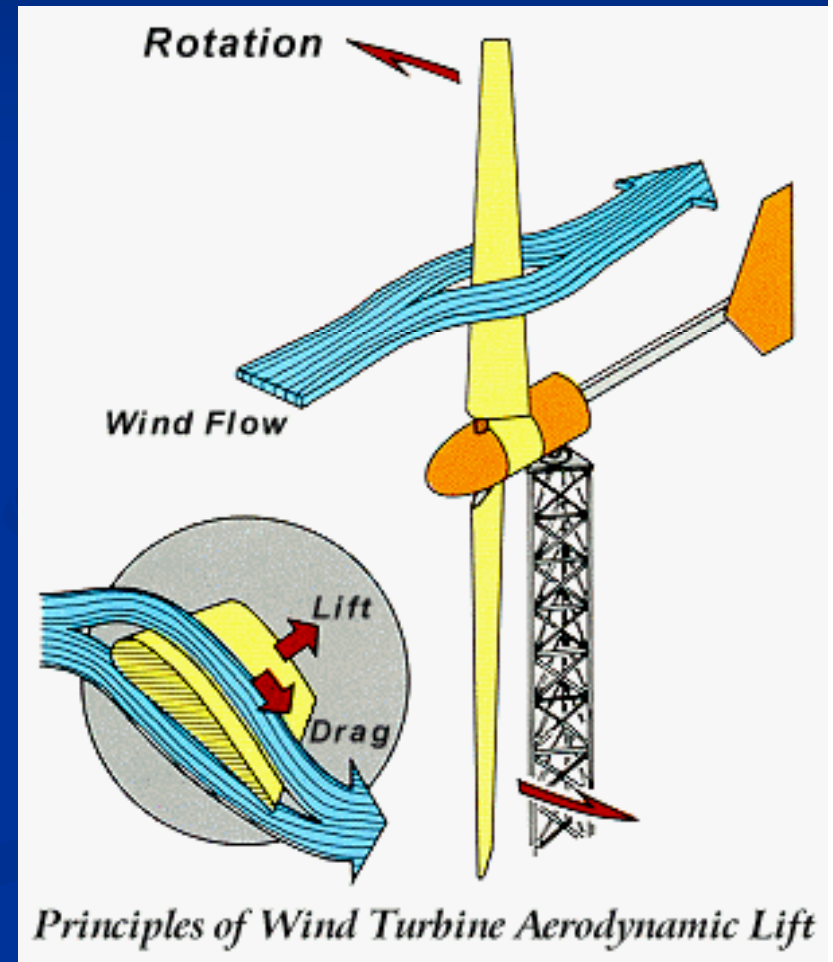
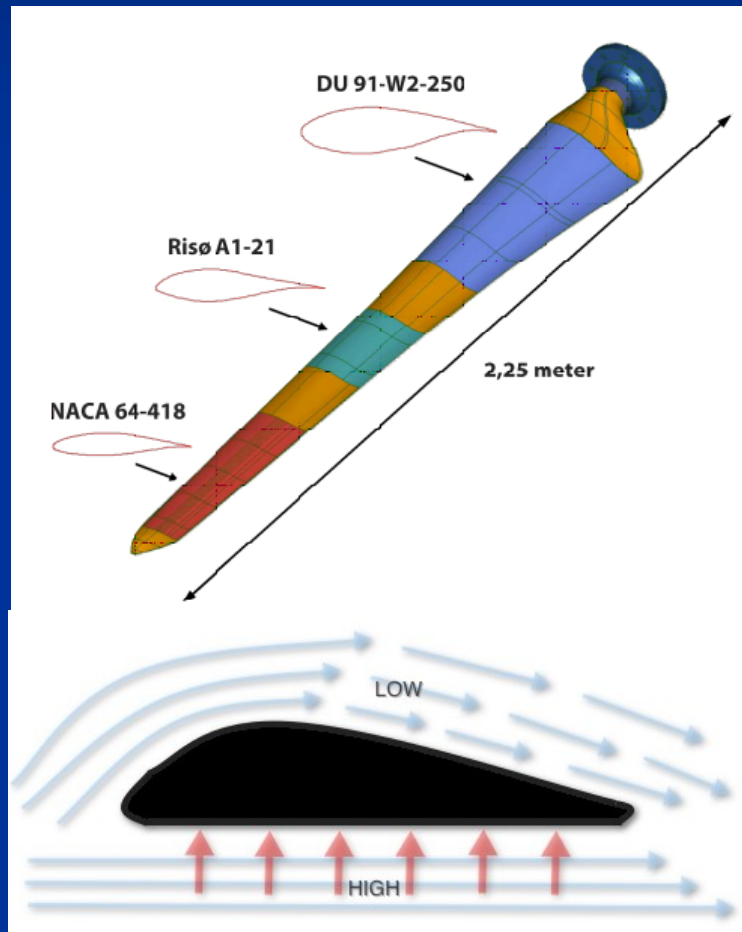
# Evolving Systems

Evolving Systems =  
Autonomously  
Assembled  
Active Structures

Or Self-Assembling  
Structures,  
which Aspire to a  
Higher Purpose;  
*Cannot be attained  
by Components Alone*

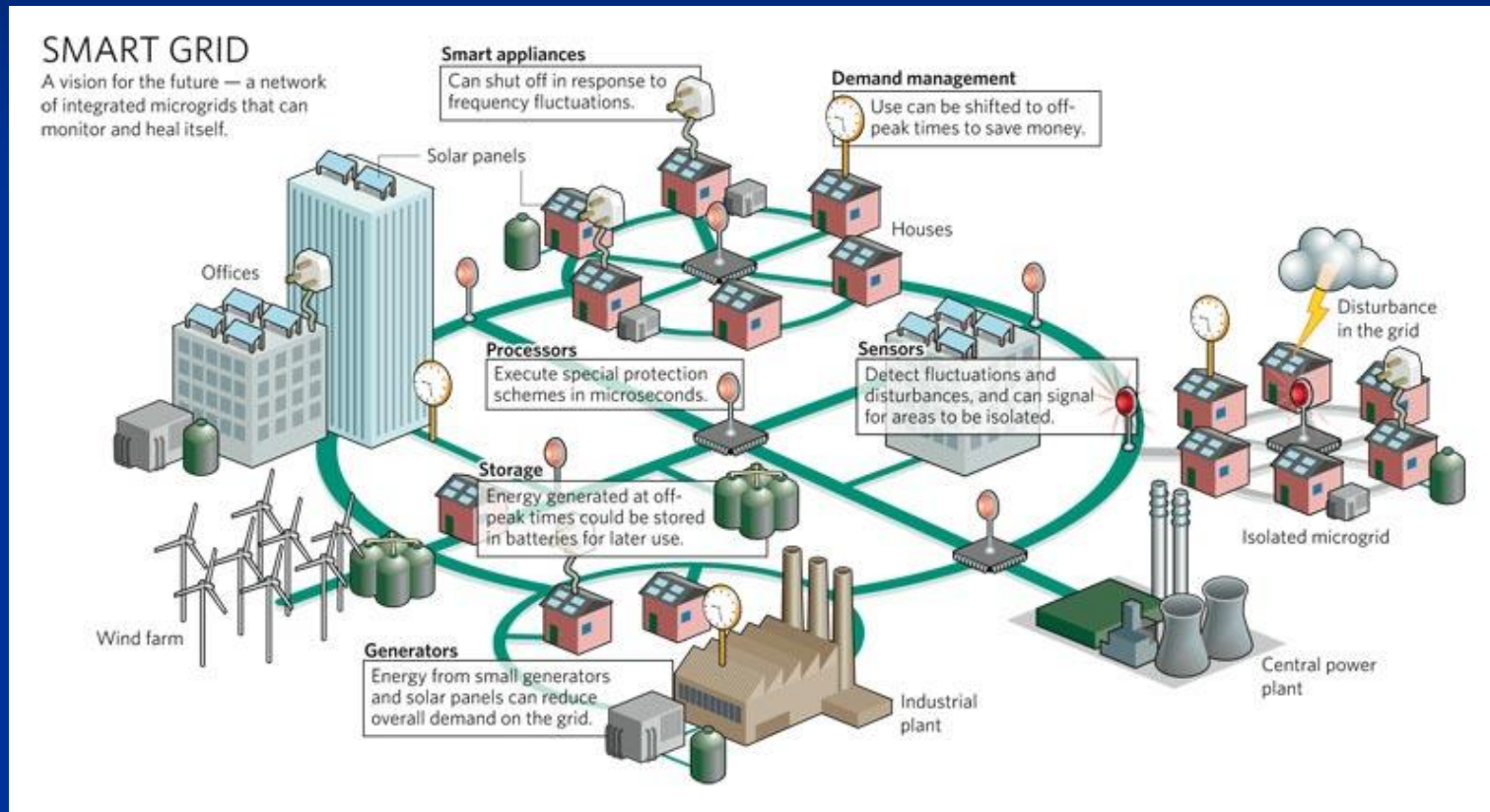


# Flow Control of Wind Turbine Aerodynamics



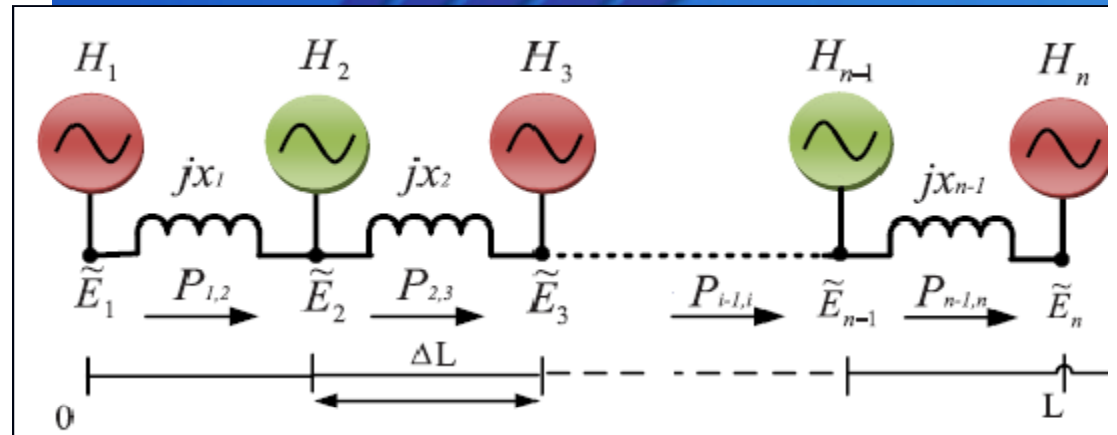
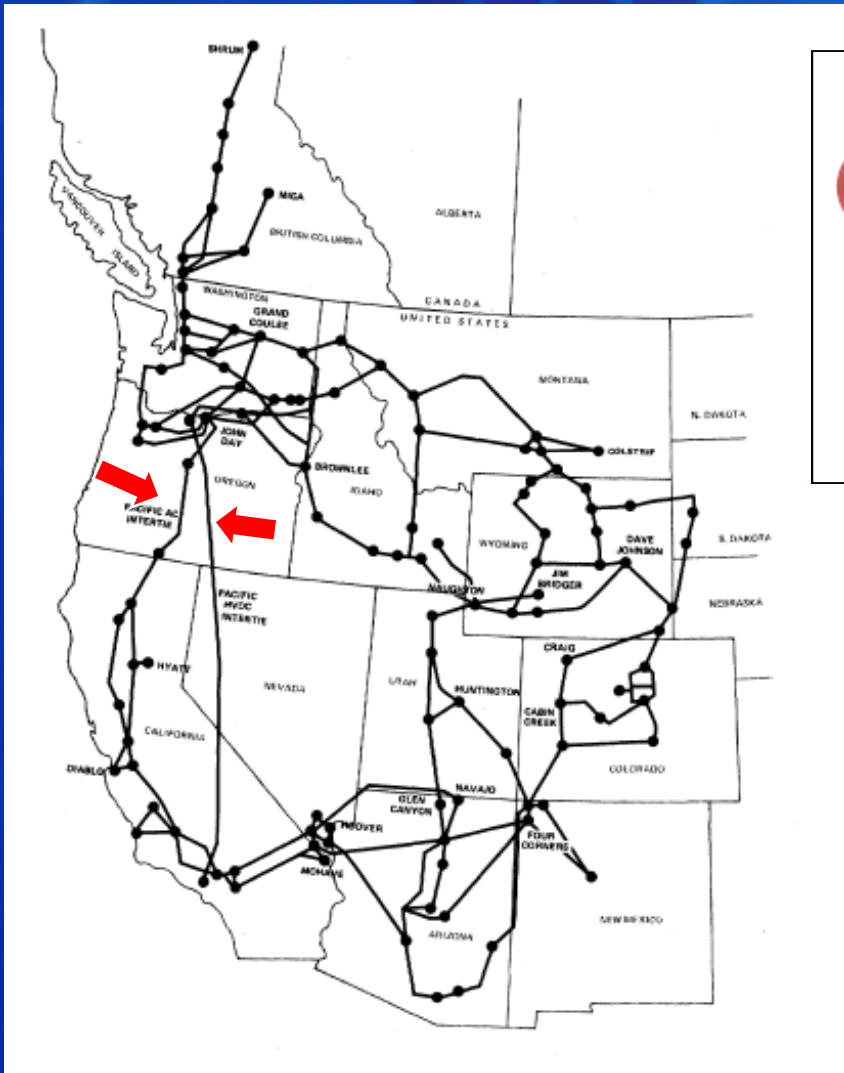


# Smart Grids: Virtual Interconnecting Forces



“It is surprising how quickly we replace a human operator with an algorithm and call it SMART”

# POWER SYSTEM AS DISTRIBUTED PARAMETER SYSTEM



Subarea Oscillations

$$\frac{\partial^2 \delta(u, t)}{\partial t^2} + \eta \frac{\partial \delta(u, t)}{\partial t} = v^2 \frac{\partial^2 \delta(u, t)}{\partial u^2}$$

# “Simplicity” via Infinite Dimensional Spaces



$$\left\{ \begin{array}{l} \frac{\partial x}{\partial t} = Ax + Bu = Ax + \sum_{i=1}^m b_i u_i \\ x(0) = x_0 \in D(A) \subset X \\ y = Cx = [(c_1, x) \quad (c_2, x) \quad \dots \quad (c_m, x)]^T \end{array} \right. \Rightarrow x(t, w_0) = \underbrace{U(t)x_0}_{\substack{\text{Evolution} \\ \text{in } X}}; \forall t \geq 0$$



“Boil Away” all the special properties of  $\mathcal{R}^N$

$C_0$  – Semigroup of Bounded Operators  $U(t)$  :

$$\left\{ \begin{array}{l} U(t+s) = U(t)U(s) \text{ (semigroup property)} \\ \frac{d}{dt}U(t) = AU(t) = U(t)A \text{ ( } A \text{ generates } U(t)\text{)} \\ U(t)x_0 \xrightarrow{t \rightarrow 0} x_0 \text{ (continuous at } t = 0\text{)} \end{array} \right.$$

J. Wen & M. Balas, “Robust Adaptive Control in Hilbert Space”,  
J. Mathematical. Analysis and Applications, Vol 143, pp 1-26, 1989.

J. Wen & M. Balas, “Direct Model Reference Adaptive Control in Infinite-Dimensional Hilbert Space,”  
Chapter in Applications of Adaptive Control Theory, Vol.11,  
K. S. Narendra, Ed., Academic Press, 1987

# The Devil Lurks in the Details

*Nonlinear Adaptive Controller*



# Semigroups

Closed Linear  
Operator

$$\text{Solve } \begin{cases} \frac{\partial x}{\partial t} = Ax \\ x(0) = x_0 \in D(A) \end{cases} \Rightarrow x(t) = U(t)x_0$$

$$\dim X = N < \infty$$

$$\Rightarrow U(t) = e^{At} = \sum_{k=0}^{\infty} A^k \frac{t^k}{k!}$$

$C_0$  - Semigroup

$U(t) : X \rightarrow X$  bounded operators  $t \geq 0$

Generator :  $Ax = \lim_{t \rightarrow 0^+} \frac{U(t)x - x}{t}$  with  $D(A) \equiv \{x / \lim_{t \rightarrow 0^+} \text{ exists} \}$  dense in  $X$

$$\text{LaPlace Transform } \begin{cases} L(U(t)) = (\lambda I - A)^{-1} \equiv R(\lambda, A) \text{ Resolvent Operator} \\ L^{-1}(R(\lambda, A)) = U(t) \end{cases}$$

# Spectrum of A

Resolvent Set  $\rho(A) \equiv \{ \lambda / R(\lambda, A) : X \rightarrow X \text{ bounded linear op on } X \}$

Spectrum  $\sigma(A) \equiv \rho(A)^c = \sigma_{\text{point}}(A) \cup \sigma_{\text{cont}}(A) \cup \sigma_{\text{residual}}(A)$

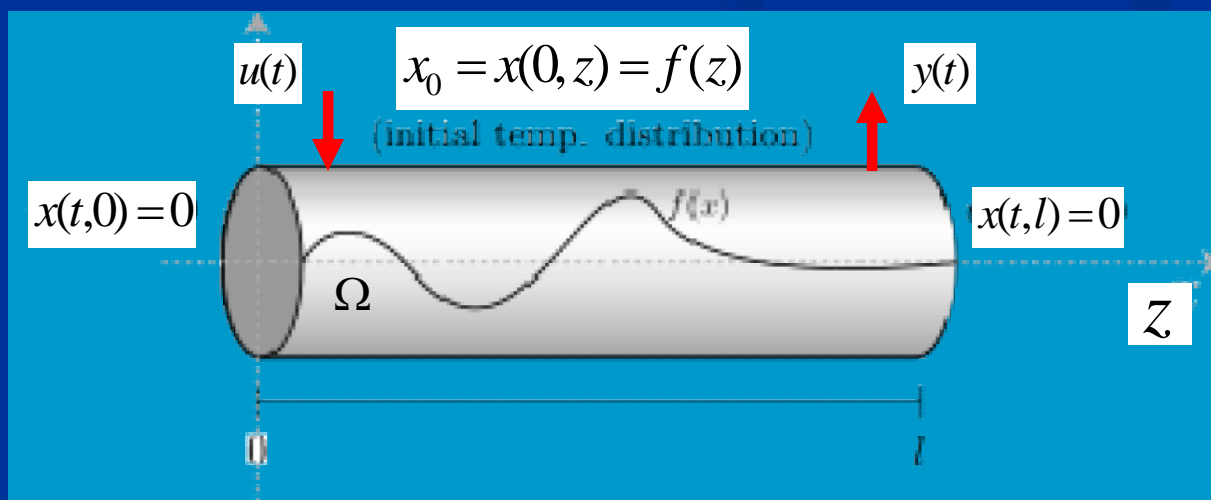
$\sigma_{\text{point}}(A) \equiv \{ \lambda / R(\lambda, A) \text{ is NOT 1-1} \} = \{ \lambda / \exists \phi \neq 0 \ni \lambda \phi = A \phi \}$

$\sigma_{\text{cont}}(A) \equiv \{ \lambda / R(\lambda, A) \text{ is 1-1, but its range is only dense in } X \}$

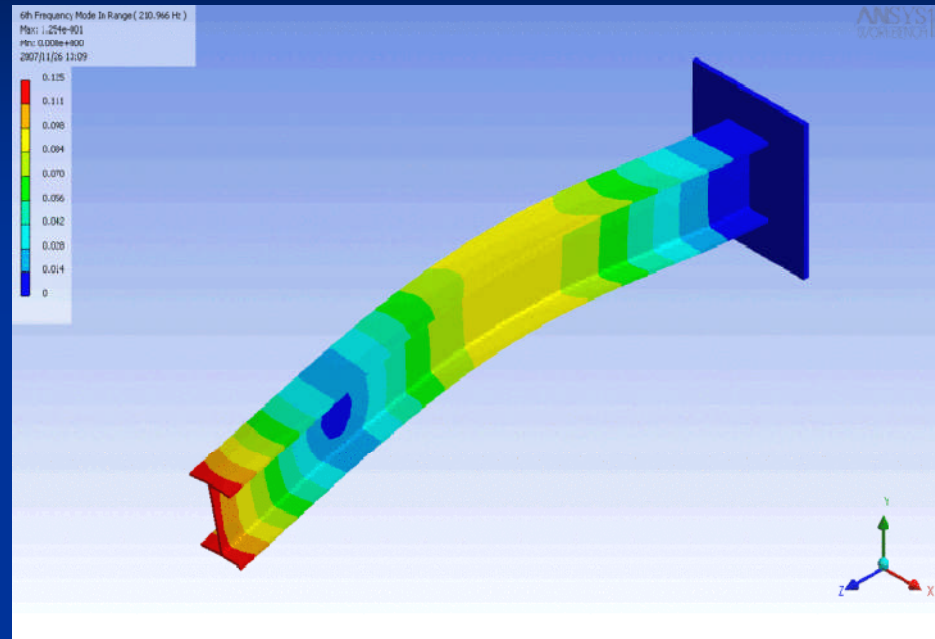
$\sigma_{\text{residual}}(A) \equiv \{ \lambda / R(\lambda, A) \text{ is 1-1, but range is a proper subspace of } X \}$

# Example: Heat Diffusion

$$\left\{ \begin{array}{l} \frac{\partial x}{\partial t} = \underbrace{\frac{\partial^2 x}{\partial z^2}}_{Ax} + bu; \\ b(z) \in D(A) \equiv \{x / \text{smooth and BC: } x(t, 0) = x(t, l) = 0\} \\ \qquad \qquad \qquad \subset X \equiv L^2(\Omega) \\ \text{with } (x, y) \equiv \int_{\Omega} x(t)y(t)dt \\ x(0) = x_0 \in D(A) \\ y = (c, x); \quad c(z) \in D(A) \end{array} \right.$$



# Euler-Bernoulli Beam



$$\frac{\partial}{\partial t} \begin{bmatrix} w \\ w_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I \\ -\frac{EI}{\rho} \frac{\partial^4}{\partial z^4} & 0 \end{bmatrix}}_A \begin{bmatrix} w \\ w_t \end{bmatrix} + \begin{bmatrix} 0 \\ b(z) \end{bmatrix} u(t)$$



# Symmetric Hyperbolic Systems

$$\frac{\partial \underline{\varphi}}{\partial t} = \underbrace{\sum_{i=1}^n \underbrace{A_i}_{\substack{|x| \text{ constant} \\ \text{symmetric}}} \frac{\partial \underline{\varphi}}{\partial z_i} + \underbrace{A_0}_{|x| \text{ constant}} \underline{\varphi}}_{A \underline{\varphi}}; \underline{x} \in D(A) \subset X \equiv L^2(\Omega; \mathbb{R}^l)$$

*Boundary*

*Conditions* :  $\Lambda(z)\varphi(z, t) = 0 \forall z \in \partial\Omega; t \geq 0$

*Theorem* :

1) Symbol :  $A(\xi) \equiv \sum_{i=1}^n \xi_i A_i$  is nonsingular  $\forall \xi \neq 0 \in \mathbb{R}^n$

2)  $A_0 + A_0^* < 0$

3)  $\dim N(A) < \infty$

4) Boundary Conditions are Coercive ( $\because \overbrace{\|\underline{\varphi}\|_1}^{\text{Sobolev Norm}} \leq \|\underline{\varphi}\| + \|A\underline{\varphi}\|$ )

$\Rightarrow A$  has compact resolvent and  $A$  generates

an exponentially stable  $C_0$  semigroup.

# Examples

## Wave Equation

$$\text{2-dim wave equation } \frac{\partial^2 x}{\partial t^2} = \underbrace{\left( \frac{\partial^2 x}{\partial z_1^2} + \frac{\partial^2 x}{\partial z_2^2} \right)}_{\Delta x} + \gamma x$$

$$\Leftrightarrow \frac{\partial \underline{x}}{\partial t} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{A_1} \frac{\partial \underline{x}}{\partial z_1} + \underbrace{\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}}_{A_2} \frac{\partial \underline{x}}{\partial z_2} + \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 \\ 0 & 0 & \gamma & 0 \end{bmatrix}}_{A_0} \underline{x} \text{ where } \underline{x} \equiv \begin{bmatrix} x_{z_1} \\ x_{z_2} \\ x \\ x_t \end{bmatrix}$$

## Smart Grid : Interarea Oscillations

$$: x_{tt} = v^2 x_{zz} - \eta x_t$$

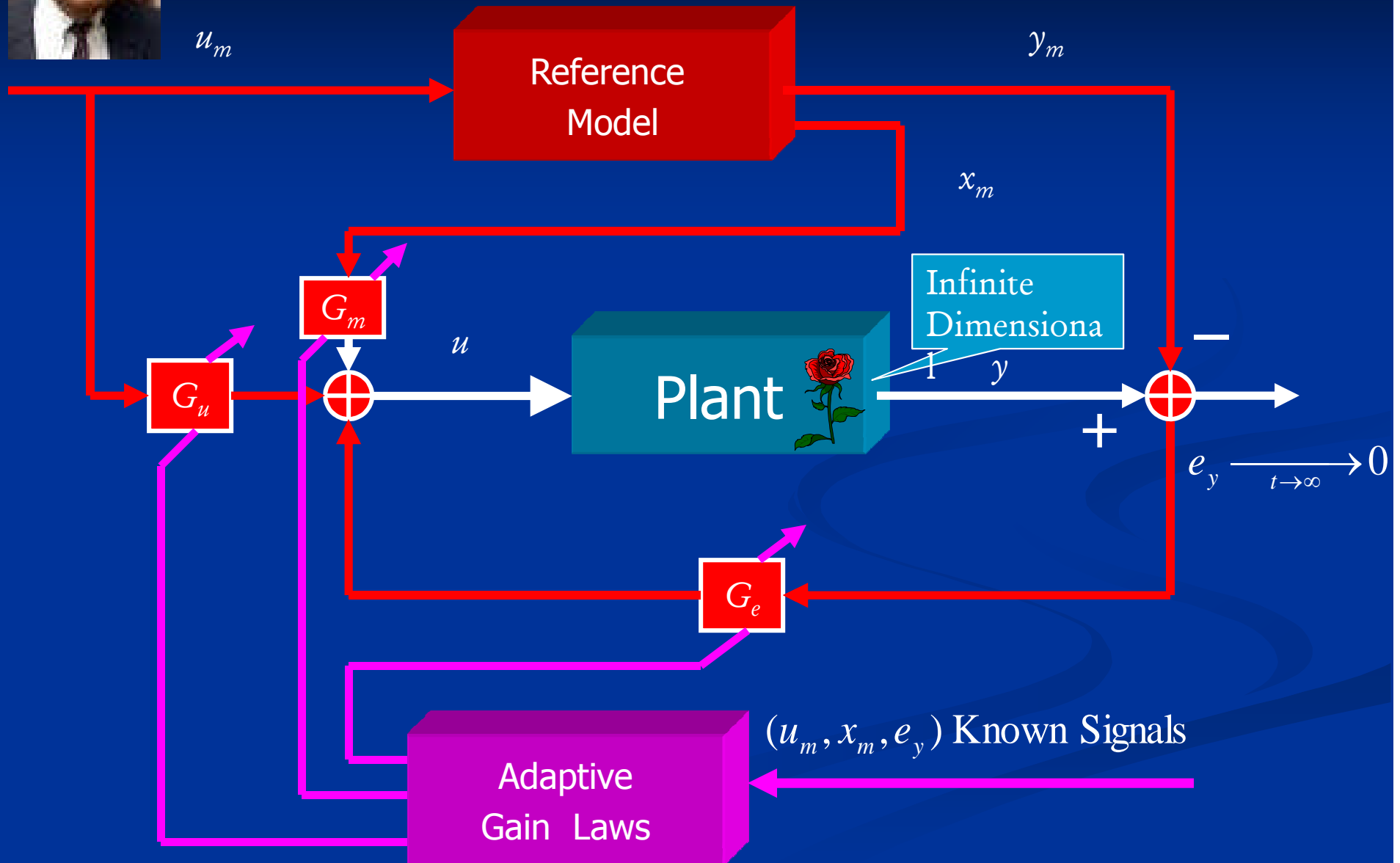
$$\Leftrightarrow \underline{x} \equiv \begin{bmatrix} x_z \\ x_t \end{bmatrix} \Rightarrow \underline{x}_t = \underbrace{\begin{bmatrix} 0 & v \\ v & 0 \end{bmatrix}}_{A_1} \underline{x}_z + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -\eta \end{bmatrix}}_{A_0} \underline{x} \equiv A \underline{x}$$

## Relativistic Fields ( Mandl & Shaw 2010)

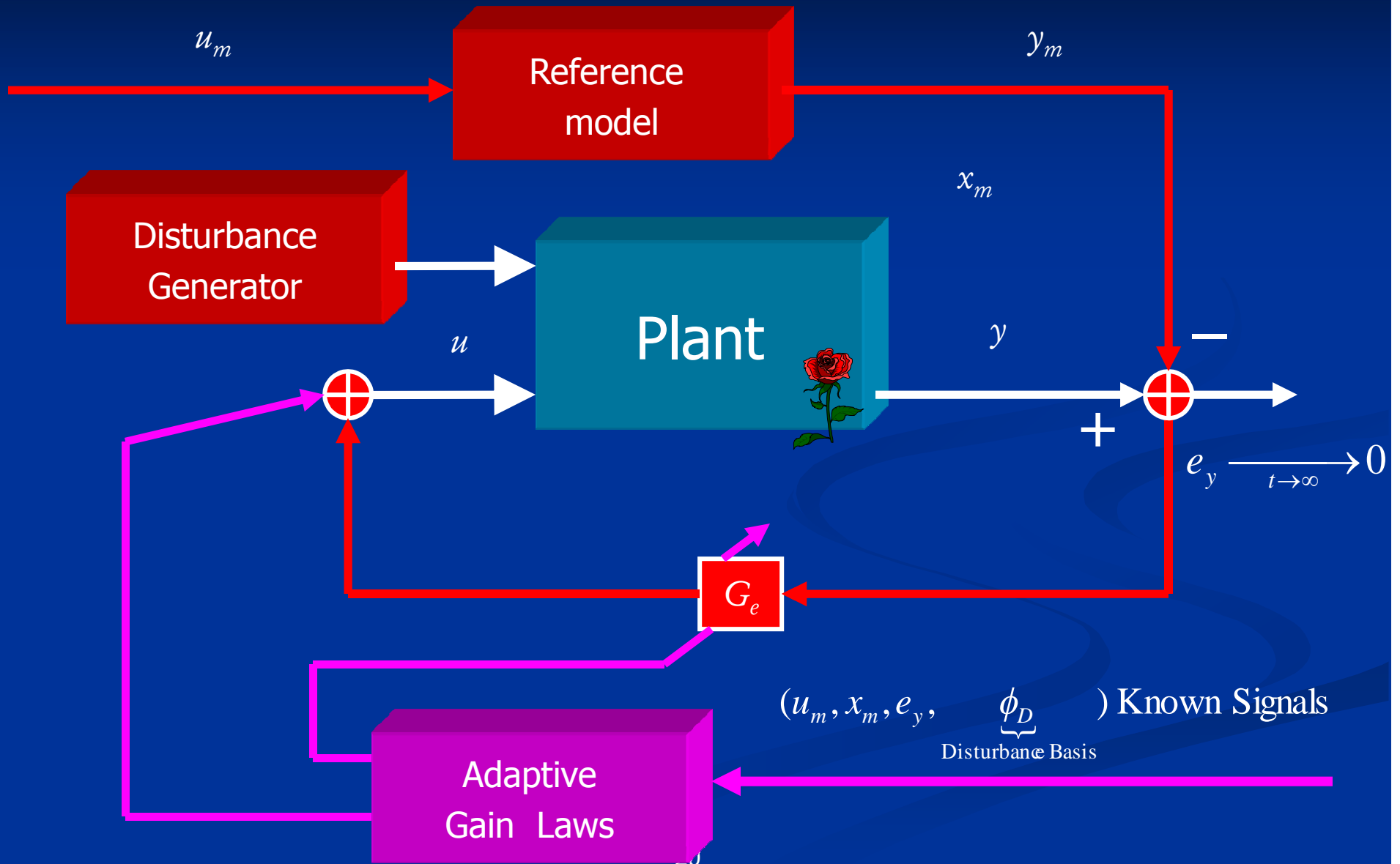
$$\text{Dirac Equation: } \frac{\partial \phi}{\partial t} = -c \left( \sum_{i=1}^3 \underbrace{A_i}_{\substack{\text{Pauli} \\ \text{Spin} \\ \text{Matrices}}} \frac{\partial \phi}{\partial x_i} \right) + \left( i \frac{mc^2}{\hbar} I_4 \right) \phi$$



# Direct Adaptive Model Following Control (Wen-Balas 1989)



# Direct Adaptive Persistent Disturbance Rejection (Fuentes-Balas 2000)



# Persistent Disturbance Example

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_A \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_B u(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_\Gamma u_D$$

Disturbances

$$u_D \equiv A_D \sin(\omega_D t + \varphi_D) = \underbrace{\begin{bmatrix} l_1 & l_2 \end{bmatrix}}_{L\theta} \underbrace{\begin{bmatrix} \sin \omega_D t \\ \cos \omega_D t \end{bmatrix}}_{\phi_D}$$

Known Basis

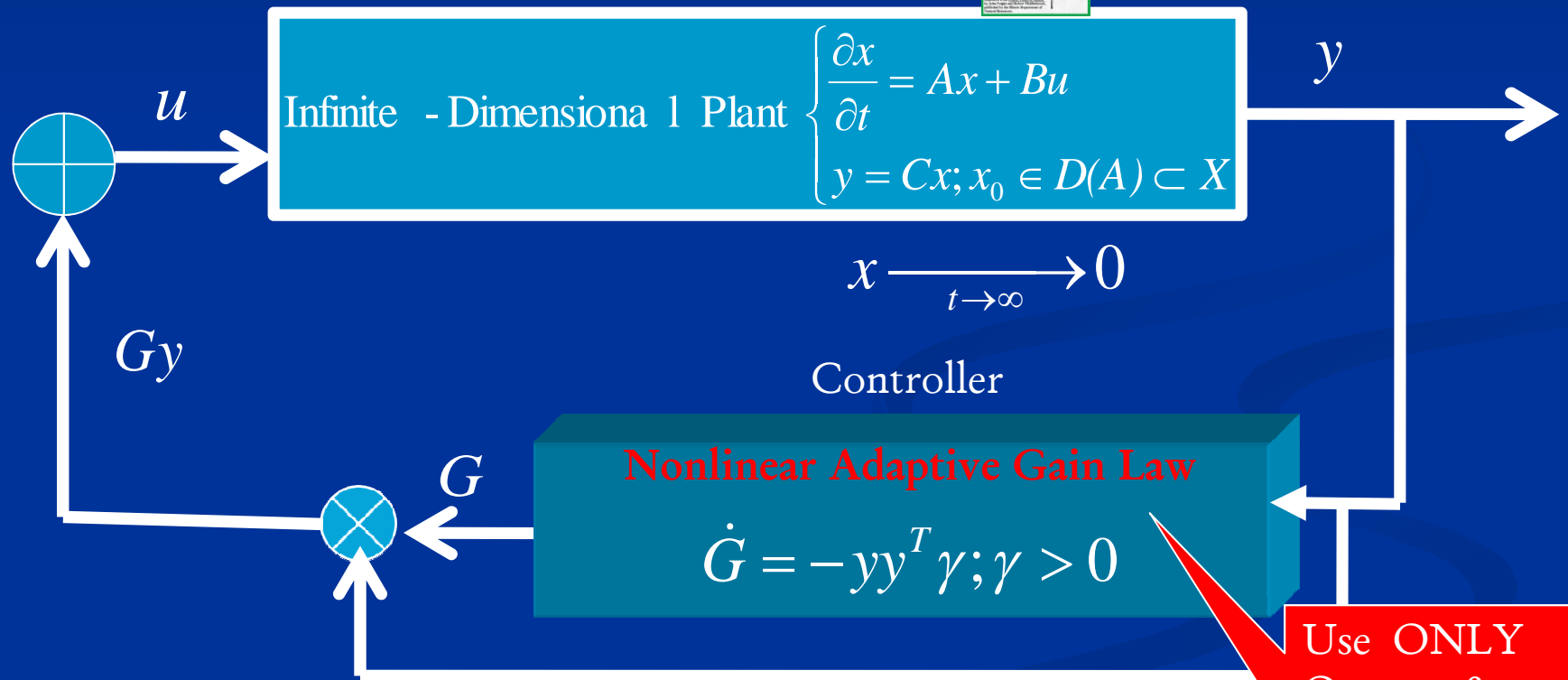
$$u_D \equiv a_0 + a_1 t + a_2 t^2 = \underbrace{\begin{bmatrix} a_0 & a_1 & a_2 \end{bmatrix}}_{L\theta} \underbrace{\begin{bmatrix} 1 \\ t \\ t^2 \end{bmatrix}}_{\phi_D}$$

Known Basis

**Unknown**

# Adaptive Control Is Not Complicated !

## Adaptive Regulation



Use ONLY  
Outputs &  
Know Almost  
NOTHING  
about the Plant

# Stability via Lyapunov- Barbalat

$$\text{Nonlinear Dynamics} \begin{cases} \dot{x} = f(x) \\ x(0) = x_0 \in \mathbb{R}^N \end{cases}$$

Find Energy - like Function :  $V(x)$

$$V(x) > 0 \text{ when } x \neq 0$$

$$V(0) = 0$$

$$\dot{V} = \text{grad}V * f(x) < 0 \Rightarrow x(t) \rightarrow 0 \text{ as } t \rightarrow \infty \text{ for all } x_0$$

Often does  
Not happen



$\dot{V} \leq 0 \Rightarrow$  All trajectories  $x(t)$  are bounded

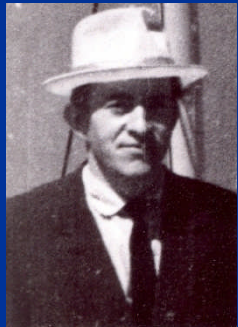
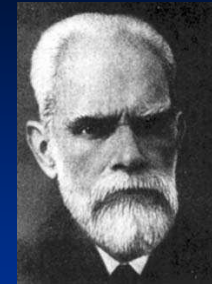
From Barbalat's lemma :

$$\dot{V}(t) \leq 0 \text{ and uniformly continuous} \Rightarrow \dot{V}(t) \rightarrow 0 \text{ as } t \rightarrow \infty$$

# Infinite-Dimensional Lyapunov-Barbalat Theory: PDE & Delay Systems



## X Hilbert or Banach Space



$$\text{Let } \begin{cases} V(t, x, \Delta G) \equiv V(t, x) + \frac{1}{2} \text{tr}(\Delta G \gamma^{-1} \Delta G^T) \\ \text{with } x(t) = U(t)x_0 \in X; t \geq 0 \end{cases}$$

Linear or  
Nonlinear

Evolution

Theorem: If  $\begin{cases} \alpha(\|(x, \Delta G)\|) \leq V(t, x, \Delta G) \leq \beta(\|(x, \Delta G)\|) \\ \dot{V}(t, x, \Delta G) \leq -W(x) \leq 0 \end{cases}$   
and  $\frac{dW(x(t))}{dt} = \underbrace{\left( \frac{\partial W}{\partial x} \right)}_{\text{Frechet Derivative}} \frac{\partial x(t)}{\partial t}$  is bounded, then  $W(x(t)) \xrightarrow{t \rightarrow \infty} 0$  and  $\Delta G$  bounded.

If  $W(x)$  is coercive in the partial state  $x$ , or  $W(x) \geq \gamma(\|x\|)$ , then  $x(t) \xrightarrow{t \rightarrow \infty} 0$ .



# Linear System Strict Dissipativity ( Balas-Frost)

$$\underline{\text{Energy Storage Function}} : \begin{cases} V(x) \equiv (x, Px) > 0; \forall x \neq 0 \\ V(0) = 0 \end{cases}$$

A Linear Dynamic Infinite-Dimensional System is STRICTLY DISSIPATIVE when

$$\begin{aligned} & \exists P : X \xrightarrow[\text{Positive}]{\substack{\text{LinearOp} \\ \text{Self-Adjoint}}} X \\ & p_{\min} \|x\|^2 \leq V(x) \equiv (Px, x) \leq p_{\max} \|x\|^2 \ni \\ & \left\{ \begin{aligned} & \text{Re}(PAx, x) \equiv \frac{1}{2} [(PAx, x) + (x, PAx)] \leq \underbrace{-\alpha \|x\|^2}_{W(x)}; \forall x \in D(A) \\ & PB = C^* \end{aligned} \right. \end{aligned}$$

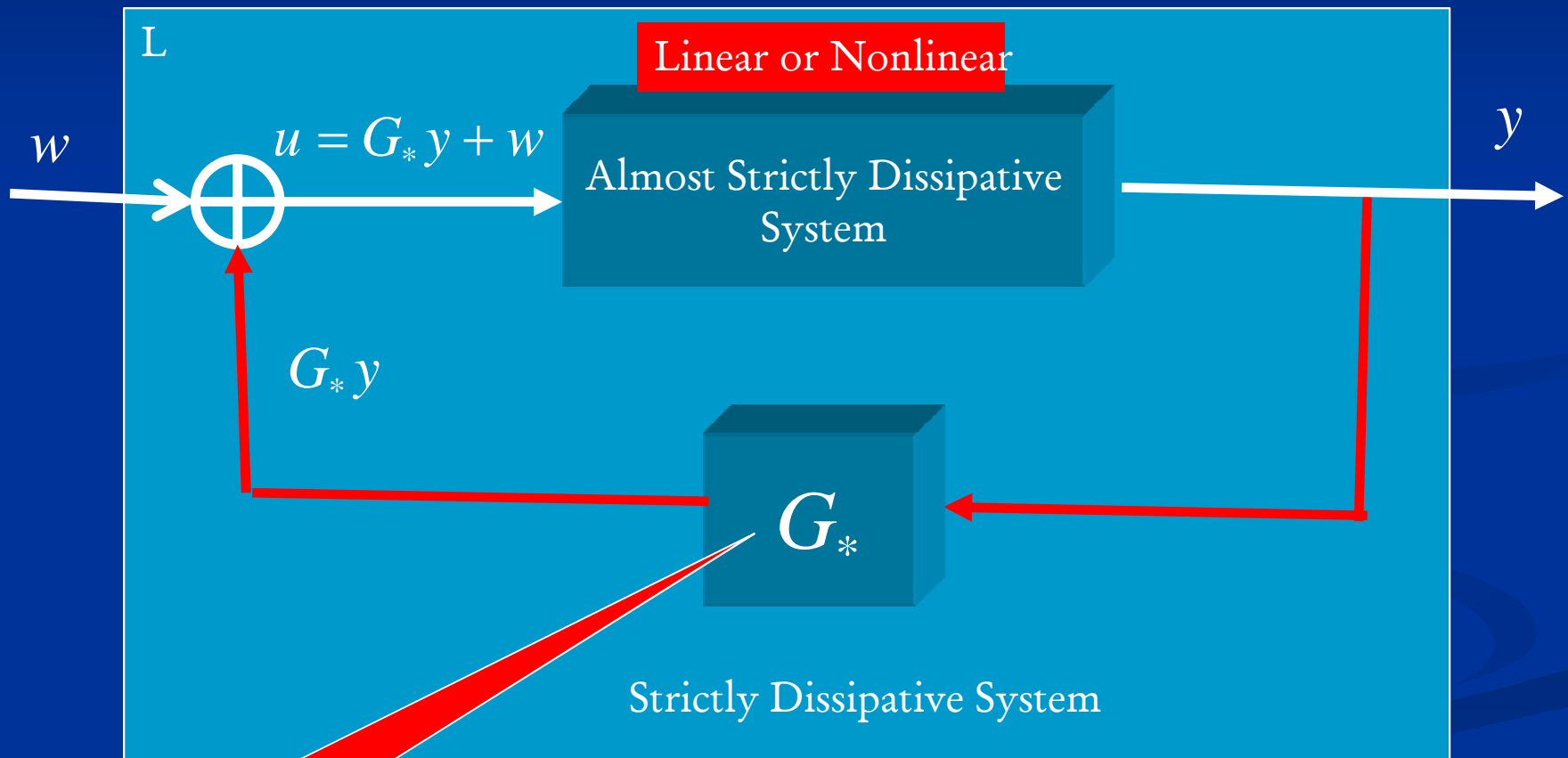
DISSIPATIVE when  $\alpha=0$

$$\Rightarrow \underbrace{\frac{dV}{dt}}_{\substack{\text{Energy} \\ \text{Storage} \\ \text{Rate}}} \leq \underbrace{(y, u)}_{\substack{\text{External} \\ \text{Power}}} - \underbrace{\alpha \|x\|^2}_{\substack{\text{Internally} \\ \text{Dissipated} \\ \text{Power}}}$$

## Almost Strictly Dissipative (ASD) Systems

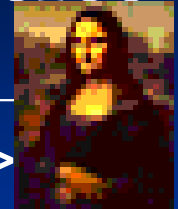
$(A(x), B(x), C(x))$  ASD means

$\exists G_* \ni (A_c(x) \equiv A(x) + B(x)G_*C(x), B(x), C(x))$  Strictly Dissipative



Need not know  
the value!

# Finite- Dimensional LINEAR ASD: Two Simple Open-Loop Properties



High Frequency Gain is Sign-Definite (CB >

Open-Loop Transfer Function is Minimum Phase  
(all transmission zeros stable)



Almost Strictly Dissipative



$$\text{Adaptive Regulation } \begin{cases} u = Gy \\ \dot{G} = -yy^T \gamma; \gamma > 0 \end{cases}$$

produces  $x(t) \xrightarrow[t \rightarrow \infty]{} 0$

with bounded adaptive gains  $G(t)$

# An Infinite-Dimensional Version

$$\begin{cases} \frac{\partial x}{\partial t} = Ax + Bu = Ax + \sum_{i=1}^m b_i u_i; A \text{ generates a } C_0 \text{ semigroup} \\ x(0) = x_0 \in D(A) \subset X \\ y = Cx = [(c_1, x) \quad (c_2, x) \quad \dots \quad (c_m, x)]^T; b_i, c_j \in D(A) \end{cases}$$

(My) Theorem: Def :  $\lambda_* \in C$  is a transmission zero of  $(A, B, C)$  when  $N(H(\lambda_*)) \neq \{0\}$

where  $H(\lambda) \equiv \begin{bmatrix} A - \lambda I & B \\ C & 0 \end{bmatrix} : D(A) \times \mathbb{R}^M \rightarrow X \times \mathbb{R}^M$  closed linear operator

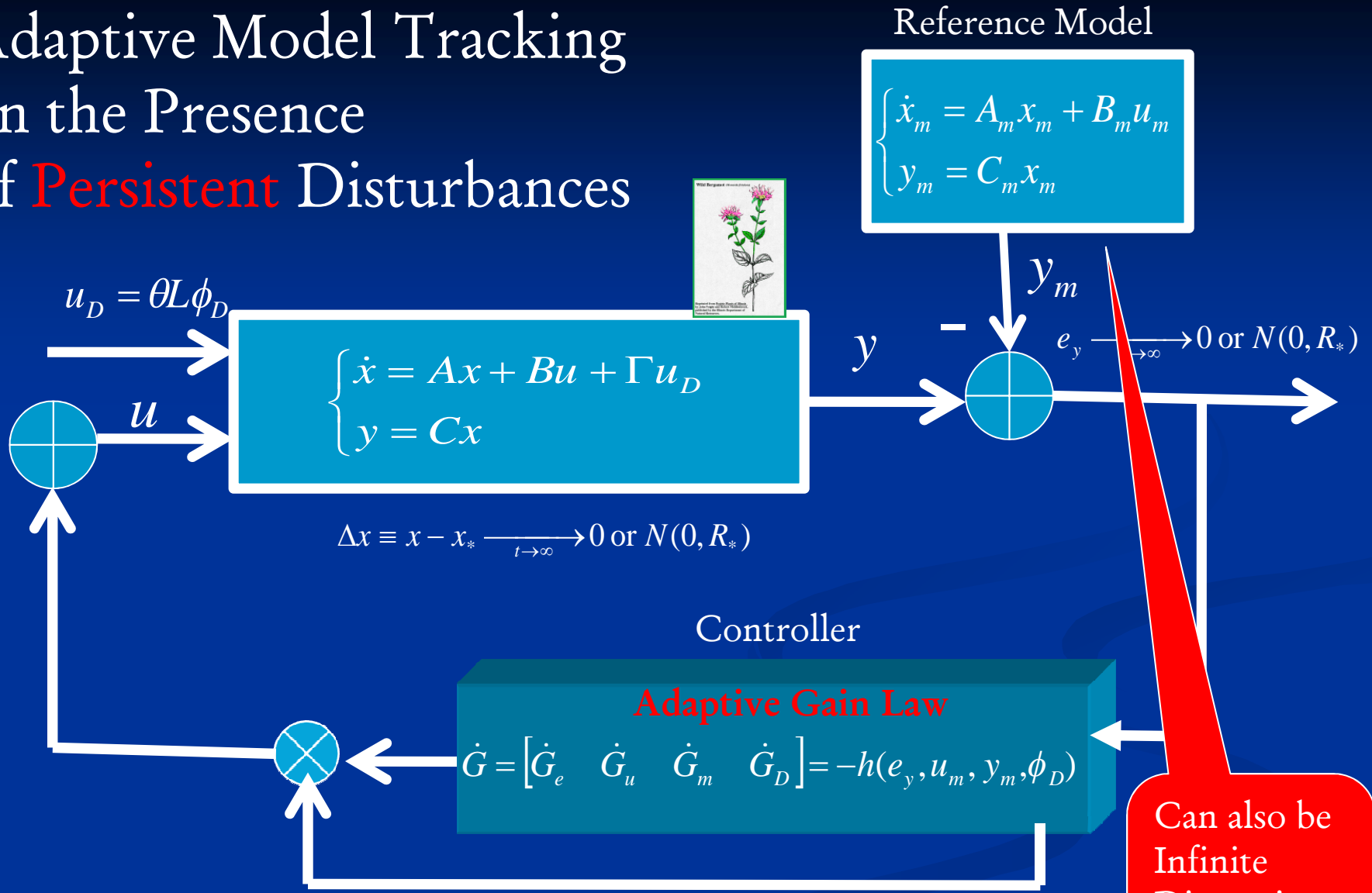
Pretty Close !!

$(A, B, C)$  is Almost Strictly Dissipative

$\Leftrightarrow CB = [(c_j, b_i)]_{m \times m}$  nonsingular and  $\text{Zeros}(A, B, C) \equiv \{\lambda / N(H(\lambda)) \neq \{0\}\} = \sigma_p(\overline{A}_{22})$  "stable"

(i.e.  $\overline{A}_{22}$  satisfies spectrum determined growth condition)

# Adaptive Model Tracking in the Presence of **Persistent** Disturbances



Can also be  
Infinite  
Dimensional

# Adaptive Control Law

$$u = \underbrace{G_u u_m + G_m w_m}_{\text{Model Tracking}} + \underbrace{G_D \phi_D}_{\text{Disturbance Rejection}} + \underbrace{G_e e_y}_{\text{Stabilization}}$$

where

$$\left\{ \begin{array}{l} \dot{G}_u = -e_y \cdot u_m^* \cdot \sigma_u; \sigma_u > 0 \\ \dot{G}_m = -e_y \cdot x_m^* \cdot \sigma_m; \sigma_m > 0 \\ \dot{G}_D = -e_y \cdot \phi_D^* \cdot \sigma_D; \sigma_D > 0 \\ \dot{G}_e = -e_y \cdot e_y^* \cdot \sigma_e; \sigma_e > 0 \end{array} \right. \begin{array}{l} \text{Gain} \\ \text{Adaptation} \\ \text{Laws} \end{array}$$

$S_{11}^* : D(A_m) \rightarrow D(A), S_{ij}^*$  and  $H_1, H_2$

# Ideal Trajectories

$$\begin{cases} \frac{\partial x_*}{\partial t} = Ax_* + Bu_* + \Gamma u_D \\ y_* = Cx_* = y_m \end{cases}$$

$$\begin{cases} x_* = S_{11}^* x_m + S_{12}^* u_m + S_{13}^* z_D = S_1 z \\ u_* = S_{21}^* x_m + S_{22}^* u_m + S_{23}^* z_D = S_2 z \end{cases}$$

Matching Conditions  $\begin{cases} AS_1 + BS_2 = S_1 \bar{A}_m + H_1 \\ CS_1 = H_2 \end{cases}$

$$\begin{aligned} S_1 &\equiv \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \end{bmatrix} : D(\bar{A}_m) \rightarrow D(A) \subset X \\ S_2 &\equiv \begin{bmatrix} S_{21}^* & S_{22}^* & S_{23}^* \end{bmatrix} : D(\bar{A}_m) \rightarrow \mathcal{R}^m, \\ \bar{A}_m &\equiv \begin{bmatrix} A_m & B_m & 0 \\ 0 & F_m & 0 \\ 0 & 0 & F \end{bmatrix} \end{aligned}$$

with  $D(\bar{A}_m) \equiv D(A_m) \times \mathcal{R}^m \times \mathcal{R}^{N_D}$   
 and  $D(\bar{A}_m)$  dense in  $\bar{X}_m \equiv X_m \times \mathcal{R}^m \times \mathcal{R}^{N_D}$ ,  
 and  $\begin{cases} H_1 \equiv [0 \quad 0 \quad -\Gamma\theta] \\ H_2 \equiv [C_m \quad 0 \quad 0] \end{cases}$

# An Infinite Dimensional Internal Model Principle

*Theorem* : Assume **CB** is nonsingular and the open loop zeros  $(\sigma(\bar{A}_{22}))$  are (exponentially) stable.

Then the zeros of the open loop plant must not overlap with the poles of the tracked reference model:

$$\begin{aligned} \sigma(\bar{A}_m) &= \sigma_p(A_m) \cup \sigma_p(F_m) \cup \sigma_p(F) \\ &\subset \rho(\bar{A}_{22}) \equiv \{\lambda \in \mathbb{C} / (\lambda I - \bar{A}_{22})^{-1} : l_2 \rightarrow l_2 \text{ is a bounded linear operator}\} \\ &(\text{or } \sigma(\bar{A}_m) \cap \sigma(\bar{A}_{22}) = \emptyset \text{ where } \sigma(\bar{A}_{22}) \equiv [\rho(\bar{A}_{22})]^c) \Leftrightarrow \end{aligned}$$

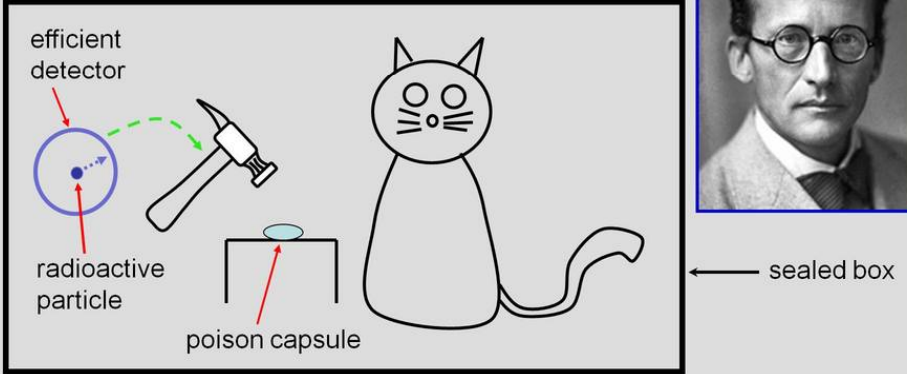
There exist unique *bounded* linear operator solutions  $(S_1, S_2)$  satisfying the Matching Conditions



# Adaptive Control in Quantum Information Systems

This might be the most fundamental application of direct adaptive control

Erwin Schrödinger's Cat (1935)



efficient detector

radioactive particle

poison capsule

sealed box

At "half-life of particle, cat is dead and alive!"  
"superposition"

$$\Psi = |\text{particle}\rangle|\text{cat}\rangle + |\text{particle}\rangle|\text{cat}\rangle$$

Ontology ( what is) vs Epistemology ( What is measured)

# Quantum Computing

A Quantum computer will operate differently from a Classical one. It will be involved w physical systems on an atomic scale, eg atoms, photons, trapped ions, or nuclear magnetic moments



Unitary  Reversible

Entanglement produces Decoherence

# Quantum Basics (Dirac & Von Neumann)

Observable  $A : X \xrightarrow[\text{self-adjoint}]{\text{bounded}} X$

Orthonormal  
Eigen -Basis for X

$$\text{Compact Resolvent} \Rightarrow Ax = \sum_{k=1}^{\infty} \lambda_k \underbrace{(x, \varphi_k)}_{P_k x} \varphi_k$$

Pure States :  $\varphi_k$  eigenfunctions of  $A$

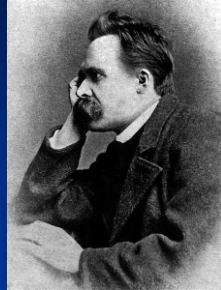
Mixed State  $\varphi \in X$  complex Hilbert Space :

$$(\varphi, \varphi) = 1 \text{ or } \|\varphi\| = 1 \Rightarrow \varphi = \sum_{k=1}^{\infty} c_k \varphi_k \ \& \ 1 = \|\varphi\|^2 = \sum_{k=1}^{\infty} |c_k|^2$$

where  $|c_k|^2 =$  probability of being in the pure state  $\varphi_k$

# Quantum Measurement

“...for when you gaze long into the abyss. The abyss gazes also into you.”



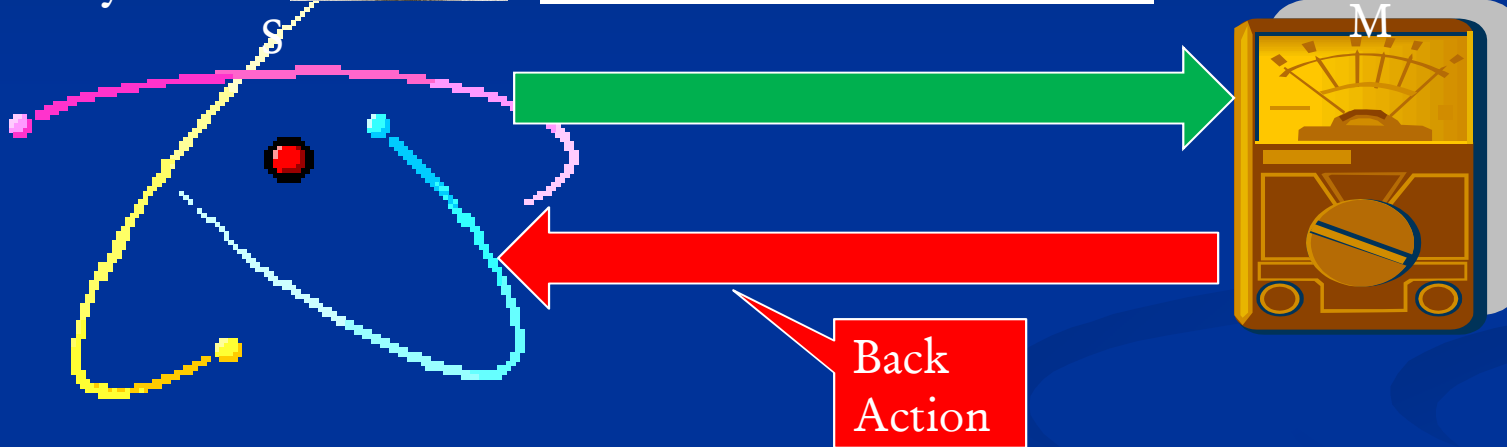
Entanglement

$$X = X_S \otimes X_M$$

$$\varphi = \sum_{k,l} \alpha_{kl} (\varphi_k^S \otimes \varphi_l^M) \neq h \otimes w$$



Nietzche!!!  
Never around when you need him



Observable  $A : X \xrightarrow{\text{bounded self-adjoint}} X$  Hilbert

Heisenberg Uncertainty Principle

$\left\{ \begin{array}{l} \text{Mean } \langle A \rangle \equiv \text{Tr}(\rho A) \\ \text{Dispersion } \Delta A \equiv \sqrt{\text{Tr}(\rho(A - \langle A \rangle)^2)} \end{array} \right.$ 
 where  $\rho$  is a state or density operator ( $\rho > 0$  &  $\text{Tr}(\rho) = 1$ )

Heisenberg Uncertainty Principle : Simultaneous Measurement of  $A$  &  $B$

$$(\Delta A)^2 (\Delta B)^2 \geq \frac{\hbar}{2} |\text{Tr}(\rho[A, B])|; \text{commutator } [A, B] \equiv AB - BA$$

# Schrodinger Wave Equation

$\varphi \in X$  complex Hilbert Space

$$i\hbar \frac{\partial \varphi}{\partial t} = \underbrace{H_0}_{\substack{\text{Self - Adjoint} \\ \text{Compact} \\ \text{Resolvent}}} \varphi + \underbrace{H_C(u)}_{\substack{\text{Disturbanc e} \\ \text{Hamiltonia n}}} \varphi$$

Diffusion with Imaginary Time :

$$t_{NEW} \equiv \frac{it_{OLD}}{\hbar}$$

$\therefore$  Discrete Real Spectrum  $\sigma(H_0) = \{\lambda_k\}_{k=1}^{\infty}$

$-\infty$

$\Rightarrow U_0(t) : X \rightarrow X$  Unitary Group (reversible)

and  $U_0(t)\varphi = \sum_{k=1}^{\infty} e^{i\lambda_k t} \langle \varphi, \phi_k \rangle \phi_k$  with  $\langle \phi_k, \phi_l \rangle = \delta_{kl}$



Marginally Stable

# Small Quantum Systems

- We can begin to experiment with just one electron, atom or small molecule
- Need:



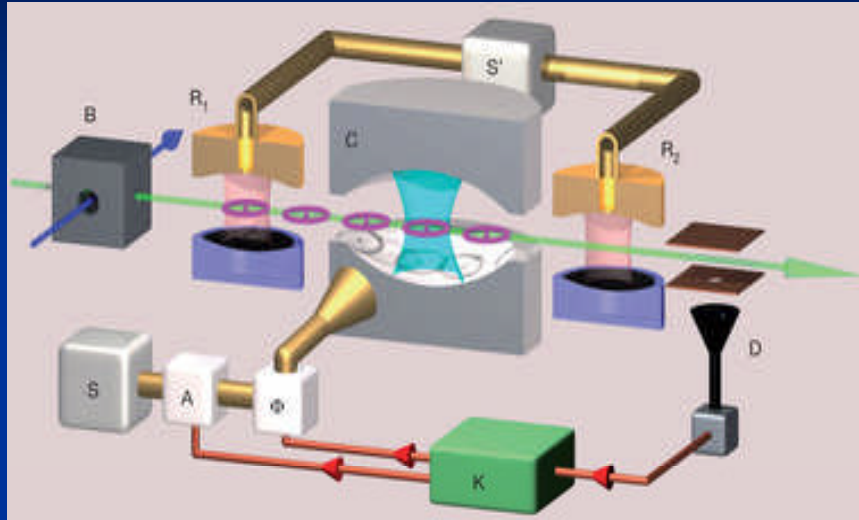
Precise control

Isolation from the environment

Simple small systems : single particles or small groups of particles

..... David Wineland NIST

# Control of Individual Quantum Systems: Quantum Feedback Loop



Physics Nobel Prize 2012  
S. Haroche & D. Wineland

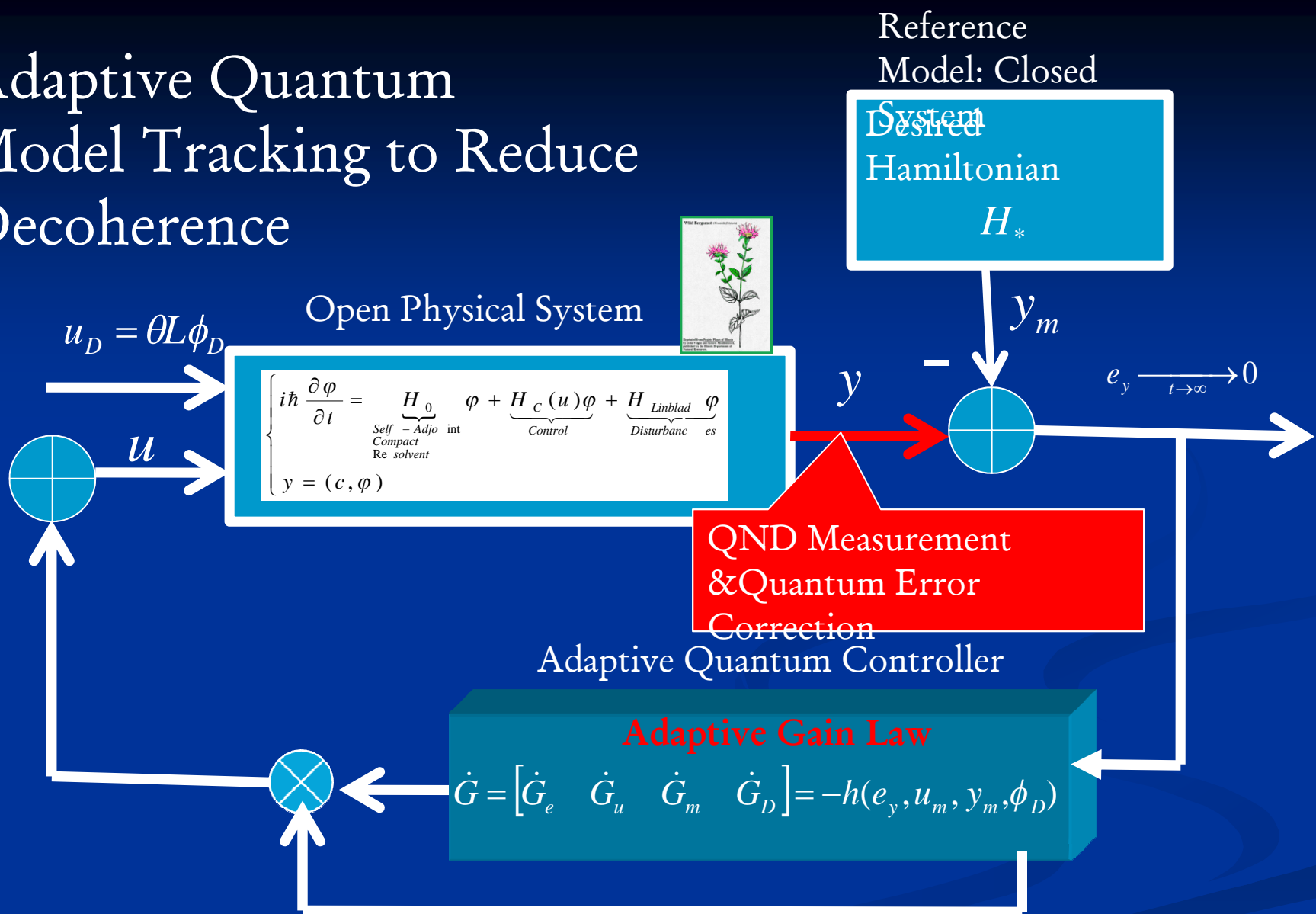
**Purpose:**

Use information from weak QND measurements to prepare photon number (Fock) states of a cavity field and protect them against decoherence.

**Method:**

Quantum feedback realized by atoms as QND probes and small coherent field injections into the cavity mode as an actuator.

# Adaptive Quantum Model Tracking to Reduce Decoherence







Famous  
Lisbon Poet

“No intelligent idea can gain general acceptance unless some stupidity is mixed in with it” .....

Fernando Pessoa, The Book of Disquiet