# Evolving Systems: An Introduction

Devolving System

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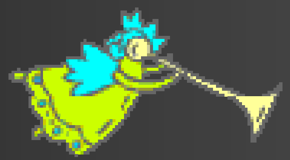
# References

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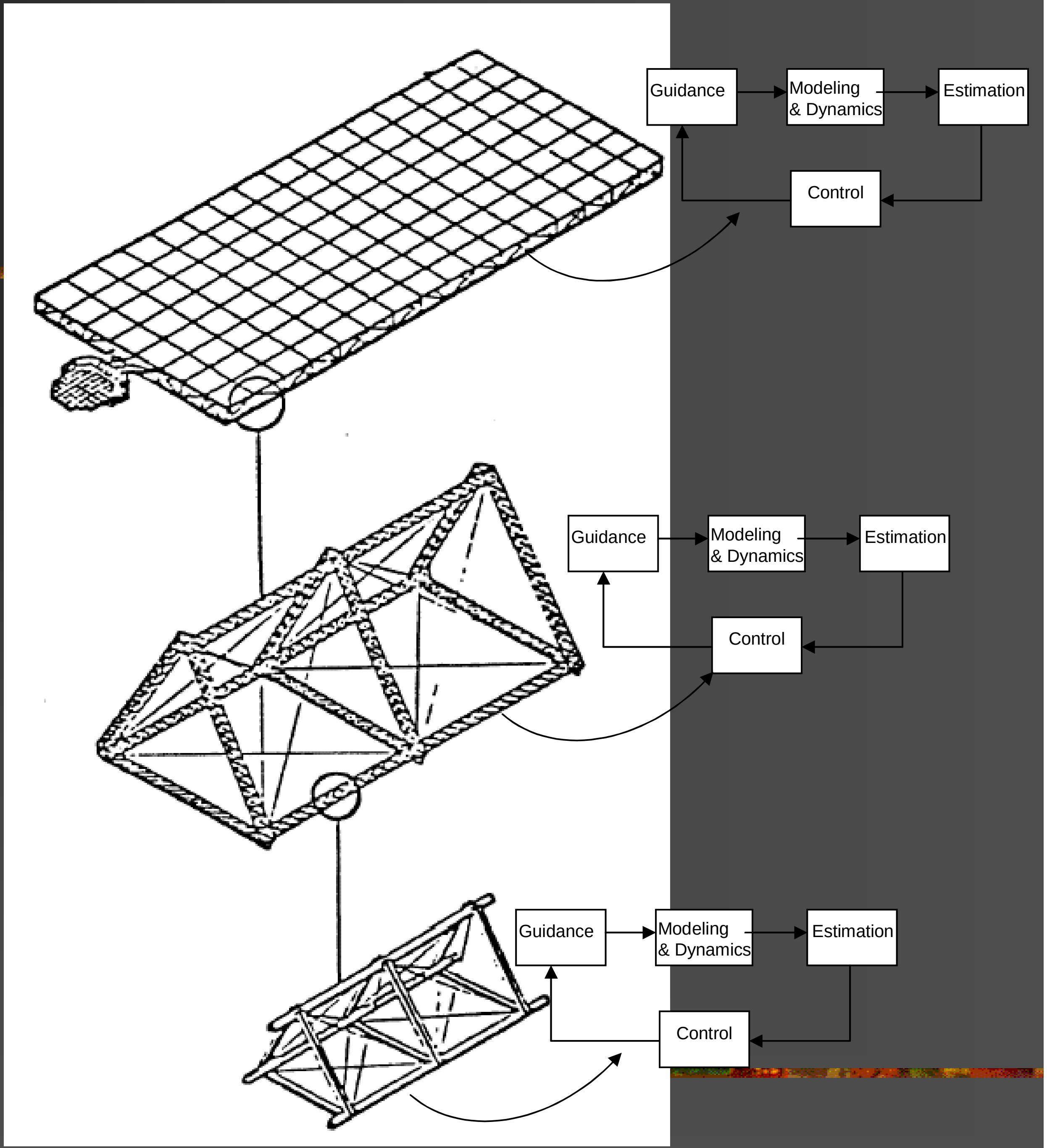
# What It Is?

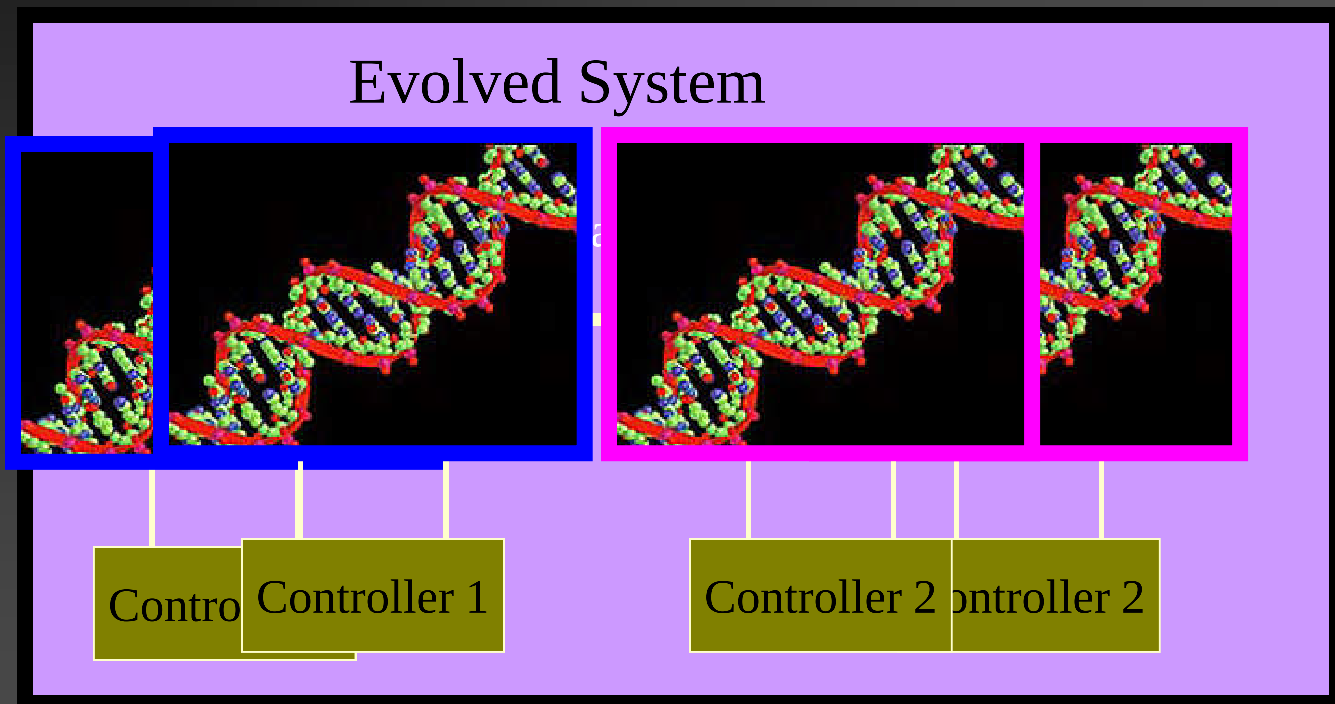
Evolving Systems=
Autonomously
Assembled
Active Structures

Or Self-Assembling
Structures,
which Aspire to a
Higher Purpose;
Cannot be attained
by Components Alone

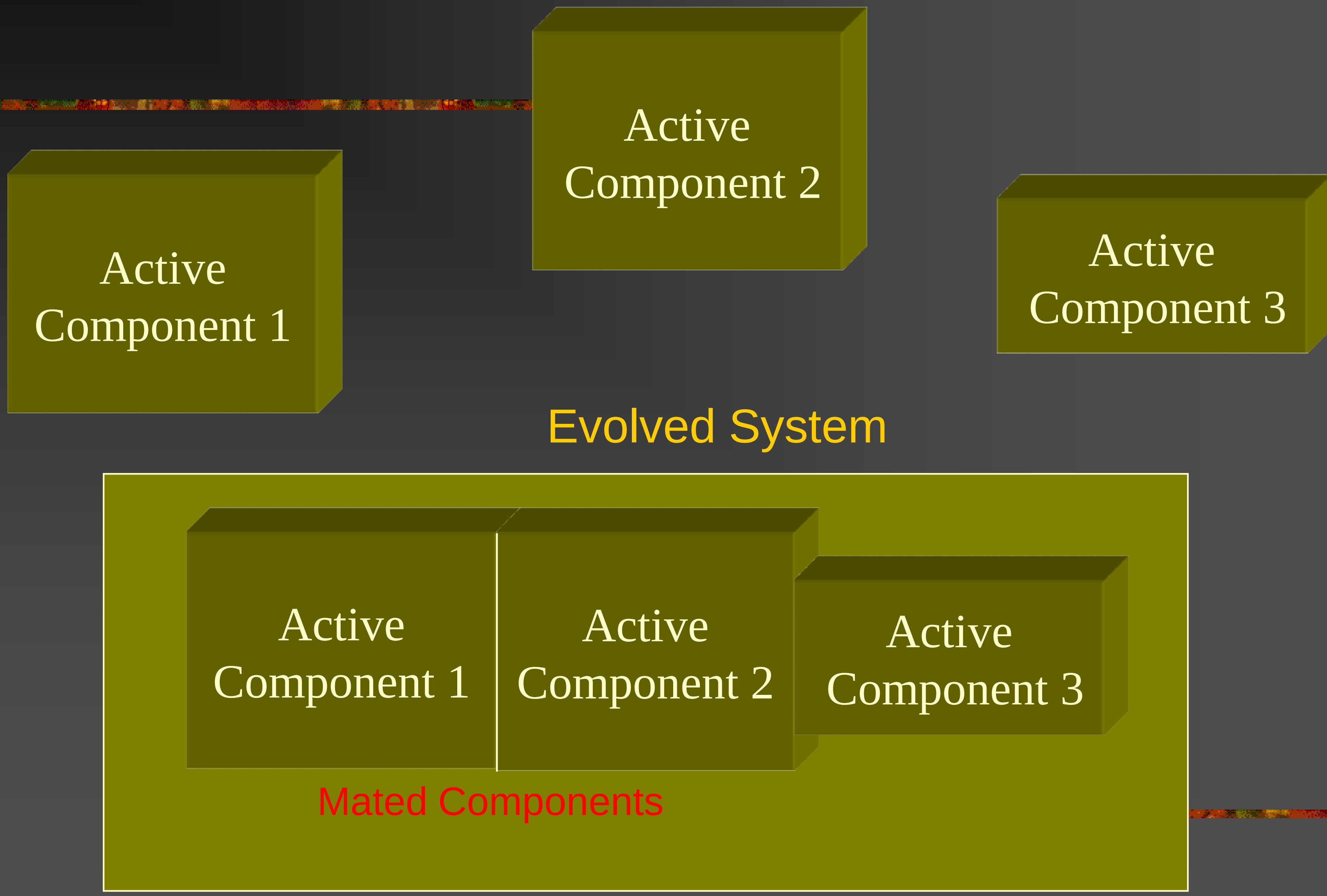



# How It Works

Susan's Slide



# The Process of Evolving Systems



It's not theories about stars; it's the actual stars that count."

………. Freeman Dyson

# Evolving Systems Applications

- Autonomous Assembly in Space

**International Space Station after 9 December 2006 Mission**

# Evolving Systems Applications

- Autonomous Rendezvous and Docking
- Servicing and System Upgrades

DARPA's Orbital Express

(ASTRO Servicing Satellite pictured on left)

# Stability is Essential During the Entire Evolution Process

Orion Crew Exploration Vehicle Docking with the ISS

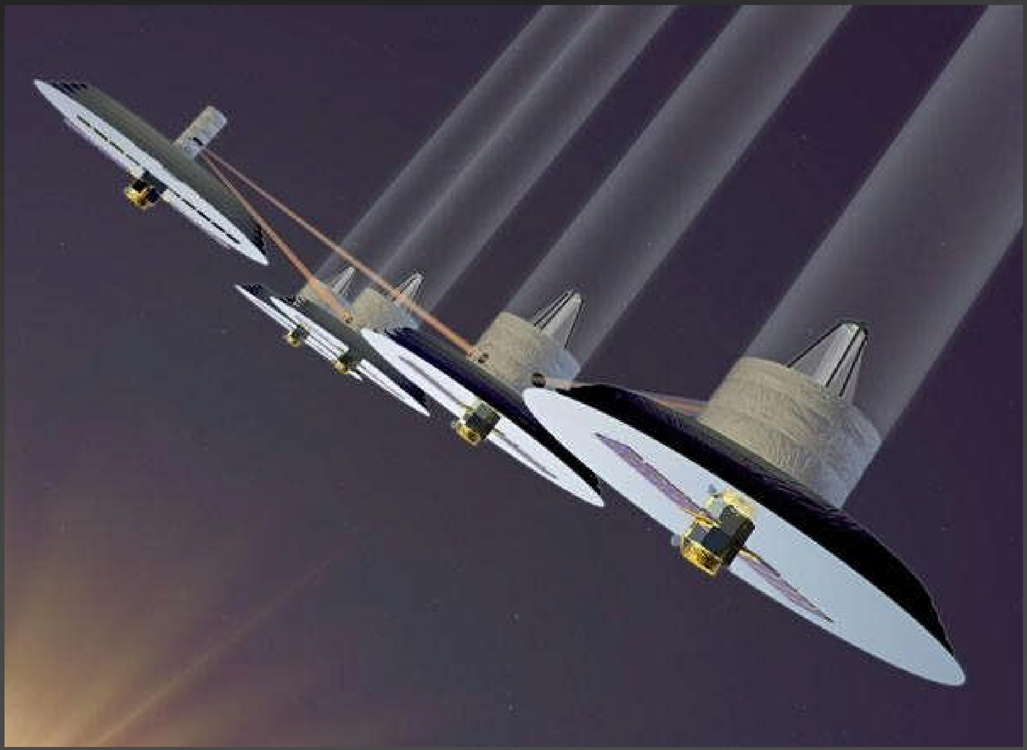
# Launch Vehicles: Devolving Systems

NASA-MSFC

Ares-Orion

$$$

# Constellations and Formations of Spacecraft (NASA-JPL)

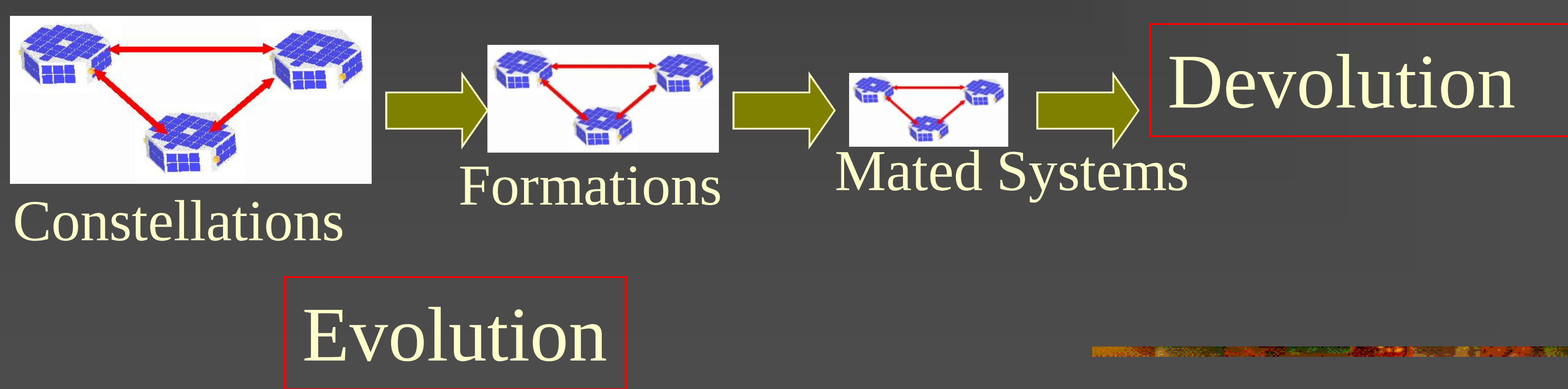


Separated Spacecraft Interferometers

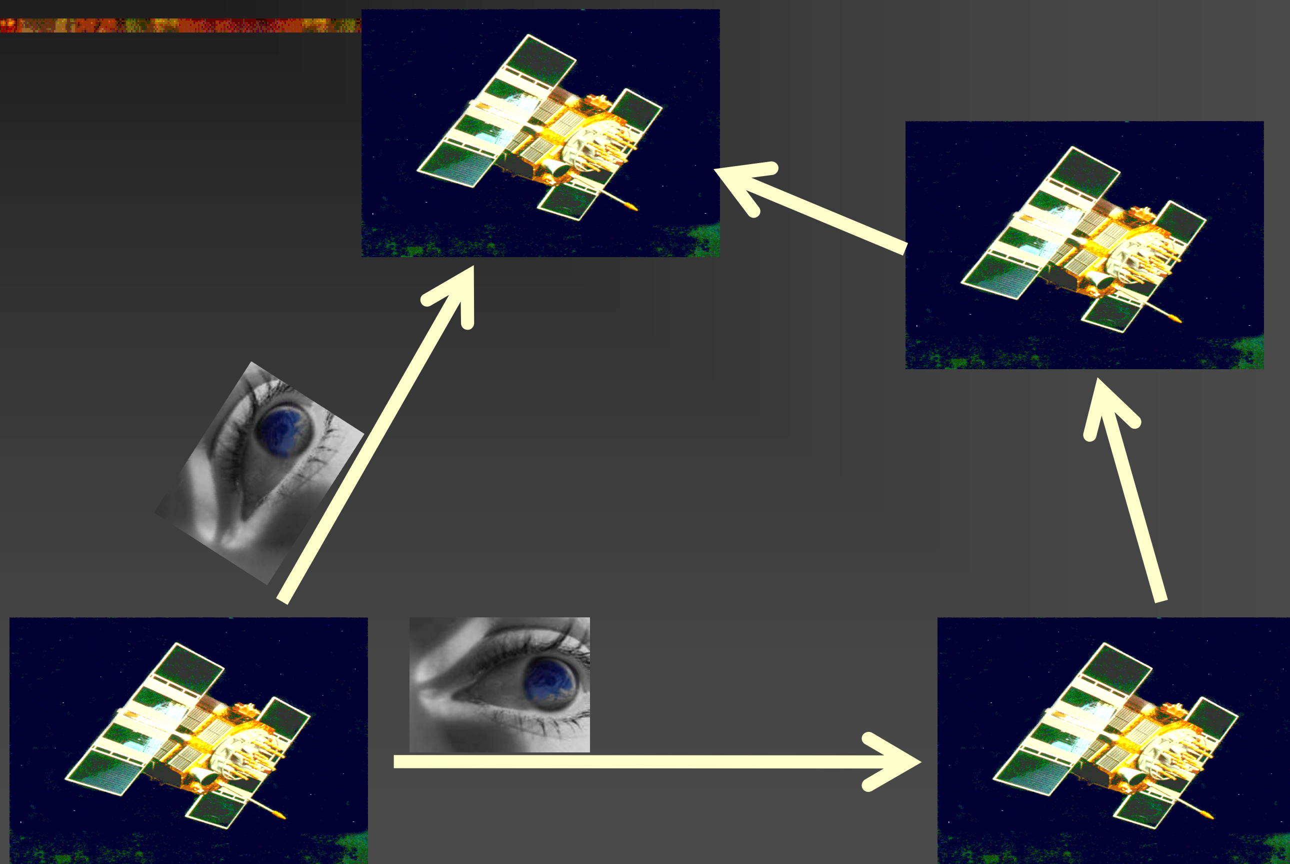
# Global Persistent Surveillance: Constellations-Formations-Evolving Systems (NRO/DARPA)

The next step in network-centric warfare will be the creation of networked sensing suites that tailor their observations to the adversary's rate of activity. These various sensors will concentrate on observing changes rather than on observing scenery …

Signal Magazine

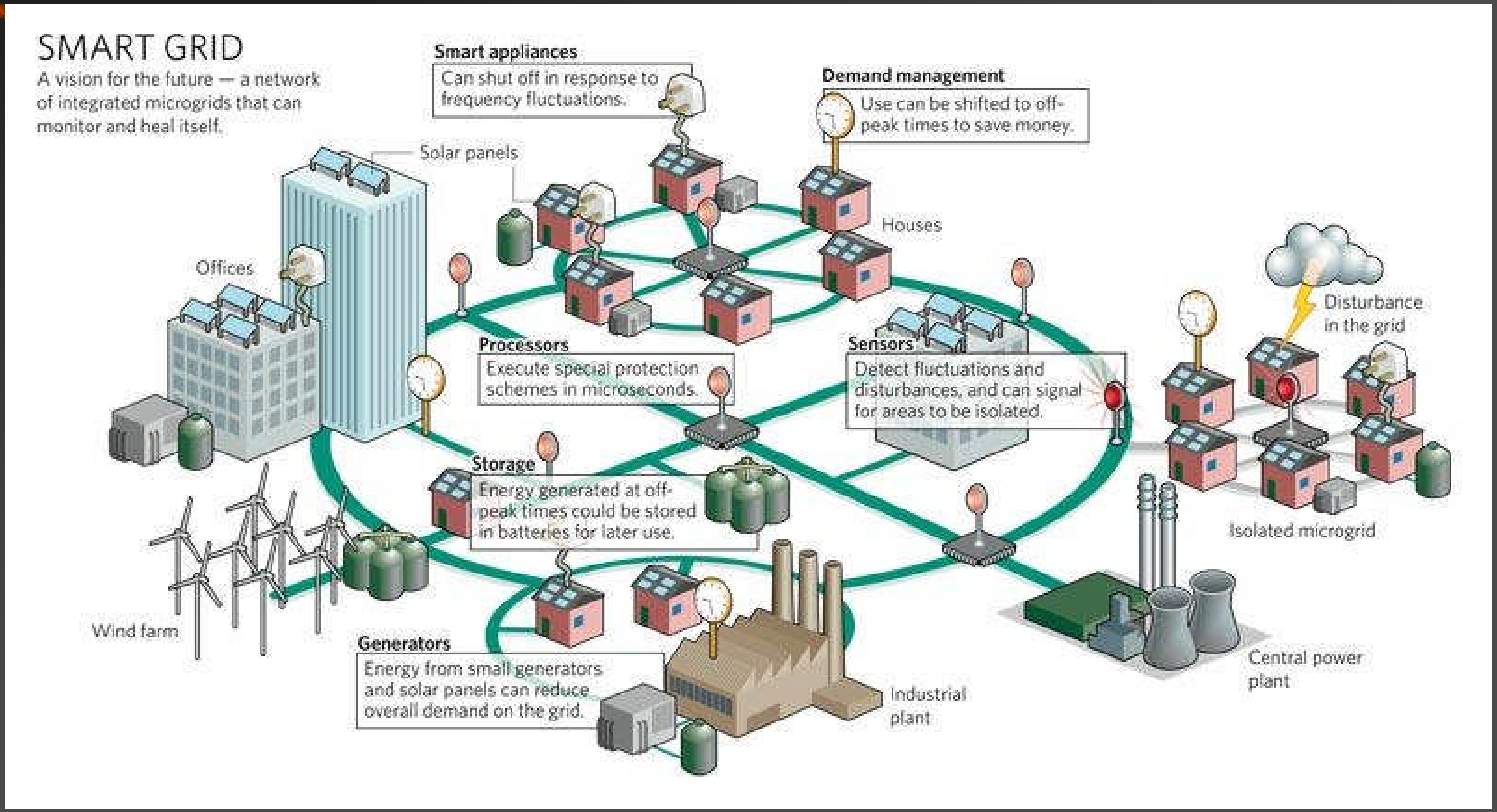


# Evolving Spacecraft Formations



Joining Spacecraft

# Smart Grids: Virtual Interconnecting Forces




"It is surprising how quickly we replace a human operator with an algorithm and call it SMART"

# Affine Nonlinear Systems

$$\begin{cases} \dot{x} = A(x) + B(x)u \\ y = C(x) \end{cases}$$

With smooth vector fields: $(A(x), B(x), C(x))$
defined on a neighborhood a smooth manifold, or $\Re^N$

# Evolving Systems: General

i th Component

$$\begin{cases} \dot{x}_i = A_i(x_i) + B_i(x_i)u_i^c \\ y_i = C_i(x_i) \end{cases}$$

Local Controller : $\begin{cases} u_i^c = h_i(z_i) + u_i \\ \dot{z}_i = l_i(z_i, y_i, u_i) \end{cases}$

$$\Rightarrow \begin{cases} \dot{x} = A(x) + B(x)u \\ y = C(x) \end{cases} \text{ Evolved System with } x \equiv \begin{bmatrix} x_1 \\ x_2 \\ \\ x_n \end{bmatrix}$$

$$\text{where } \dot{x}_i = A_i(x_i) + B_i(x_i)u_i + \sum_{j=1}^{L} \varepsilon_{ij} \underbrace{A_{ij}(x_i, x_j, u_j)}_{Interconnections}; \begin{cases} 0 \le \varepsilon_{ij} \le 1 \\ \varepsilon_{ij} = \varepsilon_{ji} \end{cases}$$

# Evolving Systems: 2 Components

$$0 \le \varepsilon_{12} = \varepsilon_{21} \equiv \varepsilon \le 1$$

Component 1

$$\begin{cases} \dot{x}_1 = A_1(x_1) + B_1(x_1)u_1 + \varepsilon A_{12}(x_1, x_2, u_2) \\ y_1 = C_1(x_1) \end{cases}$$

Component 2

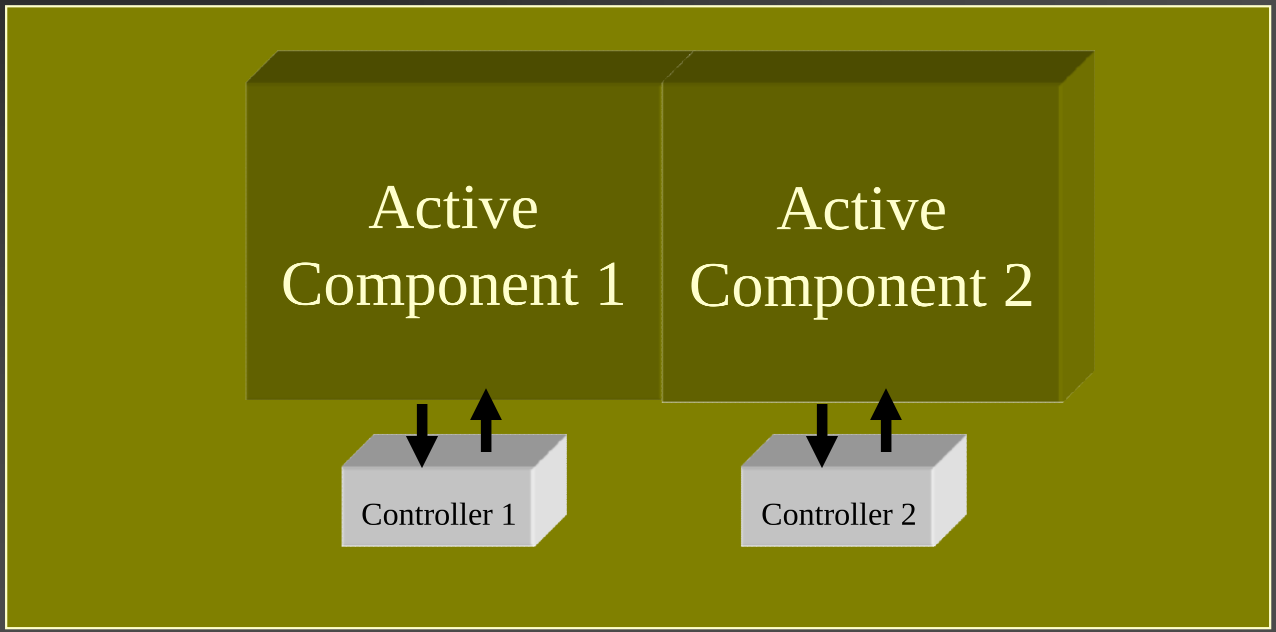
$$\begin{cases} \dot{x}_2 = A_2(x_2) + B_2(x_2)u_2 + \varepsilon A_{21}(x_2, x_1, u_1) \\ y_2 = C_2(x_2) \end{cases}$$

$$\Rightarrow \text{Evolved System} \begin{cases} \dot{x} = \overbrace{\begin{bmatrix} A_1(x_1) \\ A_2(x_2) \end{bmatrix}}^{A(x)} + \overbrace{\begin{bmatrix} B_1(x_1) & 0 \\ 0 & B_2(x_2) \end{bmatrix}}^{B(x)} u + \varepsilon \begin{bmatrix} A_{12}(x_1, x_2, u_2) \\ A_{21}(x_2, x_1, u_1) \end{bmatrix} \\ y = \begin{bmatrix} C_1(x_1) \\ C_2(x_2) \end{bmatrix} \equiv C(x) \end{cases}$$

$$\begin{cases} \varepsilon = 0 \text{ unconnecte d} \\ \varepsilon = 1 \text{ fully connected} \end{cases}$$

# Evolving Systems: Evolution=Homotopy

Evolved System




$$0 \leq \varepsilon \leq 1$$

# Genetics of Evolving Systems: Inheritance of Component Traits

Source: CNN.com

- Controllability/Observability
- **Stability**
- **Dissipativity**
- Optimality
- Robustness
- Disturbance Rejection/Signal Tracking

# How To Analyze Evolving Systems

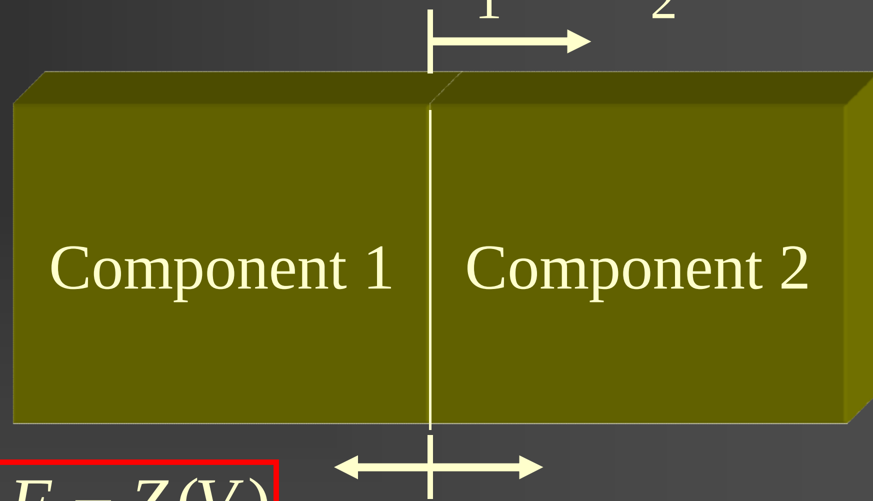
- Admittance/Impedance
- Perturbation Methods

- Graph Theory
- Differential Geometry- Lie Groups
- Other stuff

# Impedance and Admittance of Components

$$v_1 \equiv \dot{x}_1 = \dot{x}_2$$

$$x_1 = x_2$$




$$f_2 = -f_1$$

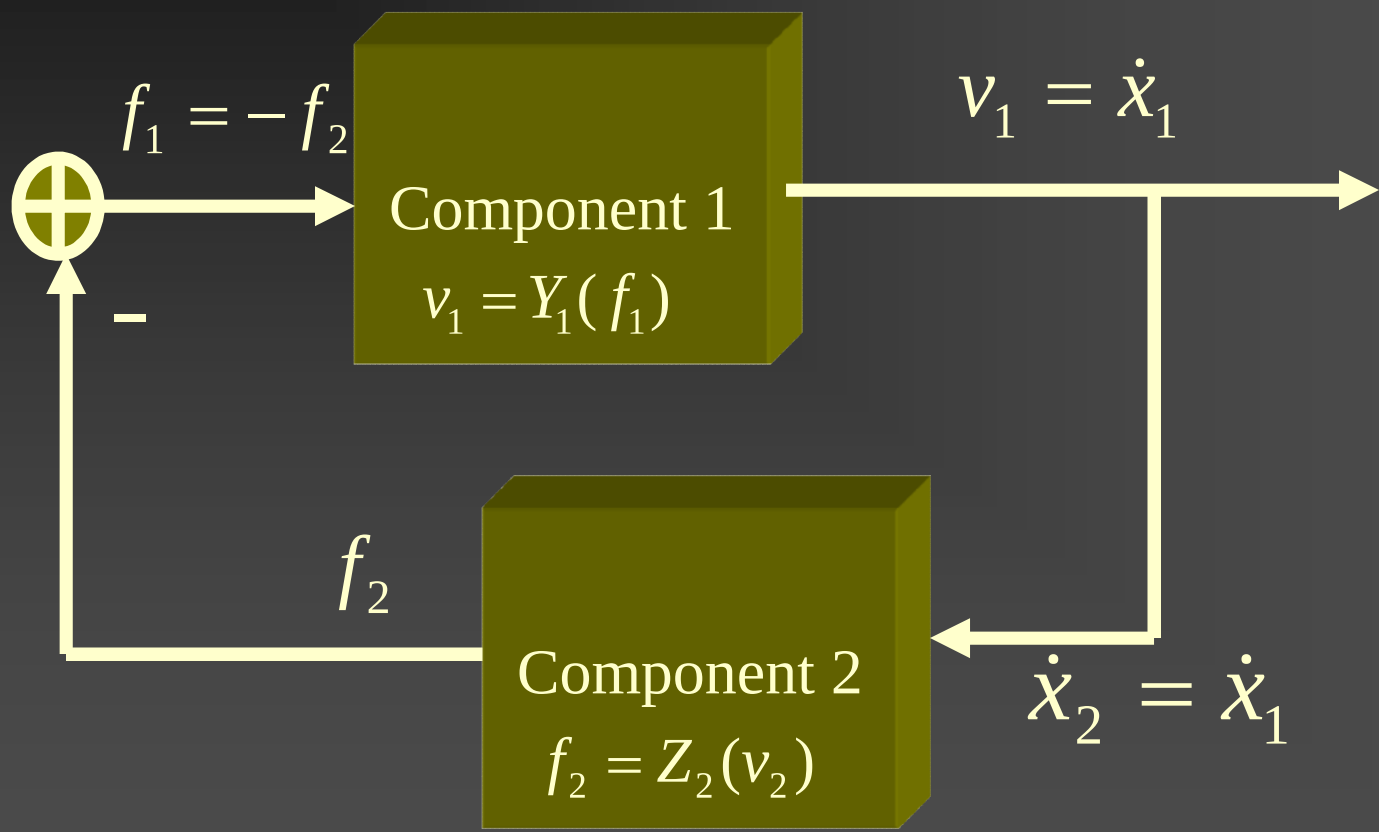
$Z \equiv$ Impedance Operator : $F = Z(V)$

$Y \equiv$ Admittance Operator : $V = Y(F)$

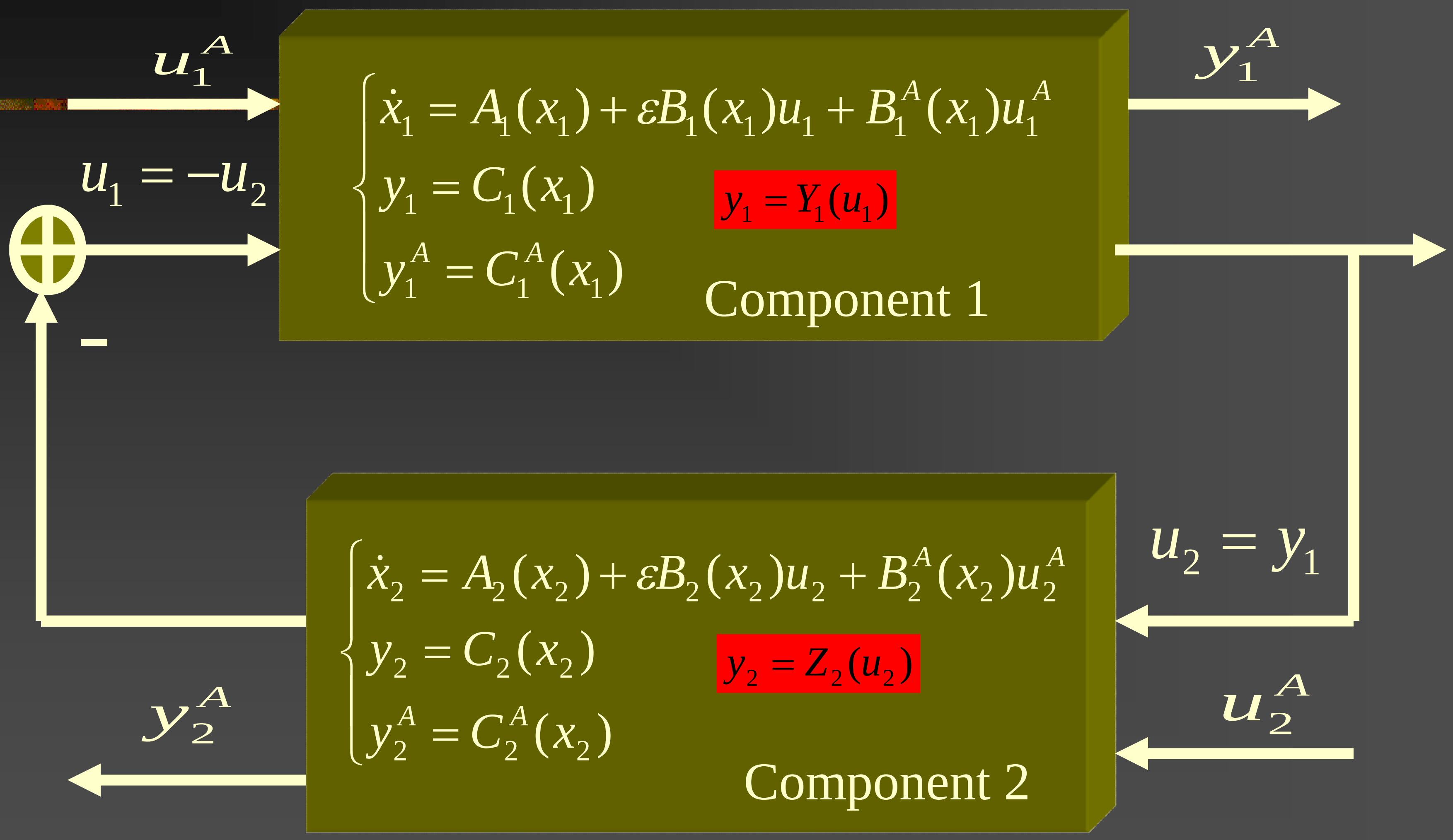
$Z \equiv Y^{-1}, \; Y \equiv Z^{-1}$

Linear Case: C. M. Harris and A. G. Piersol, Shock and Vibration Handbook, McGraw-Hill, NY , 2001

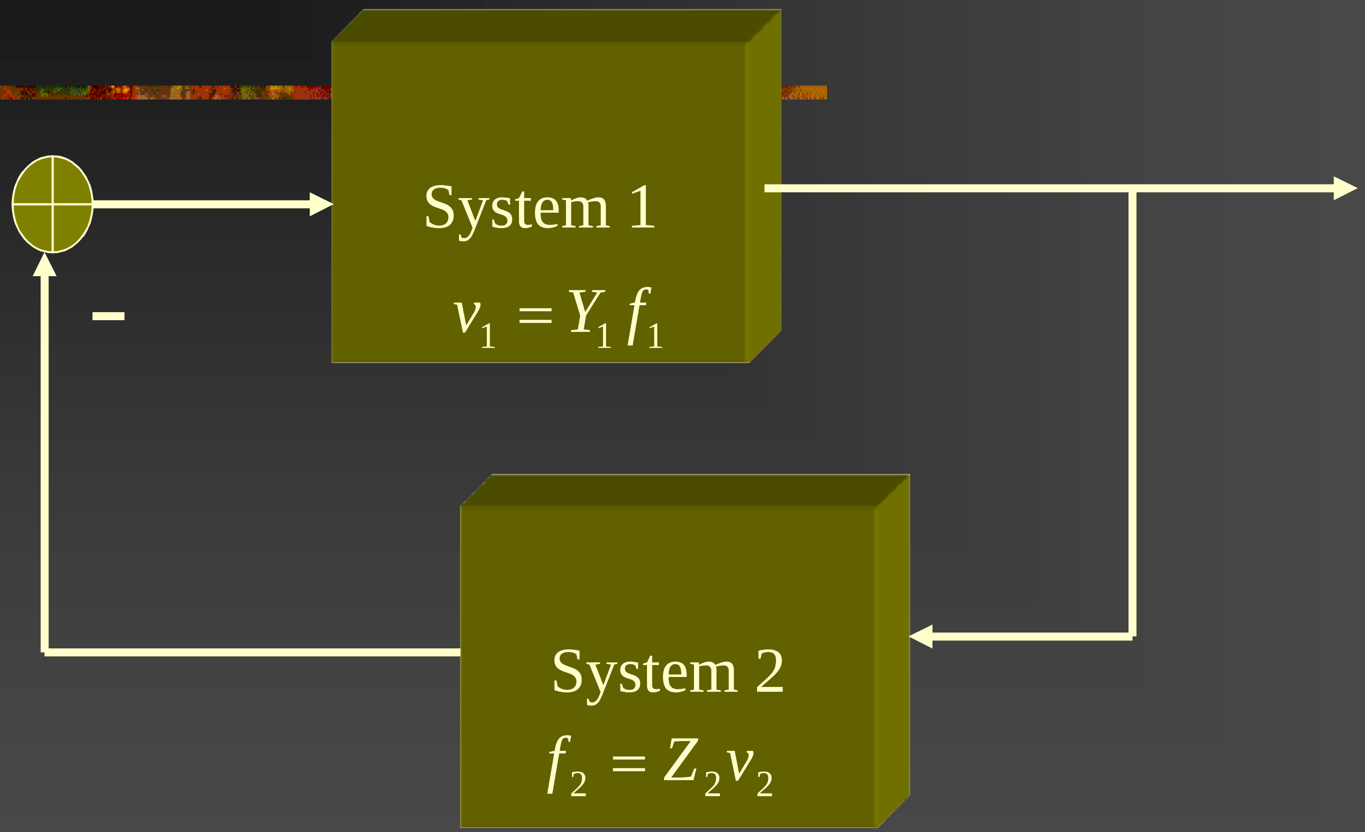
# Fully Evolved System



# Nonlinear State Space Version: Admittance-Impedance



# Characterize LINEAR Evolved System Stability




Q: Given System 1, can we find all System 2 so that connected system is stable ?

Use Youla Parametrization for nonproper Impedances/Admittances.

# Youla or Q Parameterization

**Component 1**

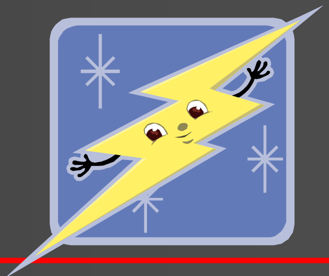
$$Y_1 u = ND^{-1}u$$

where $N, D$ are coprime and stable

**Component 2**

$$Z_2 = (Y - QN)^{-1}(X + QD)$$

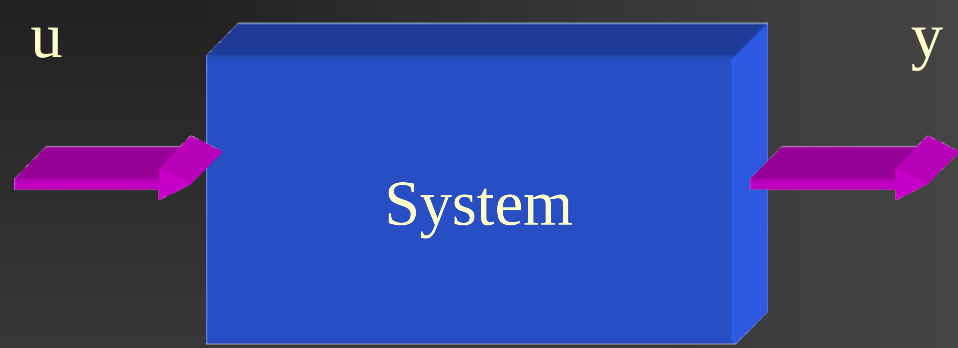
Let $X, Y$ be coprime and stable, and $NX + DY = I$, and $Q$ be any stable rational function.



All controllers that make the closed-loop stable.

M. Vidyasagar,
Control System Synthesis:
A Factorization Approach,
MIT Press, 1985

# Dissipativity: “Higher” Form of Stability



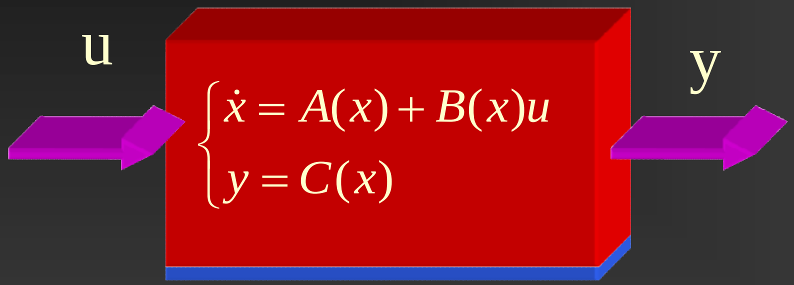

Energy Stored $\leq$ Energy Supplied

e.g. Springs and Masses

Inductors and Capacitors

Roommates/Spouses

# Definition of Dissipativity




Energy Storage Function : $\begin{cases} V(x) > 0; \forall x \neq 0; \\ V(0) = 0 \end{cases}$

$$\Rightarrow \underbrace{dV(x(t))/dt}_{\text{energy storage rate}} \equiv \nabla V \dot{x}(t) = \nabla V[A(x) + B(x)u]$$

System is Dissipative when

$$\begin{cases} L_{A(x)}V \equiv \nabla VA(x) \leq 0 \\ L_{B(x)}V \equiv \nabla VB(x) = C^T(x) \end{cases}$$

$$\leq 0 + C^T(x)u = y^T(t)u(t) \equiv \langle y, u \rangle$$

External Power

or

$$\underbrace{V(x(t))}_{EnergyStored} \leq \underbrace{V(x(0))}_{\substack{Initial \\ EnergyStored}} + \int_0^t \langle y, u \rangle d\tau$$

External Energy

# Strict Dissipativity

$$\underline{\text{Energy Storage Function}} : \begin{cases} V(x) > 0; \forall x \neq 0; \\ V(0) = 0 \end{cases}$$

System is $\underline{\text{Strictly Dissipative}}$ when

$$\begin{cases} L_{A(x)}V \equiv \nabla V A(x) \leq -S(x) \\ L_{B(x)}V \equiv \nabla V B(x) = C^T(x) \end{cases}$$

$$\Rightarrow \underbrace{dV(x(t))/dt}_{\text{energy storage rate}} \equiv \nabla V \dot{x}(t) = \nabla V[A(x) + B(x)u]$$

$$\leq -S(x) + C^T(x)u = \langle y, u \rangle - S(x)$$

Internal Power Dissipated

# Positive Realness (PR)

Given $(A, B, C)$

and $T(s) \equiv C(sI - A)^{-1}B + D$ :

There is P > 0 so that

$$\text{K - Y} \begin{cases} A^T P + PA = -L^T L \equiv Q \leq 0 \\ PB = C^T \end{cases}$$

Linear & Dissipative

A Frequency Domain Condition

$$\operatorname{Re} T(j\omega) \equiv (T(j\omega) + T^*(j\omega))/2 \geq 0 \text{ for all } \omega$$

This is often taken as the definition of Positive Real (*PR*)

For Linear Systems: PR= Dissipative= Passive

# What does Strictly Positive Real (SPR) mean?

Answer: Lotsa Things, Not all equivalent!! (See J.Wen,AC-33,1988)

Here is ONE Definition of SPR:

There is $\mu > 0$ so that

$$\mathrm{Re}\, T(j\omega - \mu) \equiv (T(j\omega - \mu) + T^*(j\omega - \mu))/2 \text{ is PR}$$

Relation to K-Y:

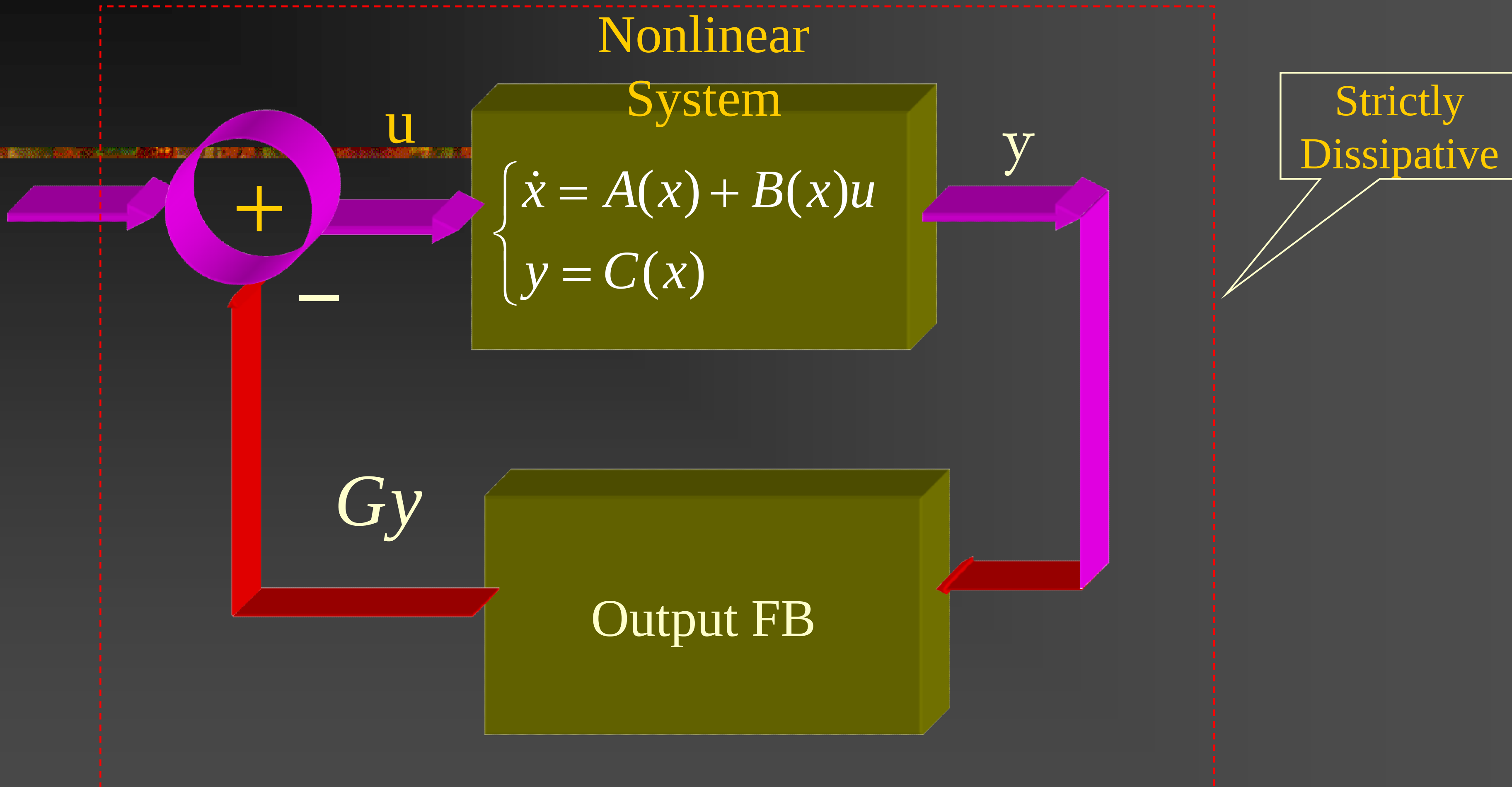
Just Change $A$ to $A + \mu I$ in K-Y

$$\begin{cases} A^T P + PA = -L^T L - 2\mu P \equiv -Q < 0 \\ PB = C^T \end{cases}$$

⟺ Linear & Strictly Dissipative

For Linear Systems: SPR=Strictly Dissipative=Strictly Passive

# Almost Strictly Dissipative (ASD)




$$\therefore (A_C(x) \equiv A(x) + B(x)GC(x), B(x), C(x)) \text{ is ASD}$$

For LTI Systems: ASD=ASPR

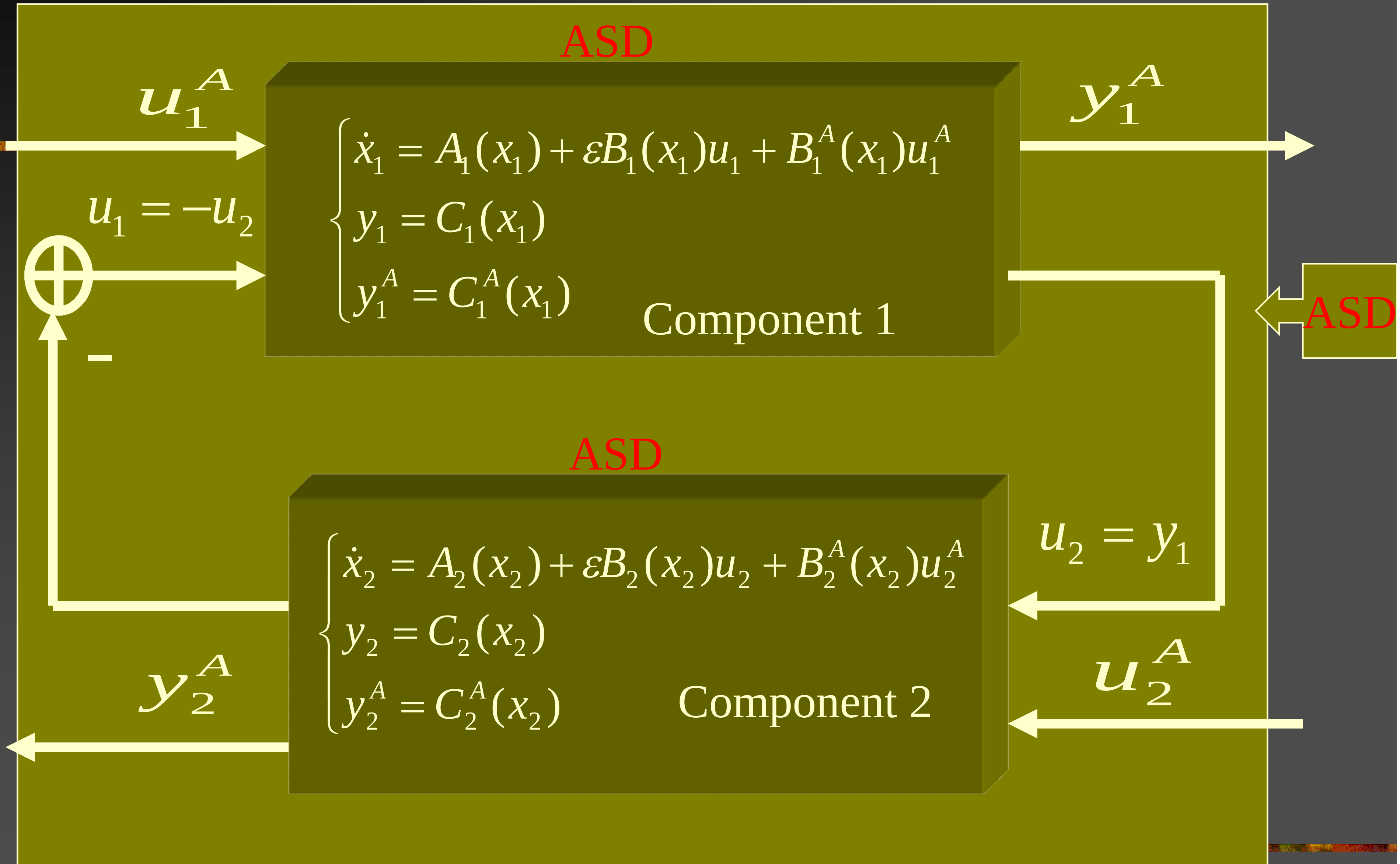
# Linear ASPR via Non-Orthogonal Projections

Balas&Fuentes :

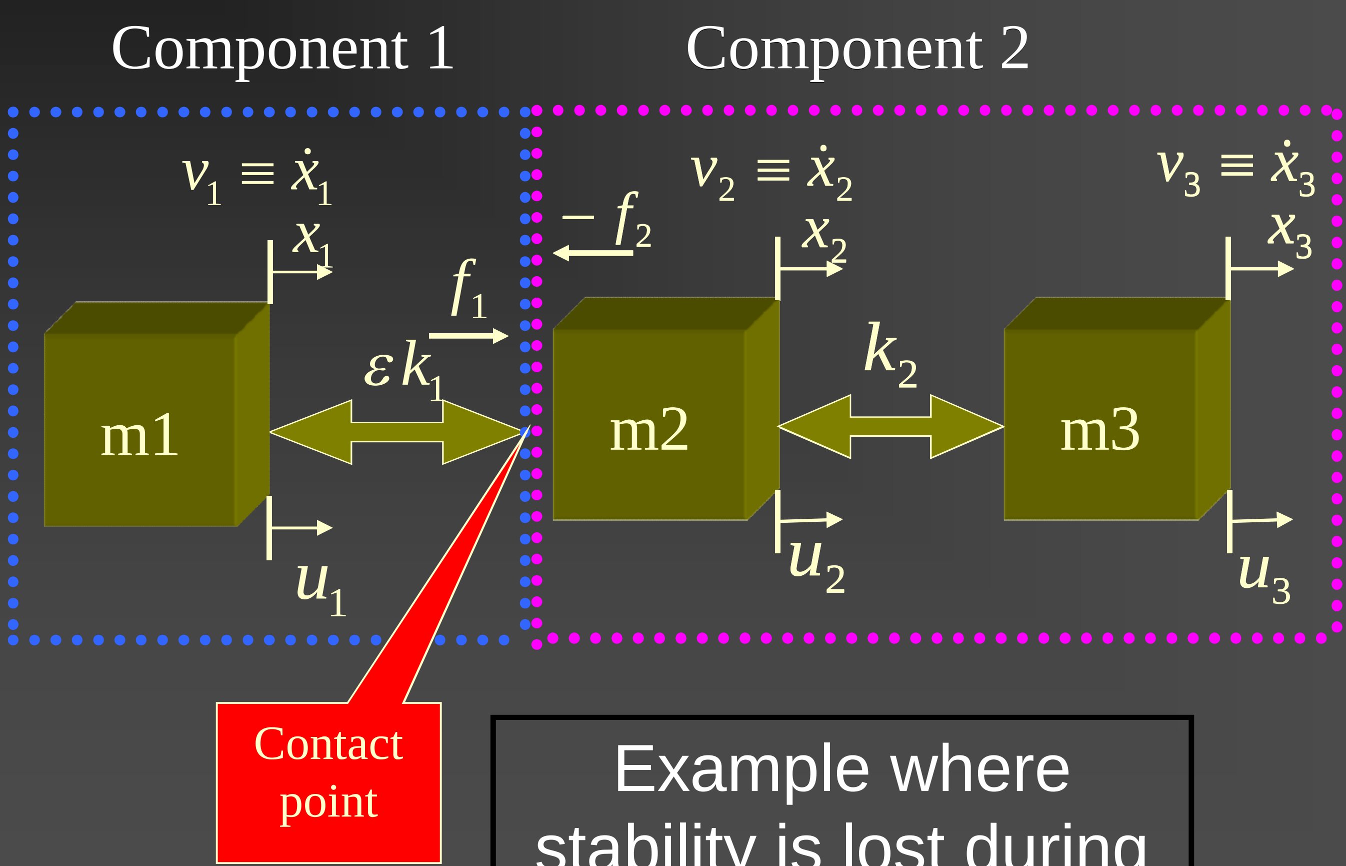
1) (A,B,C) Almost SPR if and only if CB positive definite and open-loop transfer function $P(s) \equiv C(sI - A)^{-1}B$ is minimum phase ( i.e. all transmission zeros stable)

2) Almost PR if and only if
CB positive definite,
open-loop transfer function is weakly minimum phase
(i.e. can have some marginally stable transmission zeros),
and
marginally stable zero dynamics are PR

# Inheritance of Almost Strictly Dissipative Property

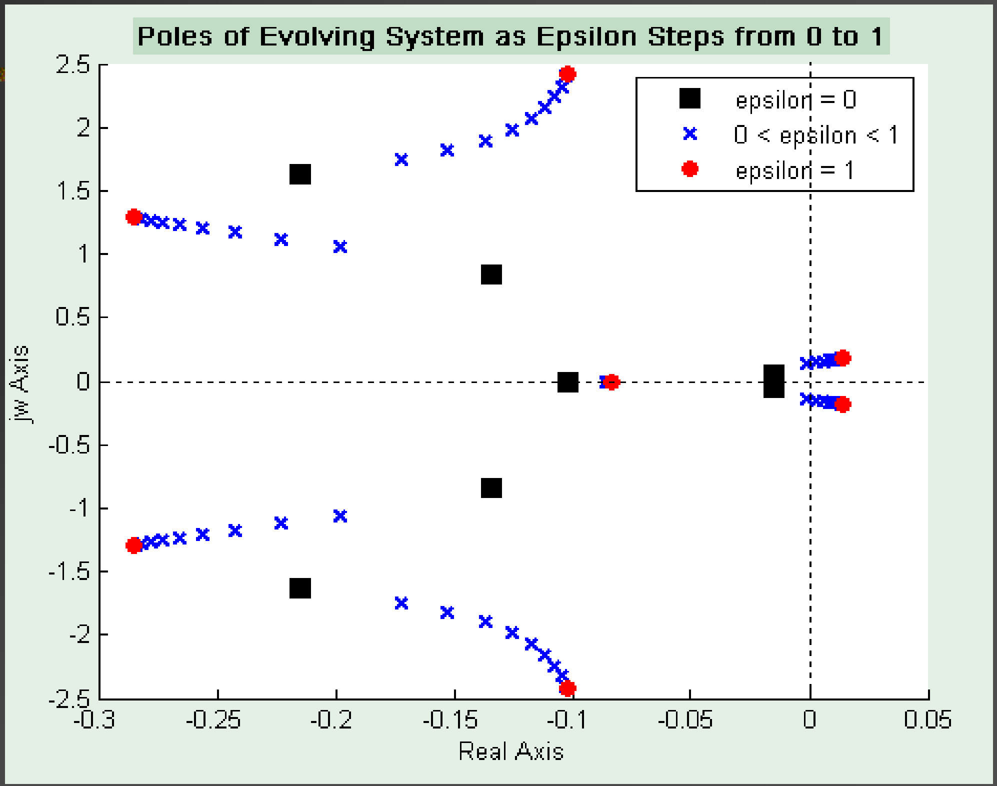


# Two Component Flexible Structure Evolving System




Example where stability is lost during evolution will follow!

# System That Does Not Inherit Stability




## Component 1:

$$\begin{cases} m_1\ddot{q}_1 = u_1 - \varepsilon k_{12}(q_1 - q_2) \\ y_1 = [q_1, \dot{q}_1]^{\mathrm{T}} \end{cases}$$

## Component 2:

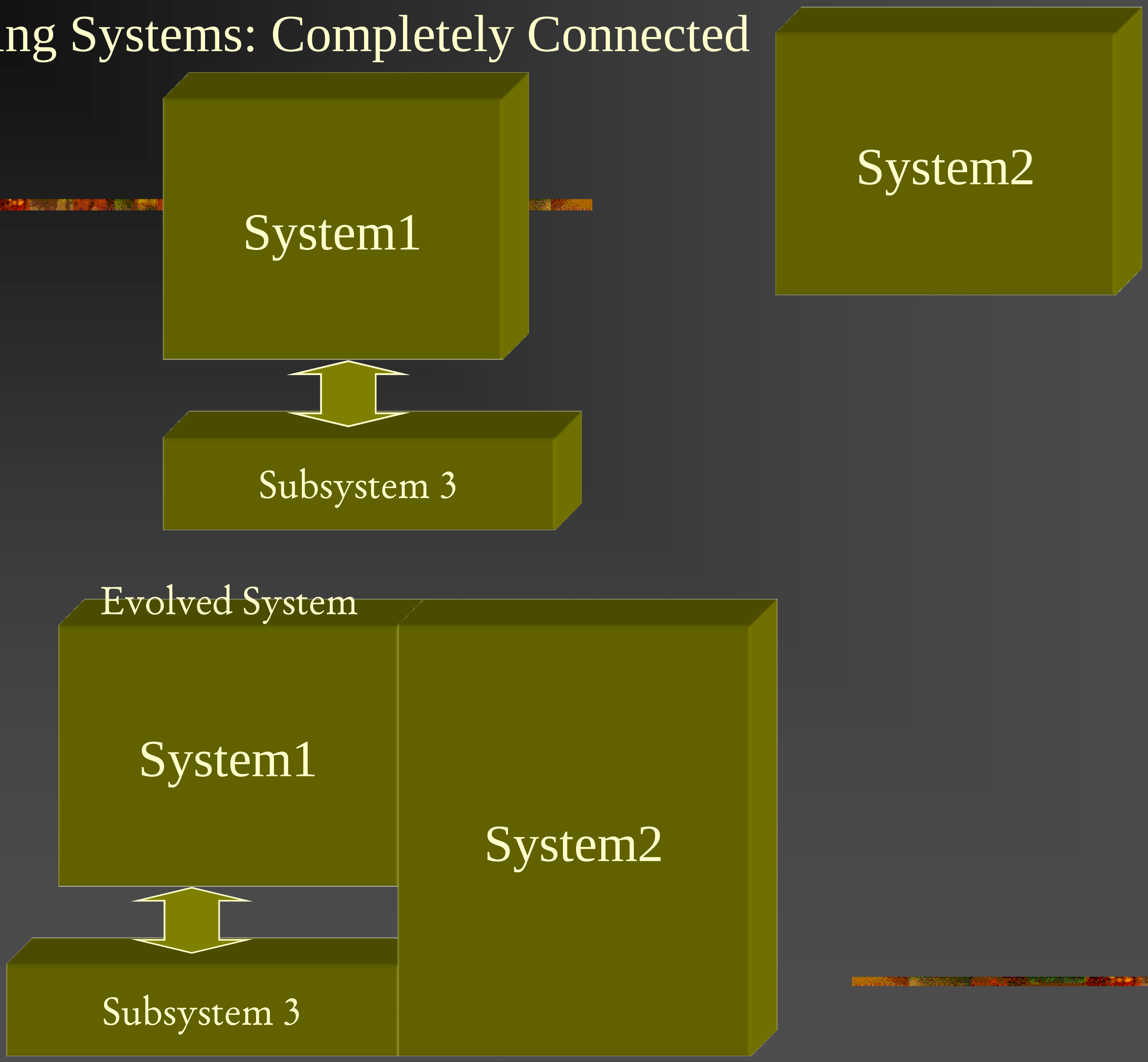
$$\begin{cases} m_2\ddot{q}_2 = u_2 - k_{23}(q_2 - q_3) - \varepsilon k_{12}(q_2 - q_1) \\ m_3\ddot{q}_3 = u_3 - k_{23}(q_3 - q_2) \\ y_2 = [q_2, \dot{q}_2]^{\mathrm{T}} \\ y_3 = [q_3, \dot{q}_3]^{\mathrm{T}} \end{cases}$$

## Component Controllers:

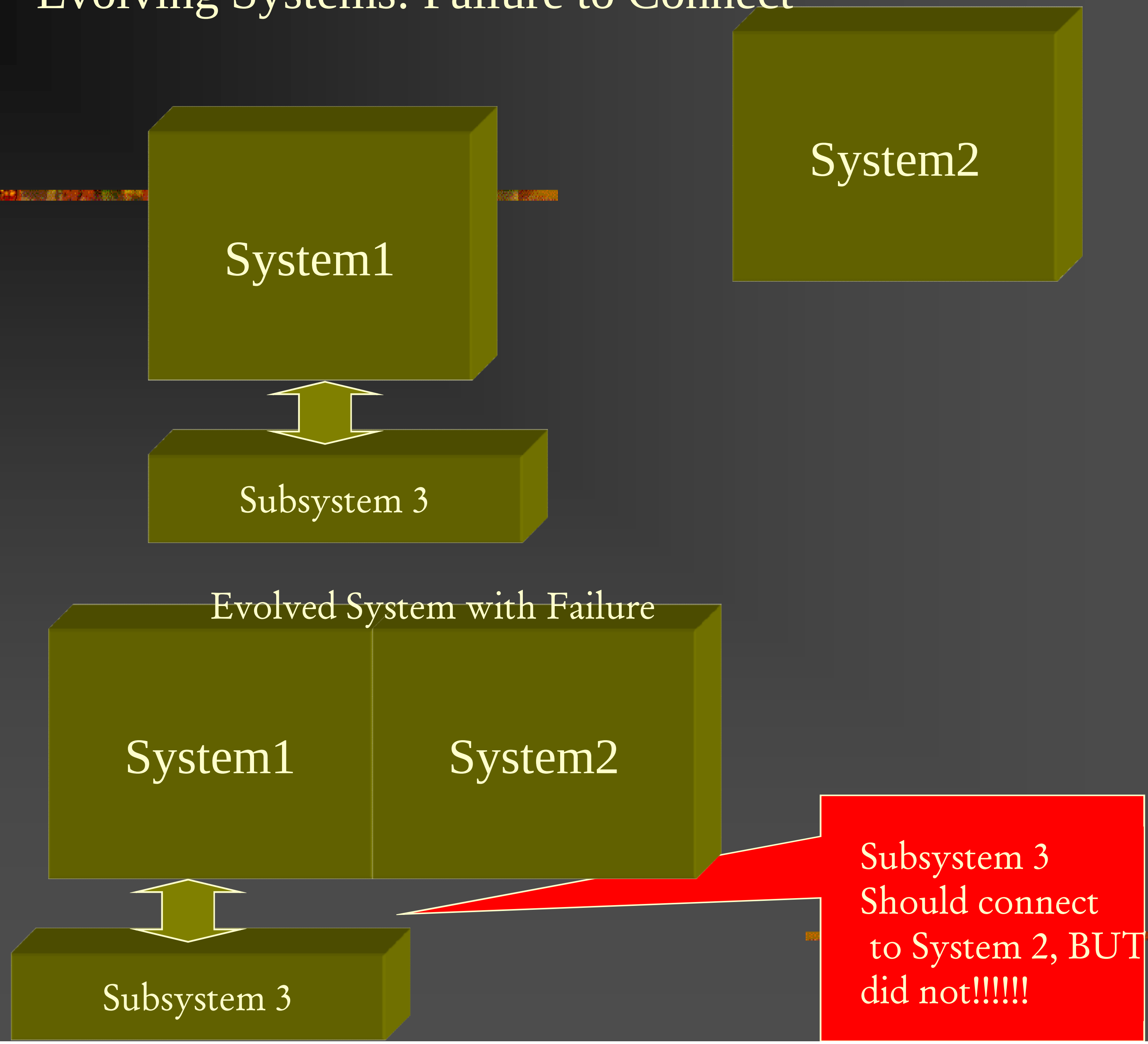
$$\begin{cases} u_1 = -(0.9s + 0.1)q_1 \\ u_2 = -\left(\frac{0.1}{s} + 0.2s + 0.5\right)q_2 \\ u_3 = -(0.6s + 1)q_3 \end{cases}$$

with $0 \le \varepsilon \le 1,\ m_1 = 30,\ m_2 = 1,\ m_3 = 1,\ k_{12} = 4$ and $k_{23} = 1$

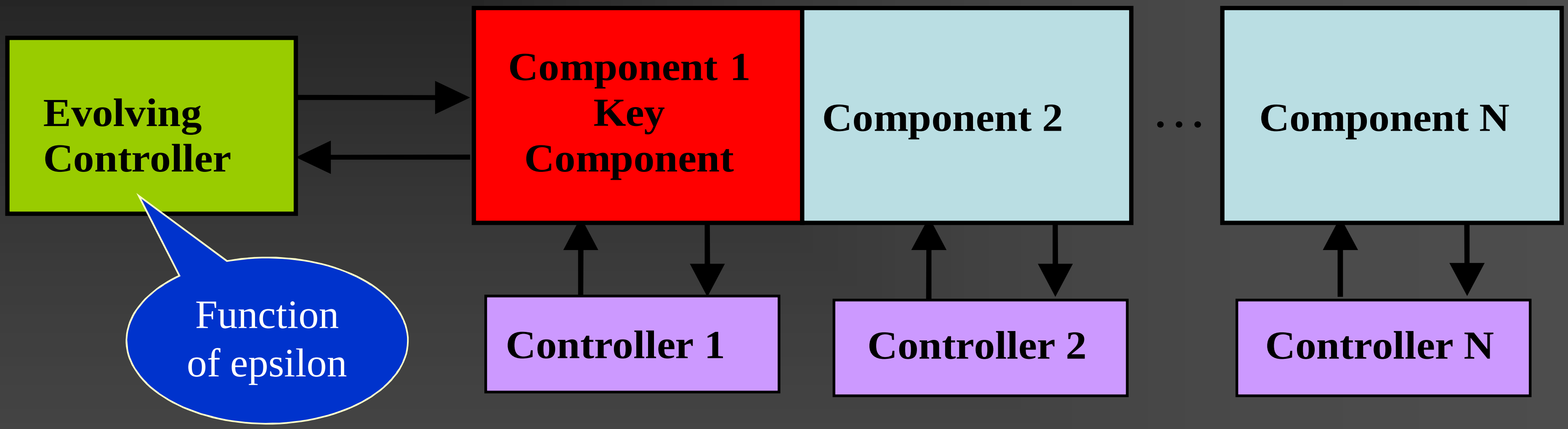
## Partial or Failed Evolutions:
Evolving Systems: Completely Connected



# Evolving Systems: Failure to Connect



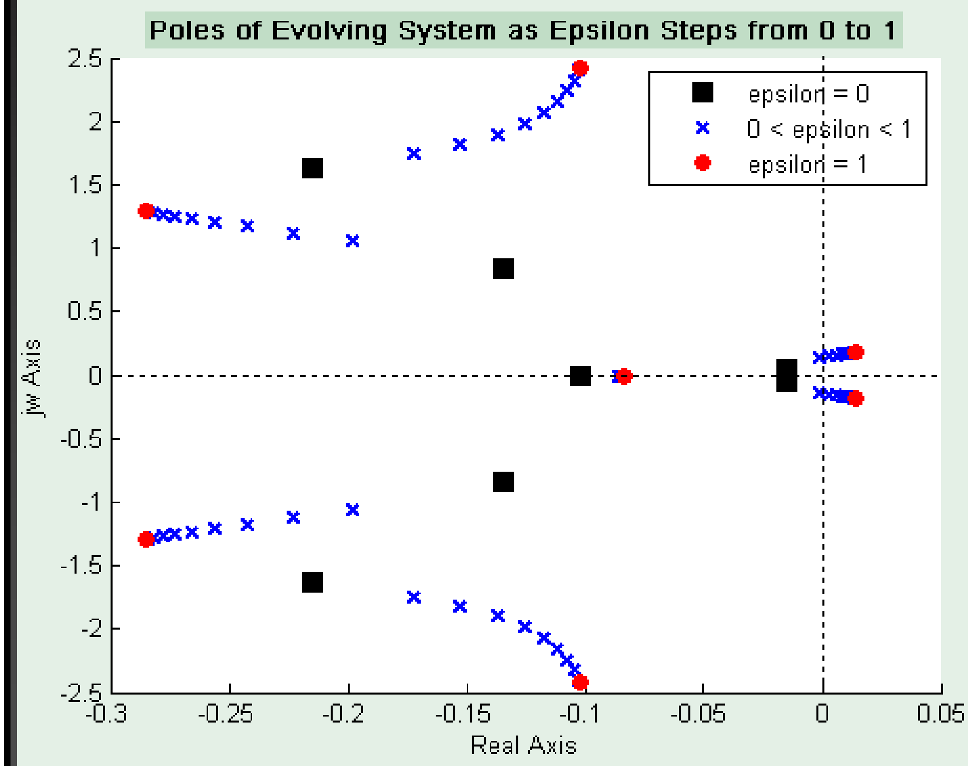
# Key Component Evolving Controller



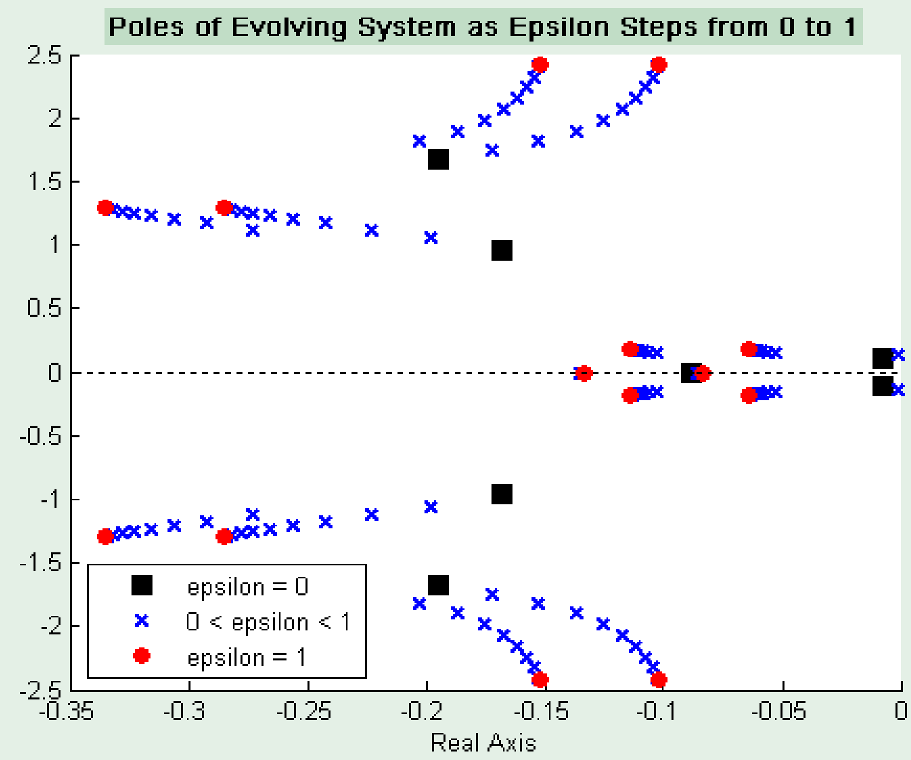

Local Controllers 1 … N and their components' input-output ports remain unchanged

# Closed Loop Poles of Example

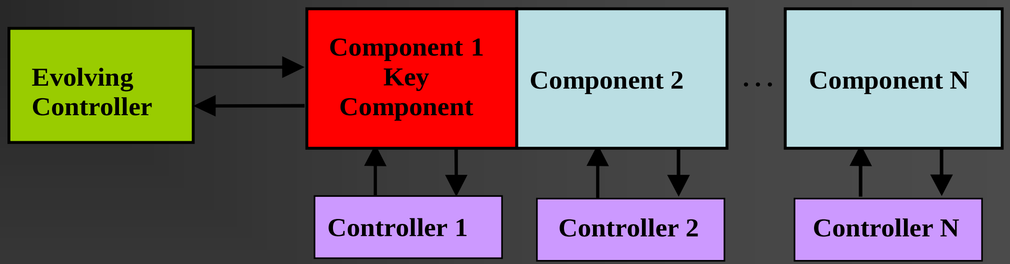
No Evolving Controllers




System with Key Component Evolving Controller

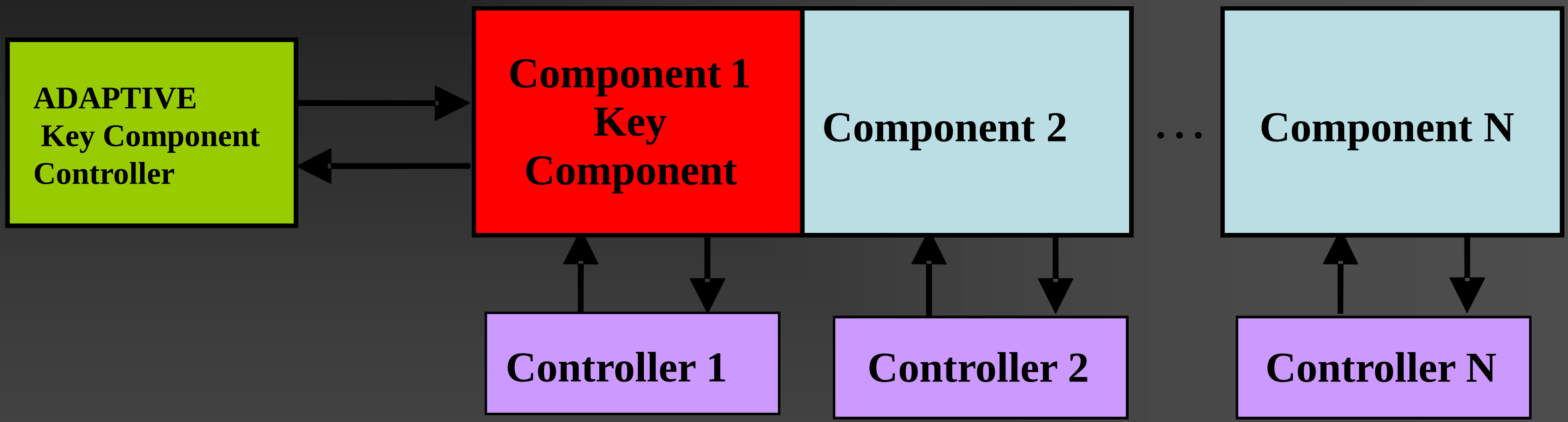


# Benefits of Key Component Evolving Controllers

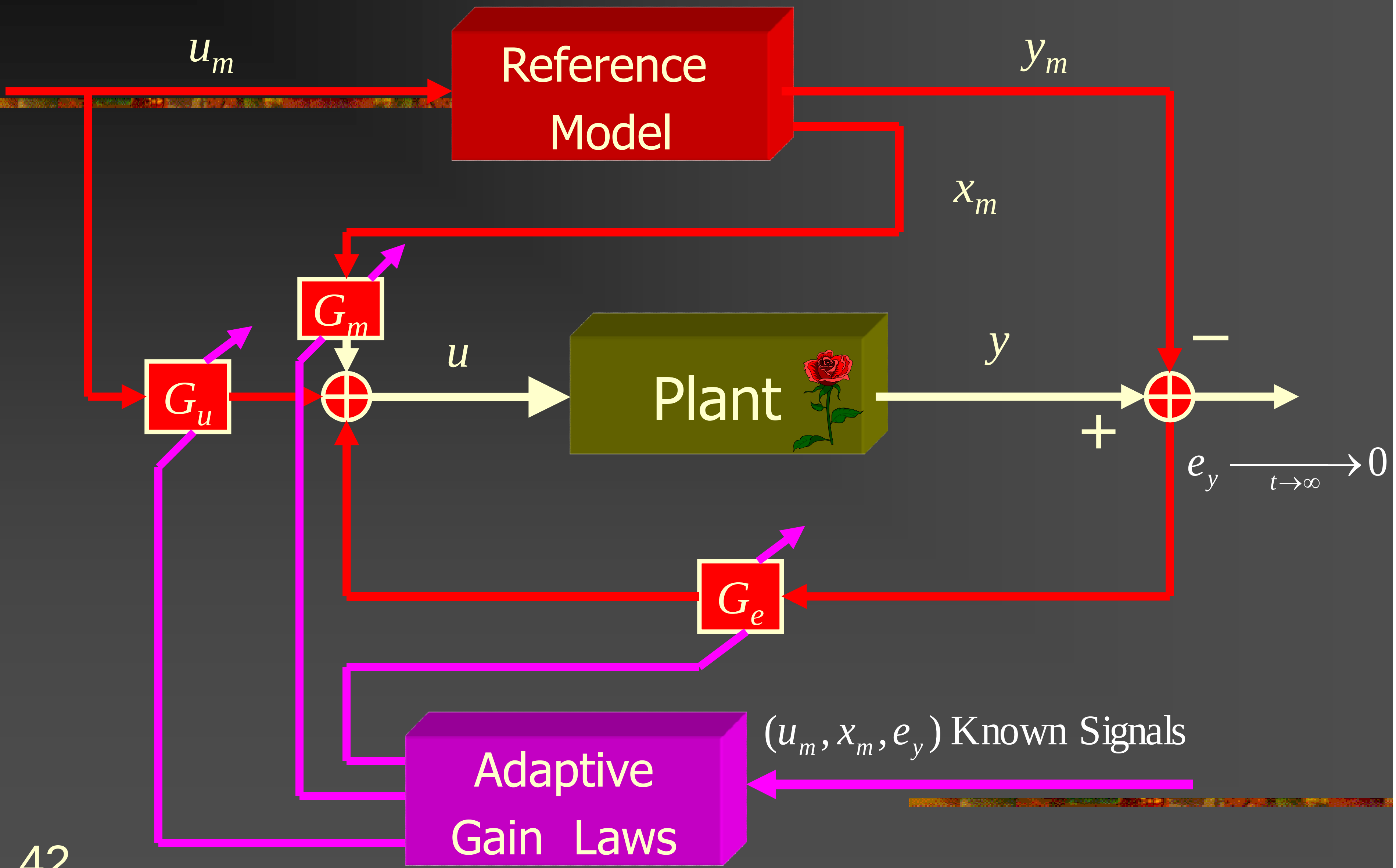



- Maintains stability during Evolution
- Interchangeability of non-key components
- Cost savings & risk mitigation
- No clear design methodology

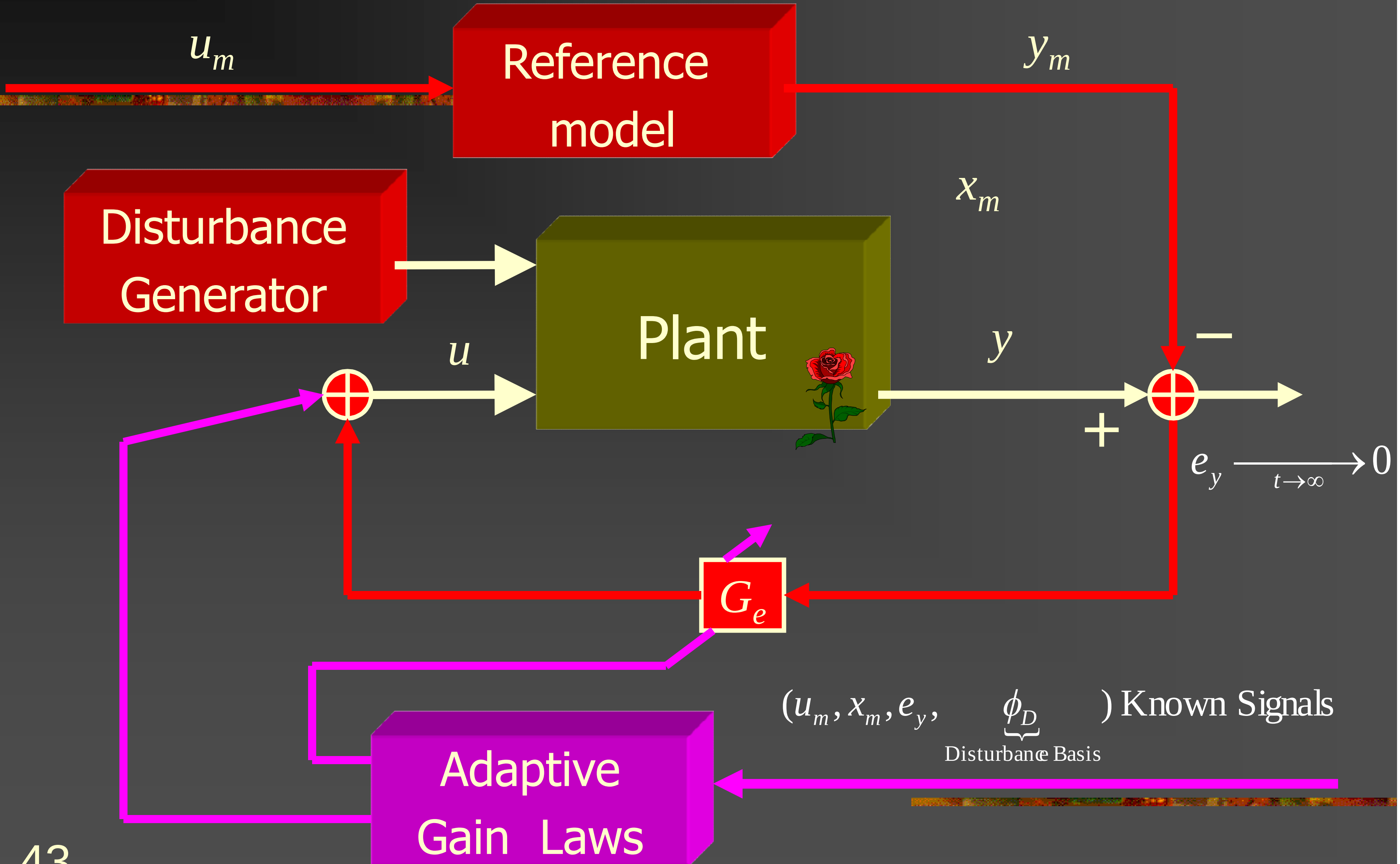
# ADAPTIVE Key Component Evolving Controller



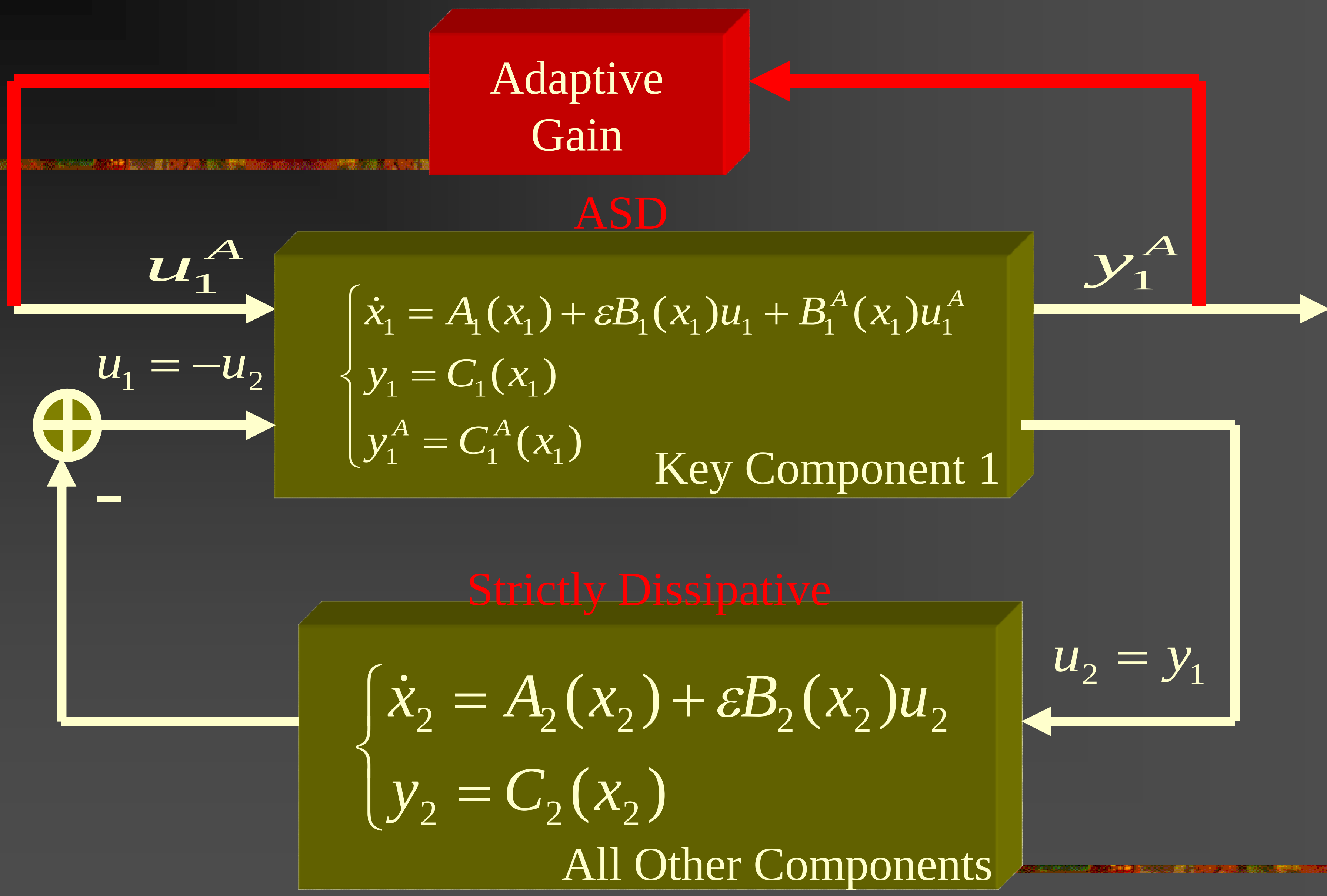
# Direct Adaptive Model Following Control (Wen-Balas 1989)

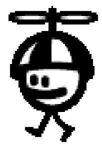


# Direct Adaptive Persistent Disturbance Rejection (Fuentes-Balas 2000)



# Adaptive Key Component Control





# A Theorem **Is** Worth A Thousand Simulations

(despite its limitations)

# Adaptive Key Component Controller Theorem

If Key Component 1 $(u_1^A, y_1^A)$ is Almost Strictly Dissipative and Component 2 $(u_2, y_2)$ is Strictly Dissipative, then

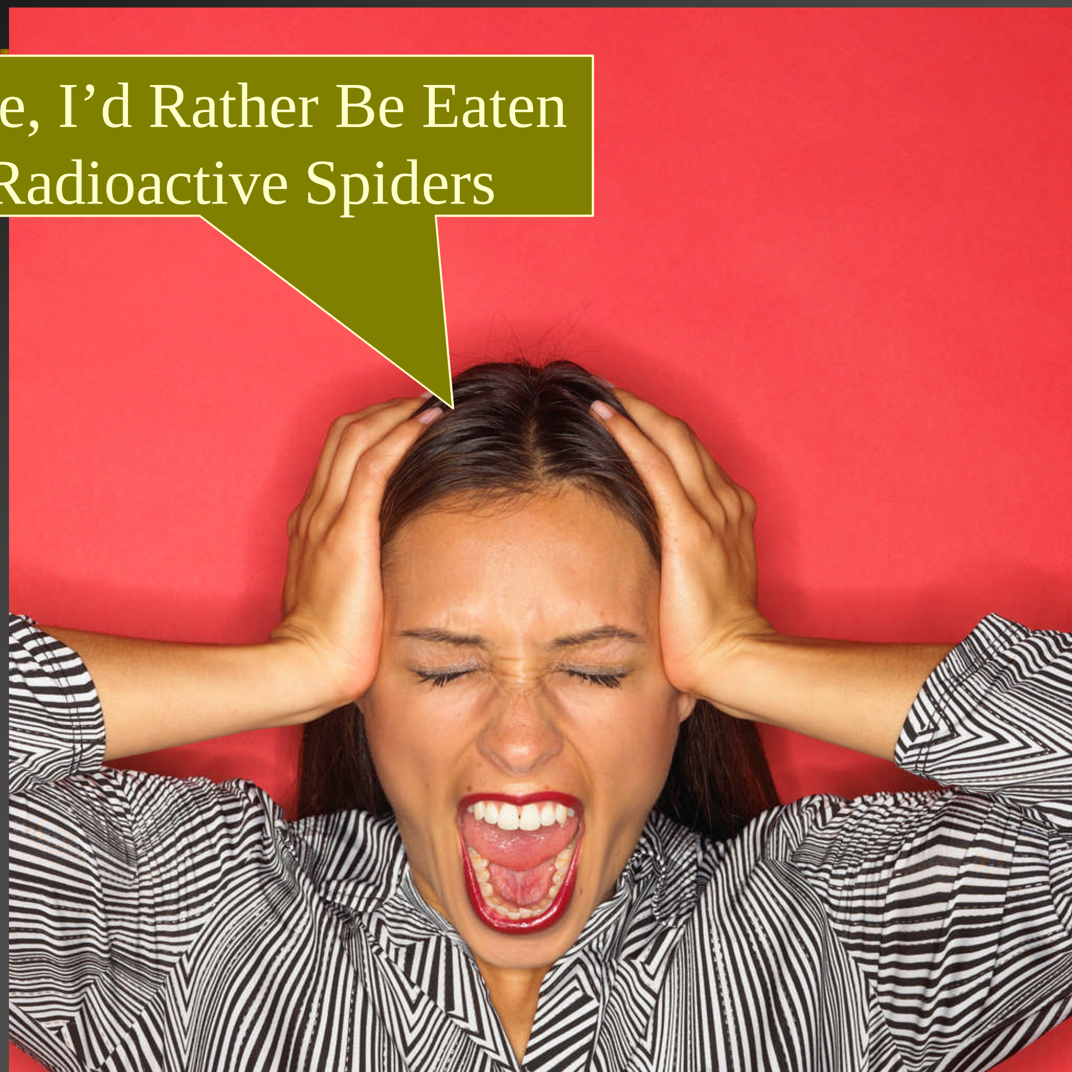
Adaptive Key Component Controller

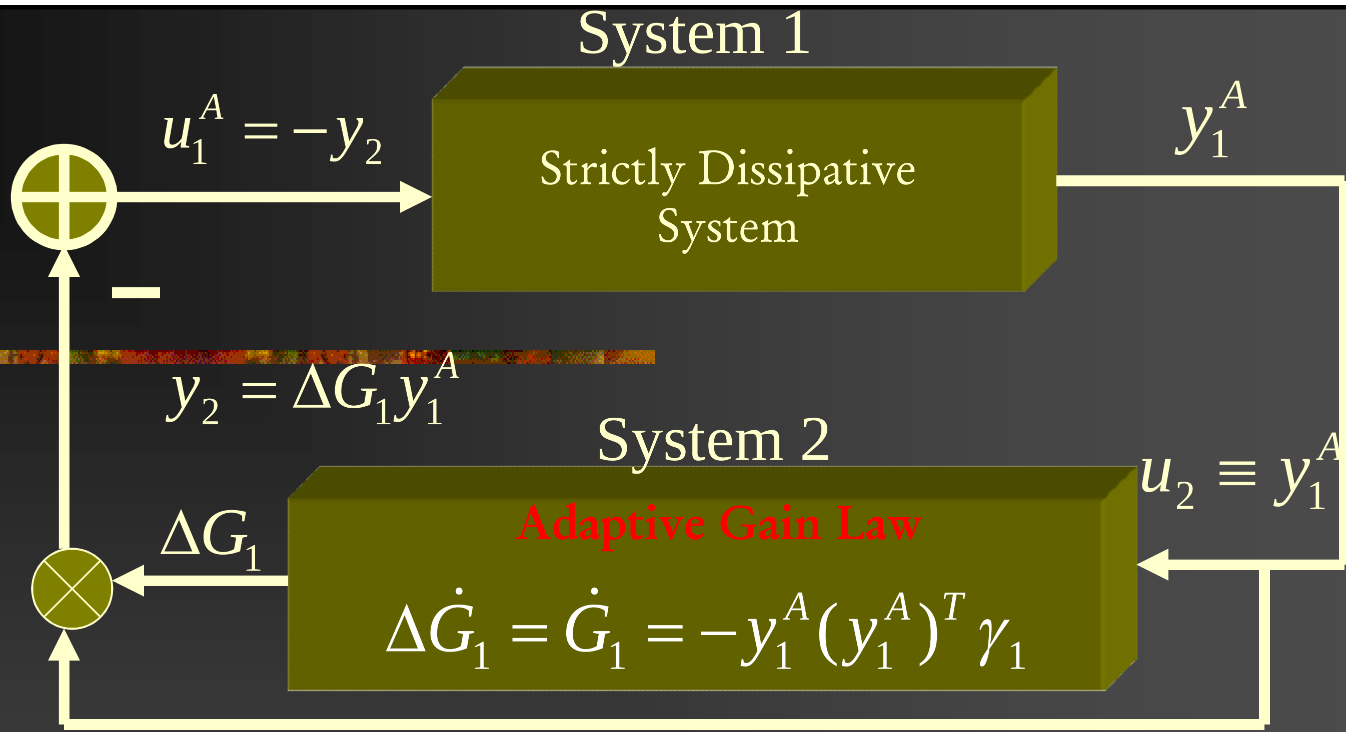
$$\begin{cases} u_1^A = G_1 y_1^A \\ \dot{G}_1 = -y_1^A (y_1^A)^T \gamma_1; \gamma_1 > 0 \end{cases}$$

Produces $x_1, x_2 \xrightarrow[t \to \infty]{} 0$ and $G_1$ is bounded

throughout the entire evolution $0 \le \varepsilon \le 1$

# Now Let's See Some Detailed Mathematical Proofs



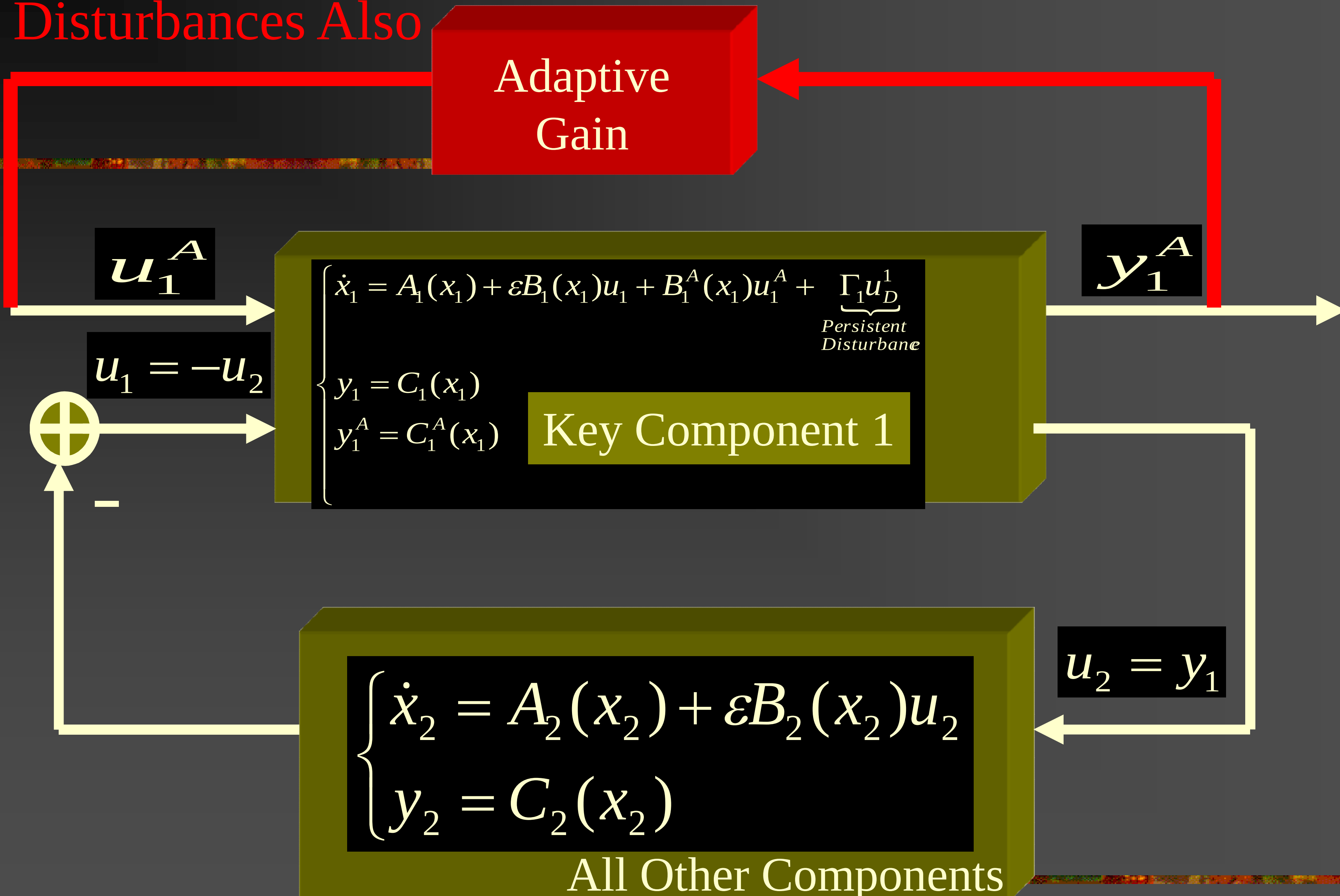



Let $\quad V_2 \equiv \frac{1}{2} trace\, \Delta G \cdot \Delta G^T > 0$

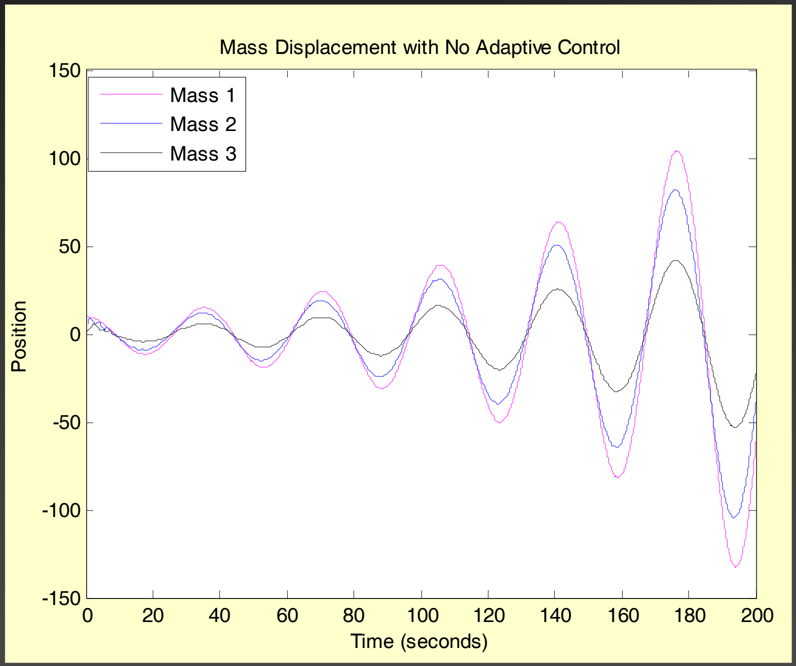
$$\Rightarrow \dot{V}_2 = trace \Delta \dot{G}_1 \gamma_1^{-1} \Delta G_1^T = trace(y_1^A (y_1^A)^T \gamma_1 \gamma_1^{-1} \Delta G^T)$$

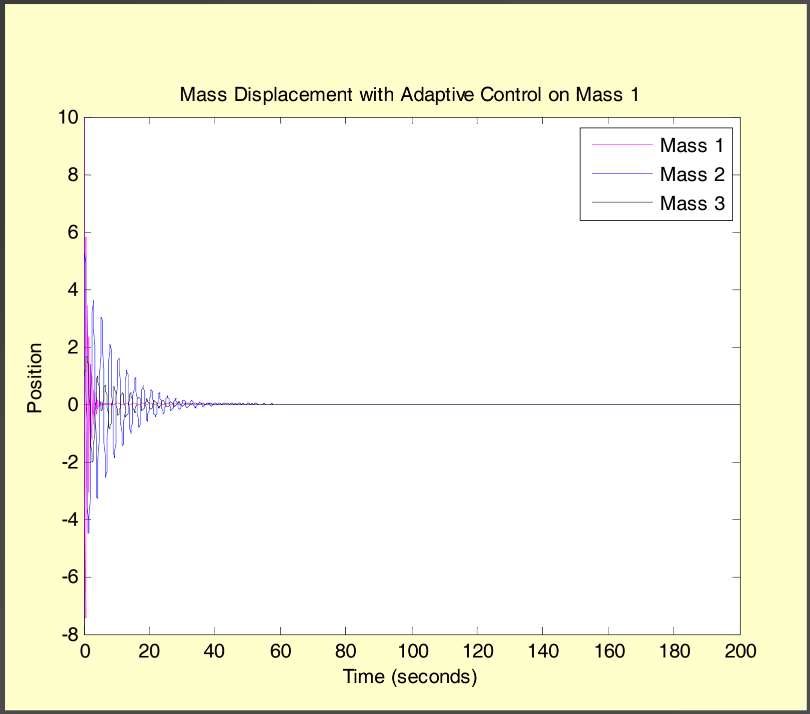
$$= (\Delta G y_1^A)^T y_1^A = \langle y_2, u_2 \rangle \qquad \therefore \text{System 2 is dissipative}$$

# Adaptive Key Component Control Can Mitigate Persistent Disturbances Also

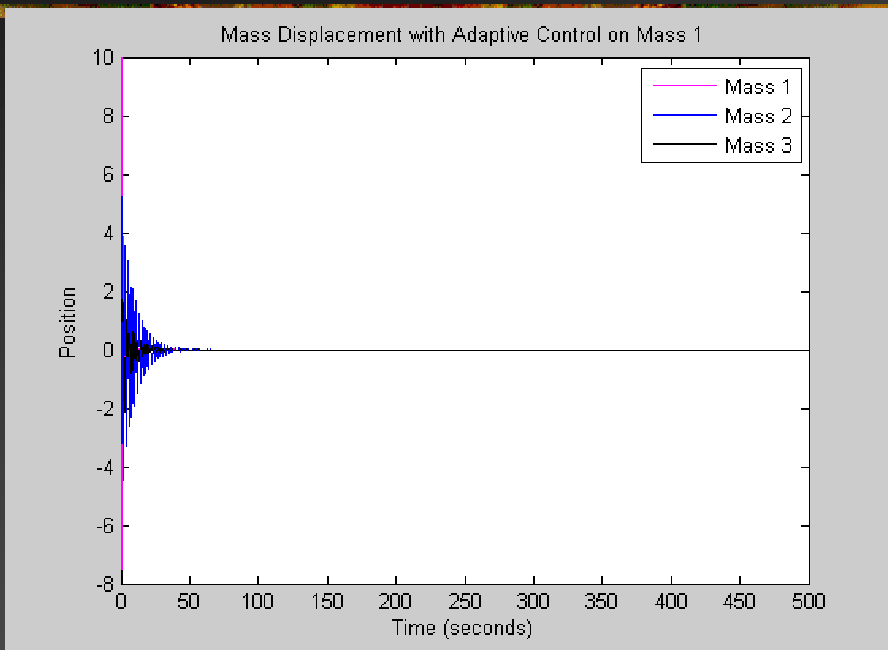


# Adaptive Key Component Control of Nonlinear Plant

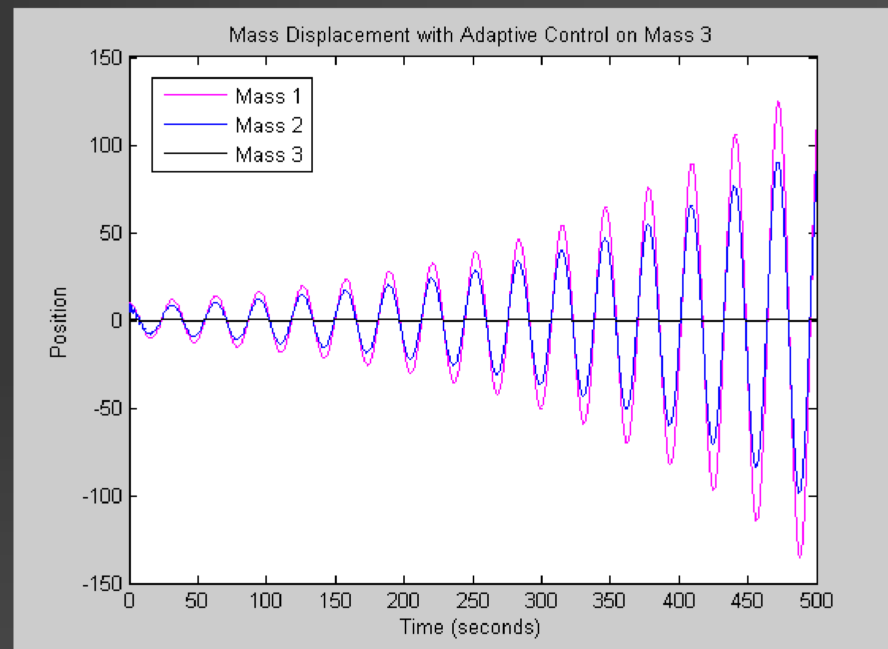


# Adaptive Key Component Controller



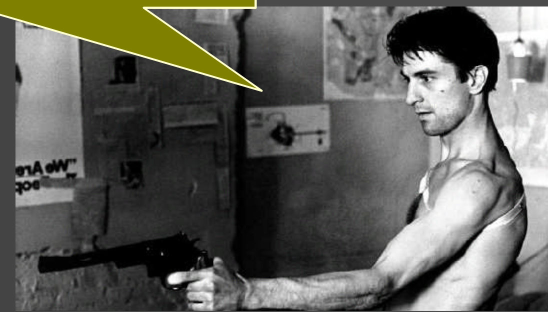

Component 1 is Key Component w/ I/O ports on mass 1

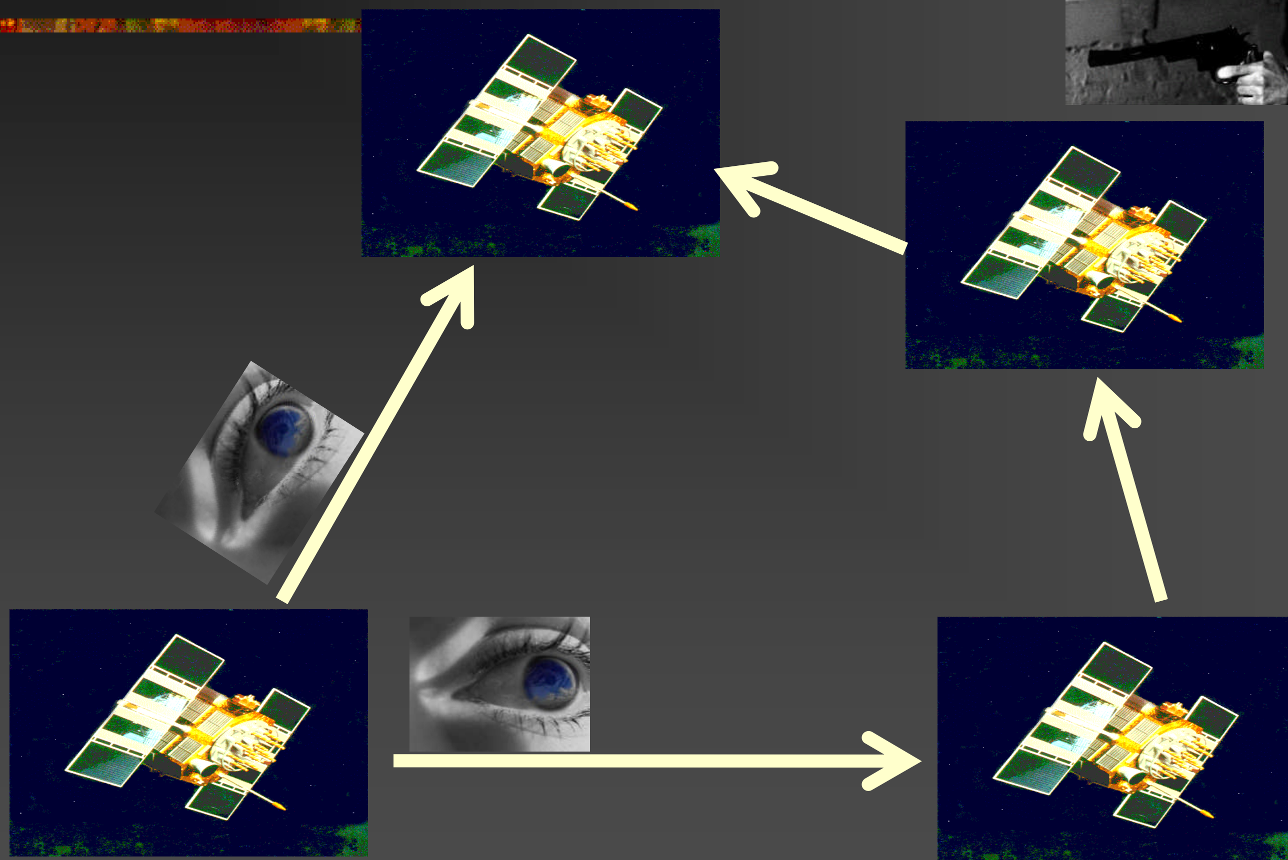



Component 2 is Key Component w/ I/O ports on mass 2

Nonminimum phase zeros:
0.0051466 + 0.20089i,
0.0051466 - 0.20089i

# Evolving Spacecraft Formations



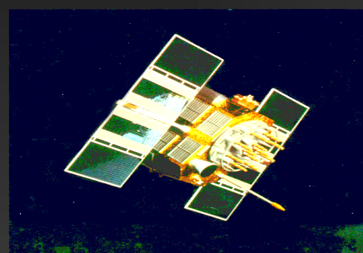



Joining Spacecraft

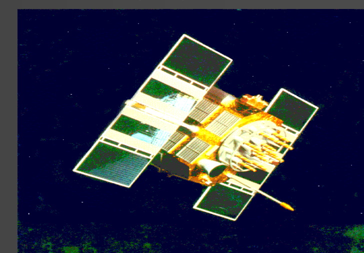
# Control Based on Relative State

$u_2 = G\xi_1$

SC2



SC1



$$\xi_1 \equiv x_2 - x_1 - r_1 \xrightarrow[t\to\infty]{} 0$$

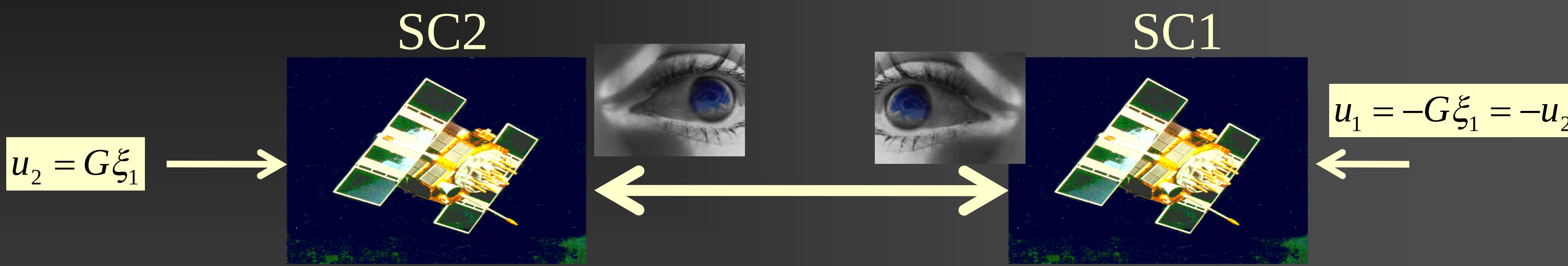
$$u_2 = G\xi_1$$

$$= \begin{bmatrix} g_P & g_D \end{bmatrix} \begin{bmatrix} q_2 - q_1 - r_P^1 \\ \dot{q}_2 - \dot{q}_1 - r_D^1 \end{bmatrix}$$

$$= \underbrace{\mathrm{g_P}(\mathrm{q_2} - \mathrm{q_1} - \mathrm{r_P^1})}_{\text{preloaded spring}} + \underbrace{g_D(\dot{q}_2 - \dot{q}_1 - r_D^1)}_{\text{preloaded damper}}$$

Uni-directional
Newton’s 3rdLaw

# Reciprocity= Usual Newton's Laws




$$\xi_1 \equiv x_2 - x_1 - r_1 \xrightarrow[t\to\infty]{} 0$$

$$u_2 = G\xi_1 = -u_1$$

$$= \underbrace{g_P(q_2 - q_1 - r_P^1)}_{\text{preloaded spring}} + \underbrace{g_D(\dot{q}_2 - \dot{q}_1 - r_D^1)}_{\text{preloaded damper}}$$

# Spacecraft Dynamics

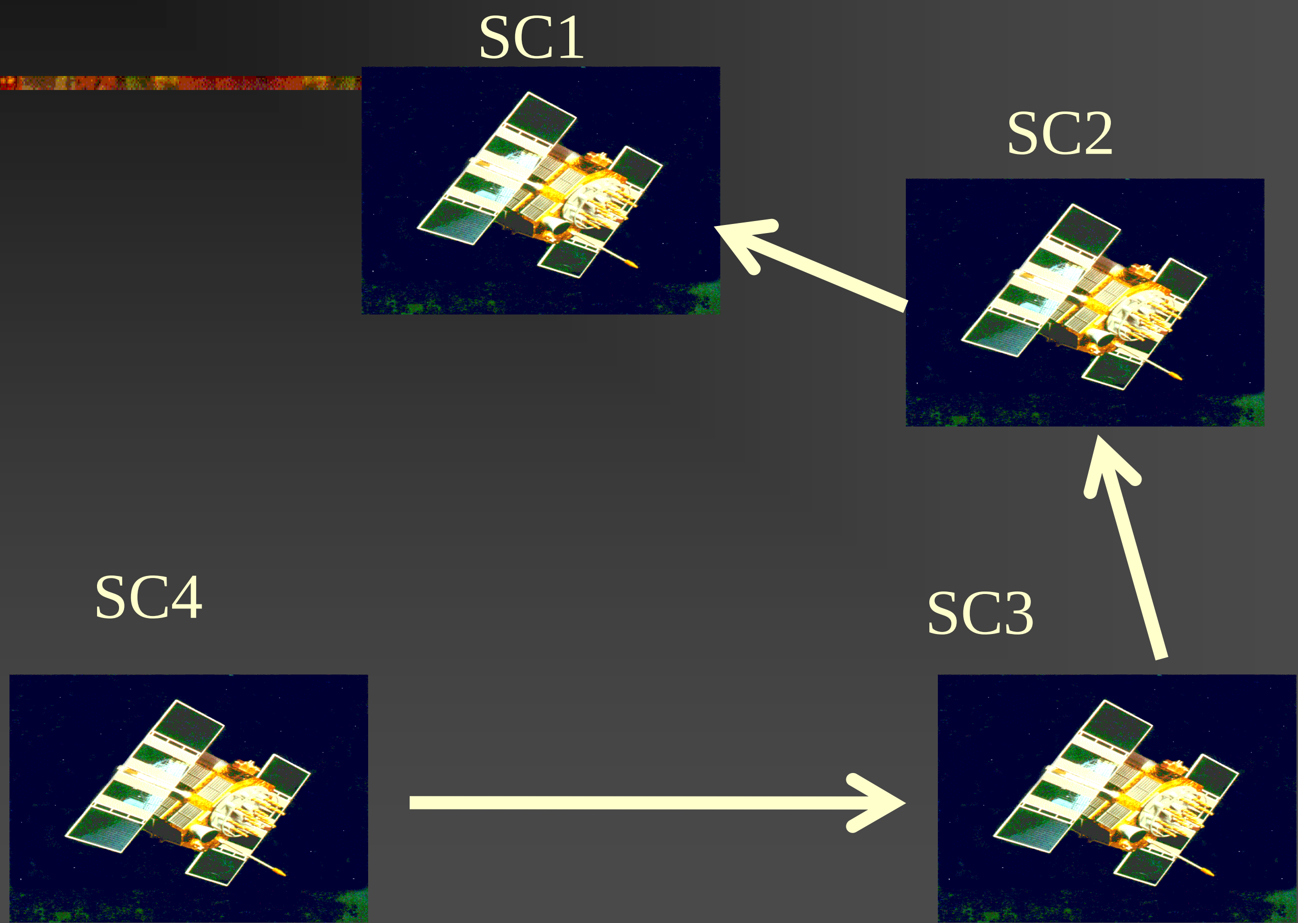
Identical Spacecraft

$$SC_k : \dot{x}_k = Ax_k + Bu_k + \Gamma u_k^D; k = 1, \ldots, N$$

$$\text{Disturbance Generator}: \begin{cases} u_k^D = \theta z_k^D \\ \dot{z}_k^D = F z_k^D \end{cases} \text{or } z_k^D = L_k \phi_D$$

$$\text{Double Integrator } A \equiv \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C \equiv \begin{bmatrix} 1 & 0.1 \end{bmatrix}$$

# Open Chain Formation



# Relative Measurements

$$SC_k : y_k = C\xi_{k-1} \equiv C(x_k - x_{k-1} - r_{k-1})$$

$$\text{Chained Output Feedback}: \begin{cases} u_k = Gy_k + G_D z_k^D = GC\xi_{k-1} + G_D z_k^D \\ u_1 = 0 \end{cases}$$

$$\Rightarrow \Delta u_k \equiv u_{k+1} - u_k = G(y_{k+1} - y_k) = GC(\xi_k - \xi_{k-1}) + G_D \Delta z_k^D$$

$$\therefore \dot{\xi}_k = A\xi_k + B\Delta u_k + \Gamma \Delta u_k^D$$

$$= (A + BGC)\xi_k - BGC\xi_{k-1} + (\underbrace{BG_D + \Gamma\theta}_{=0 \text{ or } R(\Gamma) \subseteq R(B)})\Delta z_k^D; k \geq 2$$

$$\& \ \dot{\xi}_1 = (A + BGC)\xi_1$$

# Open Chain Stability

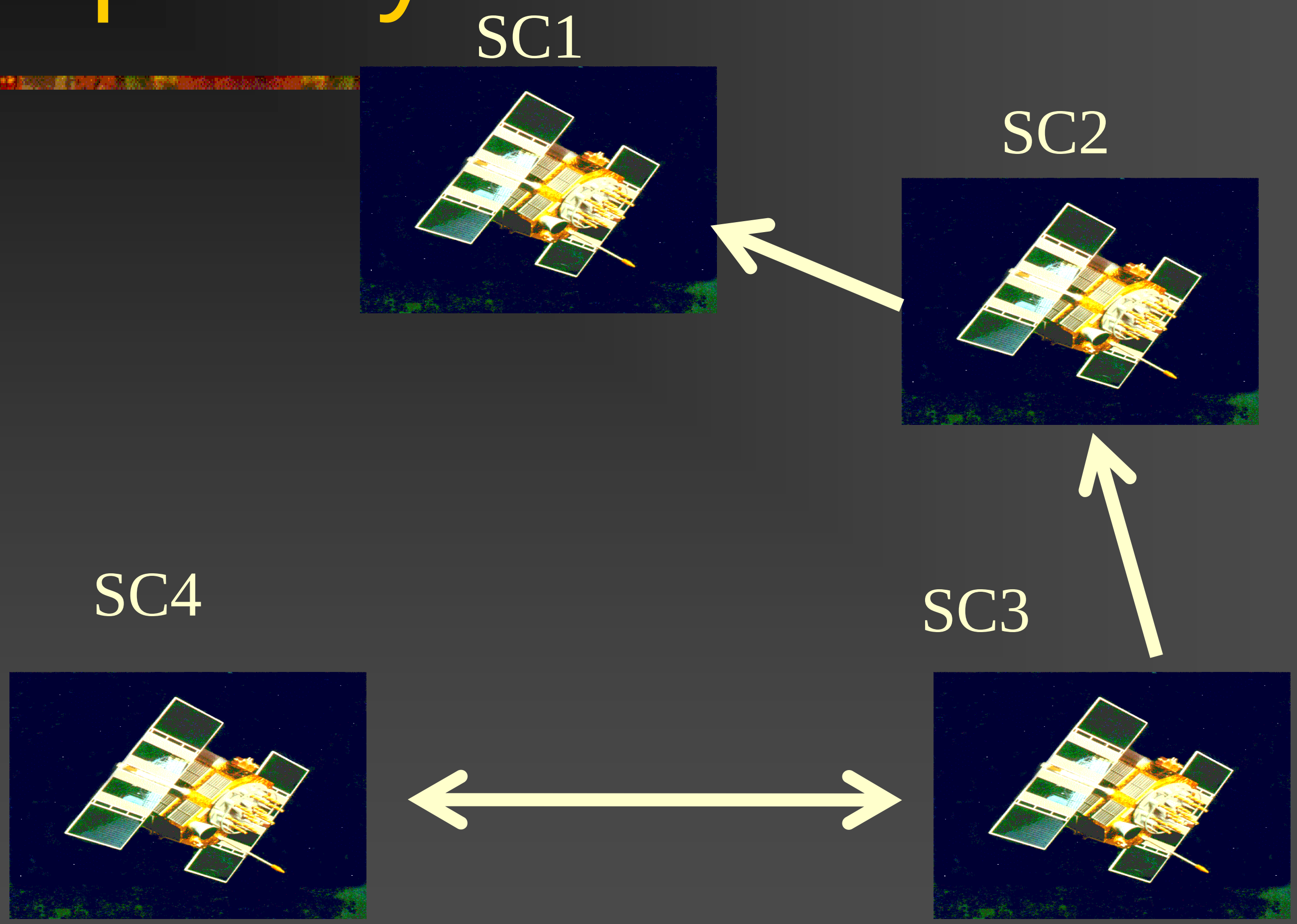
4 S/C

$$\Rightarrow \dot{\xi}_k = (A + BGC)\xi_k - BGC\xi_{k-1}$$

$$\text{Let } \xi \equiv \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} \Rightarrow \dot{\xi} = \underbrace{\begin{bmatrix} A+BGC & 0 & 0 \\ -BGC & A+BGC & 0 \\ 0 & -BGC & A+BGC \end{bmatrix}}_{\overline{A}_C} \xi$$

$\overline{A}_C$ stable $\Leftrightarrow$ $A + BGC$ stable

# Open Chain Formation with Reciprocity



# Stability Open Chain with Reciprocity

$$\text{Output Feedback}: \begin{cases} u_1 = 0 \\ u_2 = GC\xi_1 + G_D z_2^D \\ u_3 = GC\xi_2 + G_D z_3^D + (-GC\xi_3) \\ u_4 = GC\xi_3 + G_D z_4^D \end{cases}$$

Reciprocity

$$\text{Let } \xi \equiv \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} \Rightarrow \dot{\xi} = \underbrace{\begin{bmatrix} A+BGC & 0 & 0 \\ -BGC & A+BGC & -BGC \\ 0 & -BGC & A+2BGC \end{bmatrix}}_{\overline{A}_C} \xi$$

$\overline{A}_C$ stable $\Leftrightarrow A + BGC$ and ??

Can it be Unstable?

# Theorem: Open Chain with Reciprocity

$$\bar{A}_C = \underbrace{\begin{bmatrix} A+BGC & 0 & 0 \\ -BGC & A+BGC & -BGC \\ 0 & -BGC & A+2BGC \end{bmatrix}}_{\bar{A}_C} \text{ stable}$$

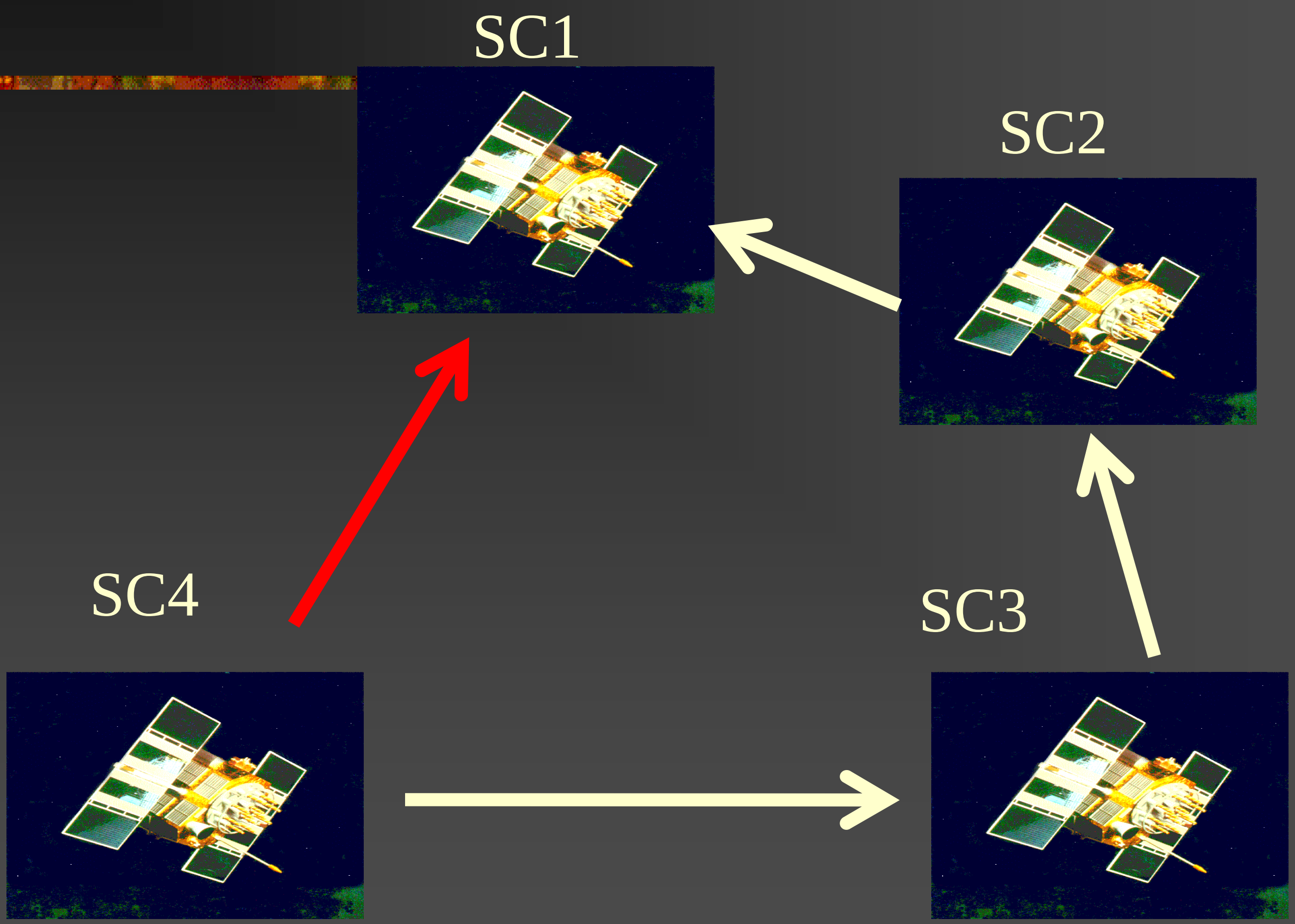
$$\Leftrightarrow A+BGC \text{ and } A+(2+\alpha)BGC, A+(1-\alpha)BGC \text{ stable}$$

$$\text{where } \alpha = -\frac{1}{2} \pm \frac{1}{2}\sqrt{5} = 0.62,\text{-}1.62 \;\; (\text{or } \alpha^2+\alpha-1=0)$$

$$\text{Proof}: \begin{bmatrix} I & -\alpha I \\ 0 & I \end{bmatrix} \begin{bmatrix} A+2BGC & -BGC \\ -BGC & A+BGC \end{bmatrix} \begin{bmatrix} I & \alpha I \\ 0 & I \end{bmatrix} = \begin{bmatrix} A+(2+\alpha)BGC & (\alpha^2+\alpha-1)BGC \\ -BGC & A+(1-\alpha)BGC \end{bmatrix} \#$$

$$Cor.\;\; A \equiv \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C \equiv \begin{bmatrix} 1 & 0.1 \end{bmatrix} \Rightarrow \bar{A}_C \text{ stable } \forall G = g > 0$$

# Open Chain Formation



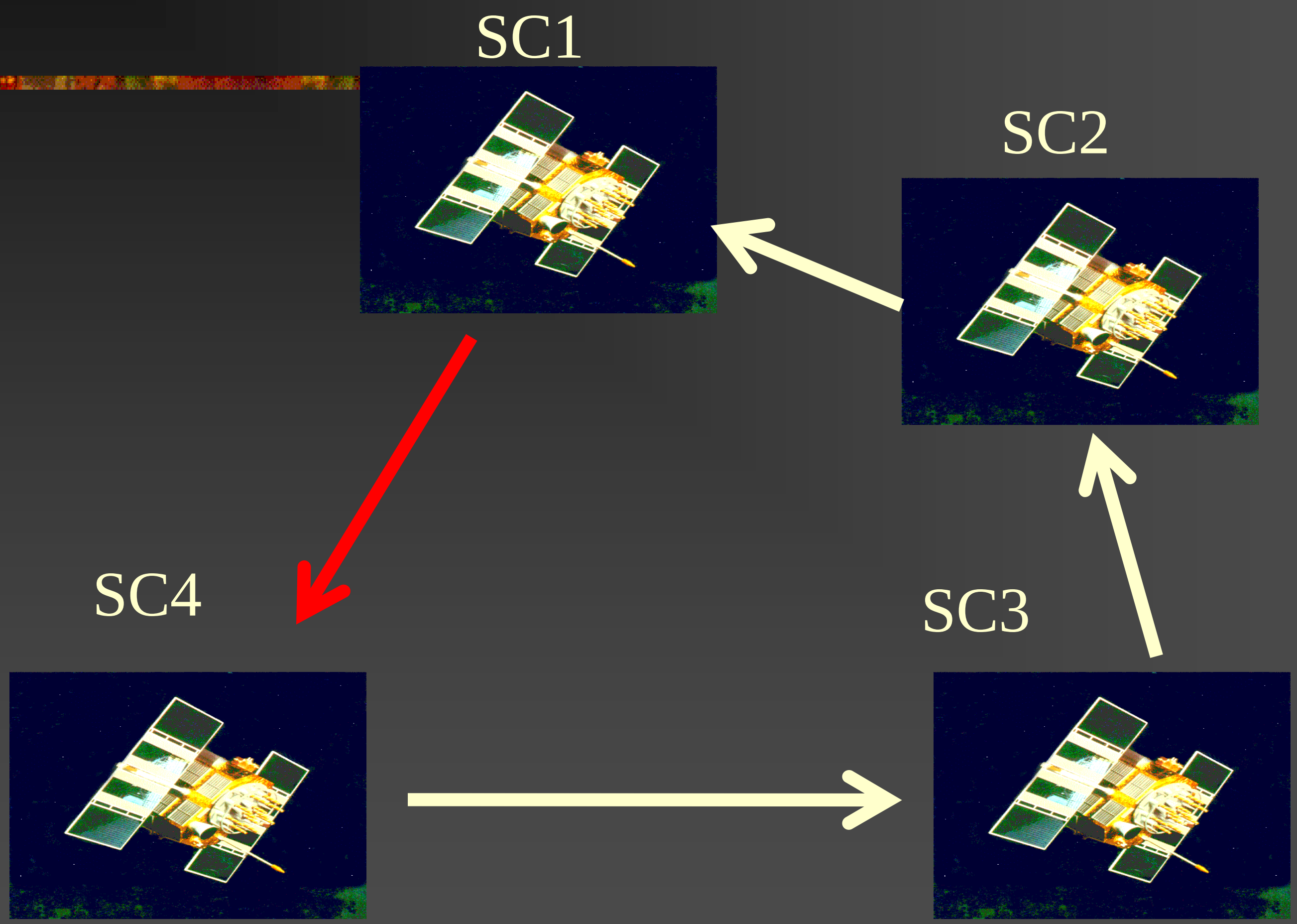
# Theorem:

$$\bar{A}_C = \underbrace{\begin{bmatrix} A+BGC & 0 & 0 \\ -BGC & A+BGC & 0 \\ BGC & 0 & A+2BGC \end{bmatrix}}_{\bar{A}_C} \text{ stable}$$

$$\Leftrightarrow A+BGC \text{ and } A+2BGC \text{ stable}$$

$$Cor.\ \ A \equiv \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C \equiv \begin{bmatrix} 1 & 0.1 \end{bmatrix} \Rightarrow \bar{A}_C \text{ stable } \forall G = -g < 0$$

# Closed Chain Formation



$$\overline{A}_C(\varepsilon)=\begin{bmatrix} A+(1+\varepsilon)BGC & \varepsilon BGC & \varepsilon BGC \\ -BGC & A+BGC & 0 \\ 0 & -BGC & A+BGC \end{bmatrix}$$

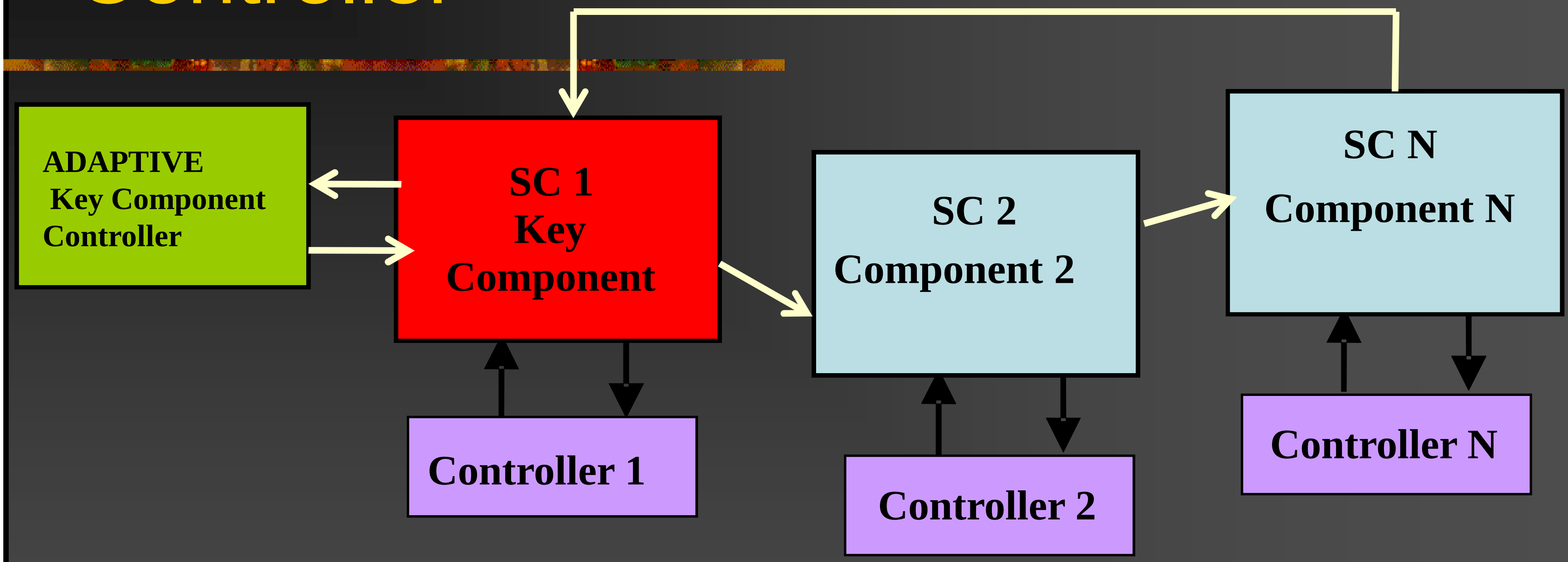
$$=\underbrace{\begin{bmatrix} A+BGC & 0 & 0 \\ -BGC & A+BGC & 0 \\ 0 & -BGC & A+BGC \end{bmatrix}}_{\overline{A}_0 \text{ stable} \Leftrightarrow A+BGC \text{ and } A+2BGC \text{ stable}}+\varepsilon\underbrace{\begin{bmatrix} BGC & BGC & BGC \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\Delta A}$$

$$\Rightarrow \exists \varepsilon_0 > 0 \ni, \forall 0 \le \varepsilon < \varepsilon_0,\ \overline{A}_C(\varepsilon) \text{ is stable.}$$

$$Cor.\ A \equiv \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C \equiv \begin{bmatrix} 1 & 0.1 \end{bmatrix}$$

$$\Rightarrow \overline{A}_C(\varepsilon) \text{ stable} \forall 0 \le \varepsilon < \varepsilon_0 = 0.02 (\text{theory} \sim 10^{-13}), \text{ and unstable for } \varepsilon = 1$$

# ADAPTIVE Key Component Controller




Was Unstable, but
Adaptive Key Component Controller Restores Stability

# Example

$$\text{Double Integrator}: A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, B=\begin{bmatrix}0\\ 1\end{bmatrix}, C=\begin{bmatrix}1 & 0.1\end{bmatrix}$$

$$C(s)\equiv -(10+\frac{1}{s})$$

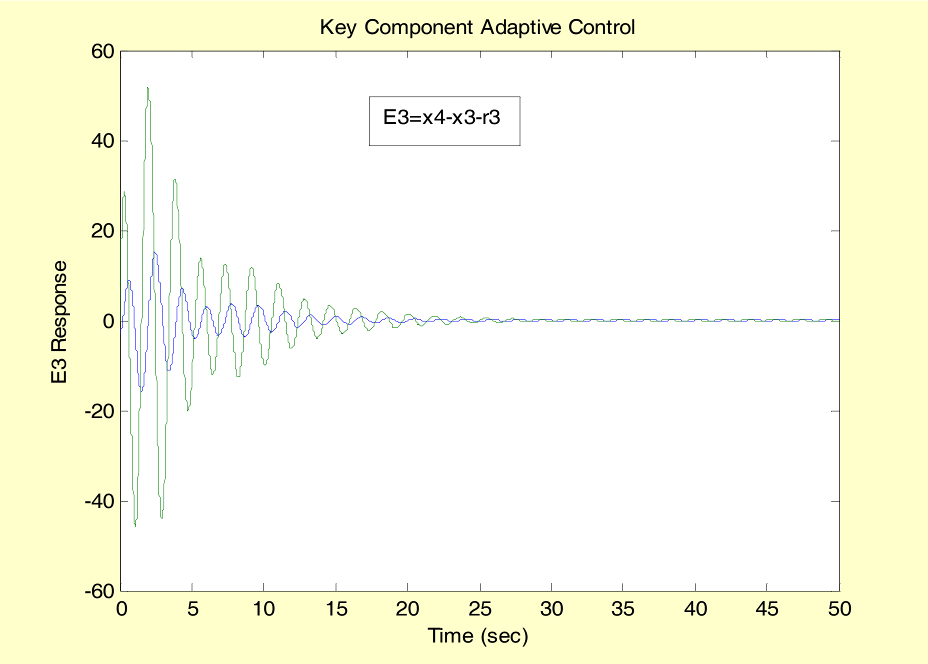
$$r_1\equiv\begin{bmatrix}3\\ 0\end{bmatrix}; r_2\equiv\begin{bmatrix}5\\ 0\end{bmatrix}; r_3\equiv\begin{bmatrix}8\\ 0\end{bmatrix}; r_4\equiv\begin{bmatrix}16\\ 0\end{bmatrix}$$

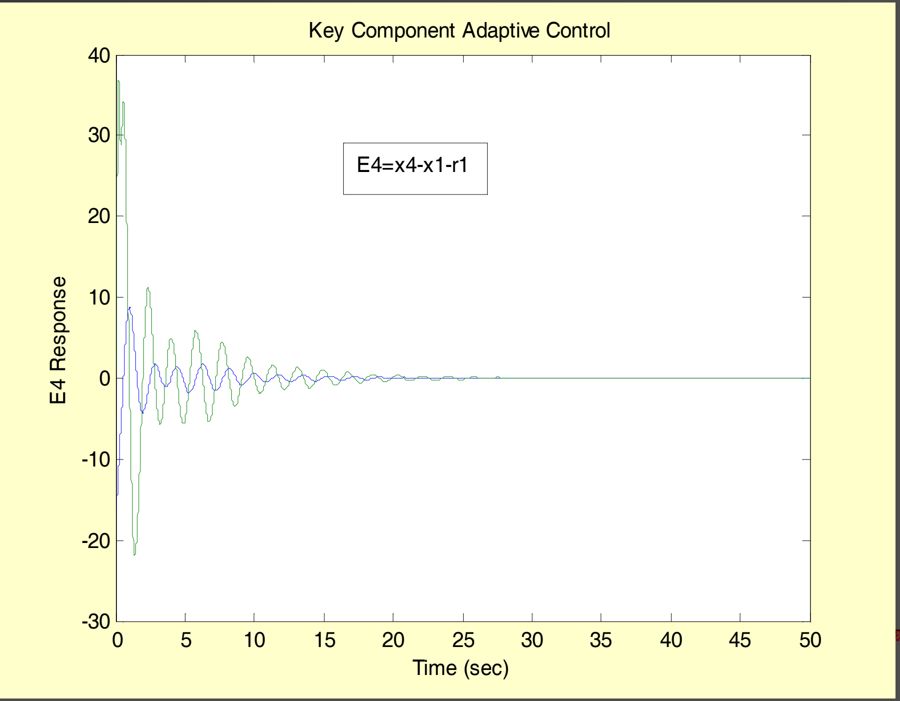
Key Component Adaptive Controller

$$u_4 = \underbrace{GC\xi_3}_{\substack{Original\\ SC4\ Control}} + \underbrace{G_e\tilde{y}_4 + G_D\phi_D}_{\substack{Adaptive\\ Control}}; \phi_D \equiv 1$$

$$\text{Adaptive Gains}\begin{cases}\dot{G}_e = -(\tilde{y}_4)^2\gamma_e; \gamma_e \equiv 100\\ \dot{G}_D = -\tilde{y}_4\phi_D\gamma_e; \gamma_e \equiv 1\end{cases}$$

$$\text{Measured Output}: \tilde{y}_4 = Cx_4$$



# Conjecto-Theorem

If individual spacecraft dynamics (A, B, C) are ASPR,

ie CB > 0 and P(s) = $C(sI - A)^{-1}B$ is minimum phase, then

Key Component Adaptive Controller (on the joining spacecraft)

$$u_N = \underbrace{GC\xi_{N-1}}_{\substack{\text{Original}\\ \text{SC N Control}}} + \underbrace{G_e\tilde{y}_N + G_D\phi_D}_{\substack{\text{Adaptive}\\ \text{Control}}};\phi_D \text{ bounded}$$

$$\text{Adaptive Gains}\begin{cases}\dot{G}_e = -(\tilde{y}_N)^2\gamma_e;\gamma_e \equiv 100\\ \dot{G}_D = -\tilde{y}_N\phi_D\gamma_e;\gamma_e \equiv 1\end{cases}$$

Measured Output : $\tilde{y}_N = Cx_N$

produces $\xi_k = x_{k+1} - x_k - r_k \xrightarrow[t\to\infty]{} 0\,\forall k$

(and rejects bounded disturbances)

with bounded adaptive gains

# Proof:

WLOG use 4 SC

Let $\xi \equiv \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} \Rightarrow \begin{cases} \dot{\xi} = \bar{A}_C(\varepsilon)\xi + \tilde{B}u_4 \\ \tilde{y}_4 = C_4 x_4 = \tilde{C}\xi \end{cases}$

$$\text{where } \bar{A}_C(\varepsilon) = \begin{bmatrix} A+(1+\varepsilon)BGC & \varepsilon BGC & \varepsilon BGC \\ -BGC & A+BGC & 0 \\ 0 & -BGC & A+BGC \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{bmatrix}}_{\bar{A}} + \underbrace{\begin{bmatrix} B & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & B \end{bmatrix}}_{\bar{B}} \underbrace{\begin{bmatrix} (1+\varepsilon)G & \varepsilon G & \varepsilon G \\ -G & G & 0 \\ 0 & -G & G \end{bmatrix}}_{\bar{G}(\varepsilon)} \underbrace{\begin{bmatrix} C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & C \end{bmatrix}}_{\bar{C}}$$

Find $G_* \ni u_4 = G_* \tilde{y}_4$

$$\Rightarrow \tilde{A}_C(\varepsilon) = \bar{A}_C(\varepsilon) + \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix} G_* \begin{bmatrix} 0 & 0 & C \end{bmatrix}$$

$$= (\underbrace{\begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{bmatrix}}_{\bar{A}} + \underbrace{\begin{bmatrix} B & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & B \end{bmatrix}}_{\bar{B}} \underbrace{\begin{bmatrix} (1+\varepsilon)G & \varepsilon G & \varepsilon G \\ -G & G & 0 \\ 0 & -G & G \end{bmatrix}}_{\bar{G}(\varepsilon)} \underbrace{\begin{bmatrix} C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & C \end{bmatrix}}_{\bar{C}})$$

$$+ \underbrace{\begin{bmatrix} B & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & B \end{bmatrix}}_{\bar{B}} \underbrace{\begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} G_* \begin{bmatrix} 0 & 0 & I \end{bmatrix}}_{\bar{G}_*} \underbrace{\begin{bmatrix} C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & C \end{bmatrix}}_{\bar{C}}$$

$$= \bar{A} + \bar{B}(\underbrace{\bar{G} + \bar{G}_*}_{\tilde{G}_*})\bar{C}$$

Clearly $\bar{C}\bar{B} > 0 \Leftrightarrow CB > 0$

and $\bar{P}(s)$ minimum phase $\Leftrightarrow P(s)$ minimum phase.

$\therefore (A,B,C) ASPR \Rightarrow (\bar{A}, \bar{B}, \bar{C}) ASPR \Rightarrow (\bar{A}_C(\varepsilon), \tilde{B}, \tilde{C}) ASPR$#

# Future Formation Stuff

- Nonlinear:

$$\begin{cases} \dot{x}_k = Ax_k + \underbrace{g(x_k)}_{\text{Lipschitz \& weak enough}} + Bu_k \\ y_k = Cx_k \end{cases}$$

- Harder but Doable

$$\begin{cases} \dot{x}_k = A(x_k) + B(x_k)u_k \\ y_k = C(x_k) \end{cases}$$

- Maintaining a Formation Shape (Tracking); Immutability of Formation Shapes

- Swarms: George Hill's Eqs
  (or Johnny-Come-Lately: Clohessy-Wiltshire)

# A Sortof Paradigm

Wonderful New Survivable System

New
Exciting
Adaptive
Controller

Old Decrepit
Broken-down
System
(eg US
power grids)